ERNEST JULIUS WILCZYNSKI—IN MEMORIAM

1. *Introduction.* Ernest Julius Wilczynski was born in Hamburg, Germany, on November 13, 1876, the son of Max and Friederike (Hurwitz) Wilczynski. He died in Denver, Colorado, on September 14, 1932, having not quite completed his fifty-sixth year. One hardly knows whether to admire most the brilliance of his scientific career, which was prematurely terminated in 1923 by illness, or the fortitude with which he endured physical infirmity for a decade. In his memory and for our own inspiration we here sketch the salient aspects of his life and character, and recall synoptically his many significant contributions to science.

2. *Life.* Wilczynski's early education partly explains what he eventually became. After he had gone to school two years in Hamburg, his family migrated to Chicago, where his father became a naturalized citizen of the United States. The boy Wilczynski attended the elementary schools and the North Division High School of Chicago, completing his high school education in three years. Then with the assistance of an uncle, Ellis Wilczynski of Hamburg, young Wilczynski returned to Germany for the purpose of entering the University of Berlin. Here he studied mathematics, physics, and astronomy under such men as Fuchs, Hensel, Planck, Pringsheim, Schlesinger, Schwarz, and Bauschinger. He received the degree of Ph.D. from the University of Berlin in 1897, being then in his twenty-first year.

Returning to the United States, and failing to secure a position in a university immediately, Wilczynski became temporarily a computer in the Office of the Nautical Almanac at Washington, D.C. It was through the influence of A. O. Leuschner, who had known him at Berlin, that he became an instructor in mathematics at the University of California in 1898. There he was successively instructor, 1898–1902; assistant professor, 1902–06; and associate professor, 1906–07. However, he was abroad as research assistant and associate of the Carnegie Institution of Washington for two years, 1903–05. He was married to Contessa Inez Macola of Verona, Italy, on August 9, 1906. She and three daughters survive him.

Wilczynski was associate professor of mathematics at the University of Illinois, 1907–10. Then he was associate professor of mathematics at the University of Chicago, 1910–14; professor, 1914–26; and professor emeritus from 1926 until his death. His health failed gradually after 1919 but he resolutely continued at his post until early in the Summer Quarter of 1923, when in the midst of a lecture he finally realized that he could go no further and, with a simple statement to that effect, walked from his class-room never to return, leaving his students amazed by the classic self-restraint with which he accepted his tragic fate. It is characteristic of the man, however, that during the long illness which followed he never lost his interest in geometry and never gave up hope and the belief that he would some day be able to return to his academic duties.

Wilczynski received during his lifetime various scientific honors and recognitions, of which the most significant are perhaps the following. He was lecturer at the New Haven Colloquium of the American Mathematical Society in 1906, with E. H. Moore and Max Mason. He was at one time Vice-President of the American Mathematical Society; he was for two years Chairman of the Chicago Section, and served for a period as Associate Editor of the Transactions of this society. He was also at one time a member of the Council of the Mathematical Association of America. He won a prize of the Royal Belgian Academy of Sciences in 1909, and was elected a member of the National Academy of Sciences in 1919.

Wilczynski was a master of the difficult art of lucid mathematical exposition. He possessed an elegant style both in spoken and written English. German was his native language, and he was at home with French and Italian. He was such a clear and polished lecturer that he made difficulties seem easy.

Wilczynski's character was gentle, his manner mild. He was unselfish and was habitually thoughtful of others. He was interested in his students and in their success. His friendly, kindly, informal attitude toward his students did much to win their affection and loyalty. He believed that a student could do his best work when he was thoroughly at ease in the presence of the instructor.

The premature termination of Wilczynski's scientific career was a great loss to mathematics. When upon one occasion he spoke in commemoration of the late G. M. Green he used words which can now appropriately be quoted and employed in commemoration of their author:

"In this brief span of years he has won enduring fame. . . . , we mourn in him not the promise of a genius unfulfilled, but the sad untimely loss of a great leader of proven strength whose power and insight had been fully tested, and whose actual achievements can never perish. . . . In his death we have suffered a heavy loss, but his life and work will continue to be, for many of us, an everlasting source of strength and inspiration."

