The latter were introduced and studied by Juel and chiefly by Segre in 1889.

In his lectures Cartan presents the fundamental notions of these complex geometries and gives interpretations which connect them with Riemannian geometry. The work is divided into two parts. The first is given to the projective geometry of the complex line and its relation to hyperbolic (Lobatchewsky) geometry. The second is concerned with complex geometry in three dimensions. The last chapter deals with harmonic polynomials of complex projective space and their application to the representation of this space or of elliptic Hermitian space, by real algebraic varieties without singularities imbedded in a euclidean space of a convenient number of dimensions.

These few indications on the contents, which are altogether too extensive to give in detail, should convey an idea of the nature of Cartan's *Leçons*. Their reading does not require more than ordinary mathematical preparation, a knowledge of the cross ratio and a notion of Riemannian space.

To anyone who is interested in this field with a view to research, Cartan's book may be highly recommended as an indispensable background for further progress.

ARNOLD EMCH

NEW MATHEMATICAL TABLES

Mathematical Tables. Vol. 1. Prepared by the Committee for the Calculation of Mathematical Tables. London, British Association for the Advancement of Science, 1931. xxxv+72 pp.

Standard Four-Figure Mathematical Tables. By L. M. Milne-Thompson and L. J. Comrie. London, Macmillan and Company, 1931. xvi+245 pp.

The first activity of the British Association for the Advancement of Science in the preparation of mathematical tables appeared in a report of a committee published in 1873. "The purposes for which the Committee was appointed are twofold, viz. (1) to form as complete a catalogue as possible of existing mathematical tables, and (2) to reprint or calculate tables which are necessary for the progress of the mathematical sciences." This Committee, with changes in personnel, has been active for most of the time during the last sixty years. In the lists of members are to be found the names of Cayley, Stokes, Sir W. Thomson, Glaisher, Lord Rayleigh, Greenhill, Sylvester, Pearson, and many other well known British mathematicians. The only American to take part in the work as a committee member appears to have been A. G. Webster.

It was at first intended that tables should be published independent of the annual Report of the Association, and some tables have been so published, but most of the results have been included in the annual reports. The Association is now carrying out the original plan of separate publication, the first step of which is explained in the following quotation from the preface. "For several years the question of collecting into book form the tables from its reports has been before the Committee, but it was apparent from the first that the simple plan of reprinting existing material would produce a heterogeneous volume neither useful nor creditable. There were gaps in the ranges of the arguments of some of the functions, natural when the tabulation had been performed at different times for special purposes, but intolerable if tables were to be issued for general use. In the case of the Bessel functions, the functions tabulated did not form in any sense a complete collection. Lastly, the original tables offered no facilities for interpolation. Two years ago the Committee decided that these difficulties must not impede publication indefinitely, and that, if the Bessel functions were reserved for an independent volume, definite progress could be made. Even with this limitation the labor of extending tables, computing differences, and checking has been heavy. When this work was completed, the tables were printed and proofs were read against the manuscript copy. The tables were then stereotyped and the proofs from the plates were checked rigorously by differencing or by mechanical integrating."

The preparation of this book, containing sixteen tables, was begun by R. A. Fisher and completed by J. Henderson. The introduction contains references to other similar tables. We give below a list of the tables in this volume, with a brief description of each.

I. *Multiples of* $\frac{1}{2}\pi$. This table gives the values to 15 decimal places of $n\pi/2$ for integral values of n from 1 to 100. Table I makes it possible to bring any angle within the range of Table III.

II. *Circular Functions.* This table gives the values to 15 decimal places of sines and cosines of angles measured in radians at intervals of 0.1 from 0.0 to 50.0. Table II, sections of which have appeared in the Reports from 1916 to 1928, was originally constructed because of its applications in the asymptotic expansion of transcendental functions.

III. *Circular Functions.* This table gives the values to 11 decimal places of sines and cosines of angles measured in radians at intervals of 0.001 from 0.000 to 1.600.

IV. *Hyperbolic Functions.* This table gives the values to 15 decimal places of $\sinh \pi x$ and $\cosh \pi x$ at intervals of 0.0001 from $x=0.0000$ to $x=0.0100$. Table IV is intended for use as an auxiliary table to Table V. It was calculated by L. J. Comrie, and is here published for the first time.

