A Course of Analysis. By E. G. Phillips. Cambridge University Press, 1930. vii+361 pp.

This book is based on the first part of a course of lectures on analysis given by the author to mathematical honors students in the University College of North Wales. The presentation follows the traditional English manner, clear, rich in content, and compact. The topics considered cover a wide range and their choice and arrangement are well calculated to arouse and hold the interest of the student. The work is intended for students of moderate mathematical maturity who have been prepared by a standard course in calculus.

The first chapter is devoted to an outline of the logical foundations of analysis and the structure of the number system. Chapters on sequences and limits and the theory of continuous functions are followed by an exposition of the elements of the differential calculus. Here the distinction between derivable and differentiable is introduced. This finds an important application later in the chapter on functions of several variables. A brief introductory chapter on series designed to provide a basis for later developments is followed by an interesting chapter on the inequalities of Hölder, Minkowski, Jensen and others. Inequalities are treated throughout the work thoroughly and elegantly, a feature of special value.

The theory of integration is limited to the Riemann integral. This limitation can hardly be avoided in an introductory volume. The distinction between a primitive and an indefinite Riemann integral is carefully illustrated. Infinite integrals are briefly treated.

After extending results previously obtained to functions of several variables the author presents in the tenth chapter a readable discussion of implicit functions with an application to the theory of the exponential function and an adequate treatment of transformations and jacobians. Chapters on multiple integration with particular reference to classical formulas of mathematical physics follow. The work concludes with a short chapter on power series and functions defined by power series, with an application to the theory of the circular functions.

Lists of exercises at the chapter ends contribute materially to the value of the work as a text. As many of the exercises are beyond the power of average students, it would be beneficial to have additional exercises for practice.

In a work of this scope and brevity it is difficult to secure perfect balance. Some subjects such as infinite integrals are too briefly treated, others, for example the integration of non-uniformly convergent sequences, receive more attention than is desirable in an introductory course. While misprints are rare, the author has frequently failed in his effort to present the student with an accurate exposition. Gehman (American Mathematical Monthly, vol. 38, pp. 166–167) has pointed out a number of slips and commented on the frequent errors in the statement of theorems, particularly mean-value theorems. He calls attention to a fallacious proof on page 32 of the theorem of Weierstrass. Another notable error occurs in the proof of Theorem 3, page 344; it invalidates the demonstration of Abel's theorem on power series. In theorems involving transformations of multiple integrals, Phillips often fails to assume that the jacobian does not change sign throughout the region of integration.

In addition to the lapses in accuracy mentioned, it seems proper to call attention to the fact that this book shares with other introductory works on analysis the fault of creating an illusion of simplicity, facility, and even completeness. The addition of a thoroughgoing treatment of some fundamental topic as an example of the nature of a rigorous discussion would improve the work.

It is usual for authors of works of this sort to introduce the elementary transcendental functions for purposes of illustration in advance of their theoretical treatment. While this method undoubtedly adds to the interest of the discussion it seems likely that the student will eventually fail to distinguish clearly between the material which is logically presented and that which is illustrative. This can be avoided by an obvious rearrangement of subject-matter. This rearrangement is much to be preferred if the reader's interest can be sustained.

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Wahrscheinlichkeitsrechnung und ihre Anwendung in der Statistik und Theoretischen Physik. By Richard von Mises. Leipzig and Wien, Franz Deuticke, 1931. 574 pp.

Aside from its usefulness in technical applications, the theory of probability has recently taken on a significance secondary to that of very few mathematical issues. Philosophers have come to study it with a view to investigate its logical implications, and physicists have recently been accustomed to regarding it as the foundation of their science. Their acceptance of probability doctrines has frequently been uncritical, and it is safe to say that many difficulties in connection with the present status of quantum dynamics are consequences of an incomplete understanding of the basic probability postulates. In the midst of this situation the appearance of a book like that of von Mises is indeed a fortunate event.

It is well known that the author's views concerning the axioms of the probability theory are specific and exclusive, that they are disputed in several quarters. Nevertheless, in the opinion of the reviewer their value in applications, such as those made in the exact sciences, is unique, and it is gratifying to observe that, in this book, von Mises' frequency theory has been carried through consistently in a manner that is detailed and complete.

The book is the first volume of a series of treatises on applied mathematics. Continuations, dealing with hydromechanics, practical analysis, and applied geometry, are announced in the author's preface. The present volume contains four principal sections; the first two are concerned with the theoretical elements of the theory, the third is an application of probability to statistics and the theory of errors, the last discusses in a lucid fashion the foundation of physical statistics. Numerous problems of historical and practical interest are treated in the text, and to each section is appended a set of problems to be solved by the reader. The book is intended as a textbook. In conformity with this aim, the author has succeeded in making his exposition clear and concise. The treatment is more extended than is customary in American texts, but the greater length makes for easy reading. Indeed its reading is distinctly less difficult than that of most similar treatises, in spite of the complete logical rigor of the