3. *Scientific Work.* Wilczynski began his scientific career as a mathematical astronomer, and published fifteen papers in this field during his lifetime. His interest next turned to differential equations, a subject upon which he published eight papers. He won pre-eminence, however, as a projective differential geometer. In this field he published one book and thirty-two papers. Toward the end of his career he published four papers on functions of a complex variable. Two college texts and fifteen miscellaneous papers bring the total number of his mathematical publications to seventy-seven. This is exclusive of forty-six abstracts of papers presented to the American Mathematical Society, which were published in this Bulletin and which of themselves form an impressive outline of a large part of his work.

A bibliography of Wilczynski's publications follows, references to which will be made by year and number, as for example (1895, 1). It may be observed that, besides private publications, his work appeared in nineteen periodicals and in four languages. We shall discuss the most outstanding of these publications briefly and formulate an estimate of his original contributions to astronomical and mathematical knowledge.

During his student days at Berlin Wilczynski published five papers (1895, 1, 2), (1896, 1, 2), (1897, 1) before his thesis. The first two of these are connected with Schmidt's theory of the sun and are primarily expository in character. The others belong to a series of ten papers closely connected with his thesis (1897, 2) on hydrodynamical investigations of the solar rotation.

Besides the thesis, the reader can get a good idea of Wilczynski's work on astronomy by consulting a popular account (1898, 2) and a more mathematical account (1897, 3). He showed, among other things, that under certain hypotheses the angular velocity of rotation of particles in the sun is the same for all particles equally distant from the axis of rotation. Thus he gave a fairly satisfactory explanation of the interesting fact that the angular velocity of rotation of the sun decreases from the equator to the poles.

Perhaps the most interesting feature of Wilczynski's work on differential equations is that this subject served as a bridge over which his interest crossed the gap between astronomy and geometry. In fact, two of his papers (1901, 1, 2) on differential equations were later included in his book (1906, 1) on the projective differential geometry of curves and ruled surfaces, which established his reputation as a geometer, the first paper being incorporated in Chapter I and the second in Chapter IV.

It has often been stated that Wilczynski was the founder, or inventor, of projective differential geometry. This is not quite precise, for Halphen in the latter part of the nineteenth century was the first ever consciously to undertake and carry to fruition a systematic projective differential investigation. He was primarily interested in curves in the plane and in ordinary space. But Wilczynski was the first ever to appreciate, demonstrate, and exploit the utility of completely integrable systems of linear homogeneous differential equations for projective differential geometry. Thus he created a new method in geometry and established himself as the leader of a new school of geometers, which has been called *the American school* of projective differential geometers. His influence, moreover, was international and was particularly strong in Italy and Czechoslovakia.

Besides studying curves by his own method Wilczynski went on to develop projective differential theories of ruled surfaces in ordinary space, and of surfaces and congruences also in ordinary space. His method can be quite briefly described, but space permits scarcely more than a suggestion of some of his most outstanding geometrical results.

Wilczynski's method consists in associating with each configuration to be studied a certain completely integrable system of linear homogeneous differential equations. He next determines the most general transformation of dependent and independent variables that does not disturb the configuration. Then he carries out this transformation on the system of equations and calculates complete systems of invariants and covariants. He interprets these geometrically, and studies the configuration by means of them.

In such a brief comment as this on Wilczynski's geometrical results much will have to be left unsaid that might be said in a more elaborate discussion. But one can certainly say here that much of the theory of ruled surfaces as it exists today, particularly, what concerns the *osculating and asymptotic reguli* along a generator, the *flecnode curves, flecnode congruence, and flecnode transformation*, and the so-called *principal ruled surface* associated with a given ruled surface, is the work of Wilczynski.

In his prize memoir (1911, 2) he developed a theory of congruences in ordinary space and studied two special types of congruences in some detail, namely, congruences belonging to linear complexes and having Laplace transforms likewise belonging to linear complexes, and secondly congruences whose focal surfaces are quadratics. He introduced (1915, 1) the *axis congruence* and *ray congruence* associated with a conjugate net on a surface in ordinary space, and proved that a conjugate net has equal Laplace-Darboux invariants if, and only if, the ray curves, that is, the curves which correspond on the surface to the developables of the ray congruence, themselves form a conjugate net.

He established the theory of non-developable curved surfaces in a series of five memoirs (1907, 1), (1908, 1, 2), (1909, 3, 4). The theory of the so-called *directrix congruence* originated with him. By means of it and a certain *canonical cubic* and *canonical quadric* he interpreted geometrically the coordinate system which leads to a certain canonical expansion for one non-homogeneous coordinate of an ordinary point on a surface in terms of the other two coordinates. Subsequently he studied (1913, 2) a special class of surfaces now called *coincidence surfaces.*

At one time (1911, 1) he was very much interested in nets of curves in the plane, and when he later studied surfaces on which the directrix curves are indeterminate (1914, 3), he employed to advantage this work on nets of curves.