V. *Hyperbolic Functions.* This table gives the values to 15 decimal places of $\sinh \pi x$ and $\cosh \pi x$ at intervals of 0.01 from $x=0.00$ to $x=4.00$.

VI. *Hyperbolic Functions.* This table gives the values to 15 decimal places of $\sinh x$ and $\cosh x$ at intervals of 0.1 from 0.0 to 10.0.

VII. *Exponential Integral.* The exponential integral, which is of importance in connection with formulas for the number of primes less than x, is defined as

$$\mathrm{Ei}(x) = \int_{\infty}^{-x} \frac{e^{-u}}{u}\,du.$$

The first part of the table gives the values to 11 decimal places of $\mathrm{Ei}(x)-\log_e x$ and $\mathrm{Ei}(-x)-\log_e x$ at intervals of 0.1 from $x=0.0$ to $x=5.0$. The second part of the table gives the values to ten or eleven significant figures of $\mathrm{Ei}(x)$ and $-\mathrm{Ei}(-x)$ from $x=5.0$ to $x=15.0$.

VIII. *Sine and Cosine Integrals.* The sine integral $\mathrm{Si}(x)$ and the cosine integral $\mathrm{Ci}(x)$ are here defined as follows:

$$\mathrm{Si}(x) = \int_0^x \frac{\sin u}{u}\,du, \qquad \mathrm{Ci}(x) = -\int_x^{\infty} \frac{\cos u}{u}\,du.$$

These functions have applications in connection with the tabulation of derivatives of the Bessel functions and with various problems, especially in radio engineering. The first part of the table gives the values to 11 decimal places of $\mathrm{Si}(x)$ and of $\mathrm{Ci}(x)-\log_e x$ at intervals of 0.1 from $x=0.0$ to $x=5.0$. The second part gives the values to 10 decimal places of $\mathrm{Si}(x)$ and $\mathrm{Ci}(x)$ at intervals of 0.1 from $x=5.0$ to $x=20.0$, and the third part gives the values at intervals of 0.2 from $x=20.0$ to $x=40.0$.

IX. *Factorial Function.* This name is used in place of the more common Gamma Function, the notation being

$$x! = \Gamma(1+x) = \Pi(x).$$

The introduction explains some of the many useful formulas for the calculation of this function and the table gives the values to 12 decimal places of $x!$ at intervals of 0.01 from $x=0.00$ to $x=1.00$.

X. *Integral of Logarithmic Factorial Function.* The function here tabulated is

$$1 + \int_0^x \log_{10} t! dt,$$

and the values to 12 decimal places are given at intervals of 0.01 from $x=0.00$ to $x=1.00$.

XI. *Digamma Function.* The function $\phi(x)$ is defined by

$$\phi(x) = \sum_{r=1}^{\infty} \frac{x}{r(r+x)} \cdot$$

It is related to the logarithmic derivative of the factorial function by the formula

$$\frac{d}{dx} \log_e x! = \phi(x) - \gamma,$$

where γ is Euler's constant. Values of the logarithmic derivative are tabulated to 12 decimal places at intervals of 0.01 from $x=0.00$ to $x=1.00$ and at intervals of 0.1 from $x=10.0$ to $x=60.0$. Formulas are given in the introduction by which values of the function can be found for values of x between 1 and 10 and also for values of x beyond those given in the table.

XII. *Trigamma Function.* This is the derivative of the digamma function, that is, the trigamma function is

$$\frac{d^2}{dx^2} \log_e x! = \sum_{r=1}^{\infty} \frac{1}{(r+x)^2} \cdot$$

This is a new table, calculated by A. Lodge. The intervals and ranges are the same as in Table XI.

XIII. *Tetragamma Function.* The name signifies the third logarithmic derivative of the factorial function, that is,

$$\frac{d^3}{dx^3} \log_e x! = -2 \sum_{r=1}^{\infty} \frac{1}{(r+x)^3} \cdot$$

The table is new and was calculated by A. Lodge. The intervals and ranges are the same as in Table XI.

XIV. *Pentagamma Function*. This is the last of the series of logarithmic derivatives of the factorial function to be given in this volume. The function is

$$\frac{d^4}{dx^4}\log_e x! = 6\sum_{r=1}^{\infty}\frac{1}{(r+x)^4}\,.$$

The table was calculated by A. Lodge and J. Wishart, and, as far as is known, is the only table of the function in existence. It extends over the same region as the three preceding tables, except that the first part (from $x=0.00$ to $x=1.00$) is given to only 10 decimal places.

XV. *Hh* Functions.

XVI. $\{Hh_0(x)Hh_2(x)\}/\{Hh_1(x)\}^2$.

The functions in the last two tables in the first volume find their applications in connection with the frequency functions of statistics. The *Hh* functions are defined by the relations

$$Hh_0(x) = \int_x^{\infty} e^{-(1/2)x^2}dx, \qquad Hh_n(x) = \int_x^{\infty} Hh_{n-1}(x)dx,$$

where n is a positive integer. Each function is thus the negative differential coefficient of the succeeding one, a property which may be used for purposes of interpolation. The method of interpolation is explained in a general introduction by J. O. Irwin.

For negative subscripts we have (Hermite functions)

$$Hh_{-1}(x) = -\frac{d}{dx}Hh_0(x) = e^{-(1/2)x},$$

$$Hh_{-n}(x) = -\frac{d^n}{dx^n}Hh_0(x) = -\frac{d^{n-1}}{dx^{n-1}}e^{-(1/2)x}\,.$$

The function in Table XVI is provided for use in solving the problem of fitting a truncated normal curve. An account of the properties and applications is given by R. A. Fisher.

The functions tabulated have been calculated by J. R. Airey. In Table XV the values of $Hh_n(x)$ are given to 10 decimal places for integral values of n from 0 to 21 and for different ranges of the argument. For negative values of n from -1 to -7, the number of decimal places varies from 0 to 10.

In the printing of these tables the mechanical work is excellent. They are in clear type on good quality of paper. The size of page (8 in. by 11 in.) allows an arrangement which is nowhere crowded and yet is not too large for convenient use.

The British Association is rendering most valuable service by its support of the laborious and expensive project of constructing mathematical tables, which are of such great importance in so many fields of application, and the individuals responsible for the work are to be congratulated on the results of their labors as shown in the first volume.

The Secretary to the British Association Mathematical Tables Committee is Dr. L. J. Comrie, one of the authors of *Standard Four-Figure Mathematical Tables*. In addition to those usually included in such a collection, this book contains many new tables. Among these are an extended table of natural logarithms and of the trigonometric functions for angles in radians, and a table of inverse trigonometric and hyperbolic functions. A complete list follows. For those unfamiliar with the terminology it may be explained that Erf(x) (Table XVII) is a form of the probability integral.

I. Logarithms. II. Antilogarithms. III. Addition Logarithms. IV. Subtraction Logarithms. V. Roots, Powers and Reciprocals. VI. Some Functions of the Integers $1-100$ (n^2, $\sqrt{n}$, $1/\sqrt{n}$, $1/n$, $1/n^2$, n^3, $\log n!$, $0.6745/\sqrt{n}$, $0.6745/\sqrt{n(n-1)}$, $2\pi/n$, $1/n$). VII. Natural and Logarithmic Trigonometrical Funcions (Degrees and Decimals). VIII. Natural and Logarithmic Trigonometrical Functions (Degrees and Minutes). IX. Natural and Logarithmic Trigonometrical Functions (Radians). X. Natural and Logarithmic Hyperbolic Functions. Powers of e. XI. Multiples of the Modulus M. XII. Inverse Trigonometrical and Hyperbolic Functions. Natural Logarithms. XIII. The Gudermannian in Radians. XIV. The Inverse Gudermannian. XV. Multiples of $1/M$. XVI. The Gamma Function. XVII. Erf(x). XVIII. Coefficients of the Second Difference. XIX. Conversion of Radians to Degrees and Degrees to Radians.

The last thirty pages of the book are devoted to a collection of the formulas of elementary mathematics, including the calculus and differential equations.

A unique feature of this work is that it is published in two editions. In Edition A the logarithms of numbers less than unity have been increased by 10. In Edition B the logarithms of numbers less than unity are printed with negative characteristics.

The size of the page is only slightly smaller than that of the British Association Tables, and the mechanical features show the same high quality. A convenient adjunct is a detached sheet of cardboard containing proportional parts of integers from 11 to 175.

W. R. LONGLEY