The problem of finding a geometrical significance of *isothermal conjugacy* of a net of curves was completely solved by him for the first time (1920, 1), using a new notion, namely, the notion of a *pencil of conjugate nets.* He gave a second and even more elegant characterization later (1922, 1), to the effect that a fundamental conjugate net is isothermally conjugate if, and only if, its so-called *flex-ray curves* themselves form a conjugate net.

In the closing years of his activity Wilczynski was concerned with functions of a complex variable. He studied (1919, 2, 3) certain line-geometric representations of a function of a complex variable, and investigated certain projective differential properties of such a function in the last memoirs that he ever published (1922, 3), (1923, 1).

No account of Wilczynski's scientific work should be concluded without giving some prominence to his miscellaneous publications. These range all the way from an essay (1900, 1) on poetry and mathematics, and a popular and philosophical account (1909, 2) of the fourth dimension, to book reviews, college texts (1914, 1), (1916, 1), and commemorations.

The two commemorations that he wrote are one (1902, 1) of a former teacher, Lazarus Fuchs, and another (1919, 3) of G. M. Green of Harvard University, whose already brilliant career was cut short by death at the untimely age of twenty-seven. This commemoration is a beautifully written appreciation of the life, character, and work of Green, for whom Wilczynski had the highest regard, and with whom he had corresponded, although the two men had never met each other personally.

Some of Wilczynski's miscellaneous publications are the result of his active interest in the affairs of the American Mathematical Society. In particular, one (1902, 4) of these in the Bulletin of this Society is a report of the organization and first program of the San Francisco Section, which now no longer exists as such. Another (1909, 1) is a minority committee report.

One of the most important of Wilczynski's activities consisted in directing thesis work. There seems to be no record of the number of master's theses that he directed, but there were many of them. He directed twenty-five doctoral dissertations, two at the University of Illinois, and twenty-three at the University of Chicago. All of his doctors, except one, survive him and form an influential group of the American mathematical community.

BIBLIOGRAPHY

The following bibliography includes Wilczynski's publications exclusive of abstracts of papers presented to the American Mathematical Society. The following abbreviations taken from Bulletin 63 of the National Research Council will be used.

1. Am. J.: American Journal of Mathematics.
2. Ann. di Mat.: Annali di Matematica Pura ed Applicata.
3. Giorn. di Mat.: Giornale di Matematiche.
4. Math. Ann.: Mathematische Annalen.
5. Pal. Circ. Mat.: Rendiconti del Circolo Matematico di Palermo.
6. Wash. Nat. Ac. Sc. Proc.: Proceedings of the National Academy of Sciences.

The following abbreviations will also be used:

1. An. J.: Astronomical Journal.
2. Ap. J.: Astrophysical Journal.

WILCZYNSKI'S PUBLICATIONS

1895. 1. *Schmidt's theory of the sun*, Ap. J., vol. 1, pp. 112–26.
2. *Notes on Schmidt's theory of the sun*, Ap. J., vol. 2, pp. 69–73.

1896. 1. *Hydrodynamical investigations of the solar rotation*, Ap. J., vol. 3, pp. 101–5.
2. *Outlines of a theory of spiral and planetary nebulae*, Ap. J., vol. 4, pp. 97–100.

1897. 1. *Ueber die Rotationsbewegung der Sonne*, Astronomische Nachrichten, vol. 142, pp. 133–6.
2. *Hydrodynamische Untersuchungen mit Anwendung auf die Theorie der Sonnenrotation*, Mayer and Müller, Berlin.
3. *On the causes of the sun's equatorial acceleration and the sunspot period*, An. J., vol. 18, pp. 57–9.

1898. 1. *Theory of the motion of the spots on Jupiter*, An. J., vol. 19, pp. 61–3.
2. *On the causes of the sun's equatorial acceleration and the sun-spot period*, Publications of the Astronomical Society of the Pacific, vol. 10, pp. 97–9.
3. *On the causes of the sun-spot period*, Ap. J., vol. 7, pp. 124–30.
4. *On the depth of the reversing layer*, Ap. J., vol. 7, pp. 213–4.
5. Nippoldt, *Neue allgemeine Erscheinungen in der täglichen Variation der erdmagnetischen Elemente*, Terrestrial Magnetism, vol. 3, p. 184.

1899. 1. *On an mn^2 parameter group of linear substitutions in mn variables*, Proceedings of the California Academy of Sciences, vol. 1, pp., 59–62.
2. *On systems of multiform functions belonging to a group of linear substitutions with uniform coefficients*, Am. J., vol. 21, pp. 85–106.
3. *On linearoid differential equations*, Am. J., vol. 21, pp. 354–66.
4. *Dynamics of a nebula*, An. J., vol. 20, pp. 67–9.

5. Liznar, *Ueber die Aenderung der erdmagnetischen Kraft mit der Höhe*, Terrestrial Magnetism, vol. 4, pp. 191–2.
6. *A generalization of Appell's factorial functions*, this Bulletin, vol. 5, pp. 388–94.

1900. 1. *Poetry and mathematics*, The University Chronicle (University of California), vol. 3, pp. 191–204.
2. *On continuous binary linearoid groups and the corresponding differential equations and* Λ *functions*, Am. J., vol. 22, pp. 191–225.
3. *An application of group theory to hydrodynamics*, Transactions of this Society, vol. 1, pp. 339–52.

1901. 1. *Transformation of systems of linear differential equations*, Am. J., vol. 23, pp. 29–36.
2. *Invariants of systems of linear differential equations*, Transactions of this Society, vol. 2, pp. 1–24.
3. *Geometry of a simultaneous system of two linear homogeneous differential equations of the second order*, Transactions of this Society, vol. 2, pp. 343–62.

1902. 1. *Lazarus Fuchs*, this Bulletin, vol. 9, pp. 46–9.
2. *Reciprocal systems of linear differential equations*, Transactions of this Society, vol. 3, pp. 60–70.
3. *Covariants of systems of linear differential equations and applications to the theory of ruled surfaces*, Transactions of this Society, vol. 3, pp. 423–50.
4. *The first meeting of the San Francisco Section of the American Mathematical Society*, this Bulletin, vol. 8, pp. 429–37.

1903. 1. *Forsyth's Differential Equations*, this Bulletin, vol. 10, pp. 86–93.
2. *On a certain congruence associated with a given ruled surface*, Transactions of this Society, vol. 4, pp. 185–200.

1904. 1. *A fundamental theorem in the theory of ruled surfaces*, Math. Ann., vol. 58, pp. 249–59.
2. *On self-dual scrolls*, this Bulletin, vol. 11, p. 8.
3. *Studies in the general theory of ruled surfaces*, Transactions of this Society, vol. 5, pp. 226–52.
4. *On ruled surfaces whose flecnode curve intersects every generator in two coincident points*, Transactions of this Society, vol. 5, pp. 438–46.
5. *Invariants of a system of linear partial differential equations, and the theory of congruences of rays*, Am. J., vol. 26, pp. 319–60.

1905. 1. *General theory of curves on ruled surfaces*, Transactions of this Society, vol. 6, pp. 75–82.
2. *General projective theory of space curves*, Transactions of this Society, vol. 6, pp. 99–133.
3. *The general projective theory of space curves and ruled surfaces*, Verhandlungen des dritten internationalen Mathematiker-Kongresses, Heidelberg, 1904, pp. 361–5.

1906. 1. *Projective Differential Geometry of Curves and Ruled Surfaces*, Teubner, Leipzig.

1907. 1. *Projective differential geometry of curved surfaces* (First memoir), Transactions of this Society, vol. 8, pp. 233–60.

1908. 1. *Projective differential geometry of curved surfaces* (Second memoir), Transactions of this Society, vol. 9, pp. 79–120.

2. *Projective differential geometry of curved surfaces* (Third memoir), Transactions of this Society, vol. 9, pp. 293–315.

1909. 1. *Mathematical appointments in colleges and universities*, this Bulletin, vol. 15, pp. 492–5.

2. *The fourth dimension*, American Mathematical Monthly, vol. 16, pp. 149–53.

3. *Projective differential geometry of curved surfaces* (Fourth memoir), Transactions of this Society, vol. 10, pp. 176–200.

4. *Projective differential geometry of curved surfaces* (Fifth memoir), Transactions of this Society, vol. 10, pp. 279–96.

5. Hesse, *Vorlesungen aus der analytischen Geometrie der geraden Linie, des Punktes und des Kreises in der Ebene*, this Bulletin, vol. 15, pp. 186–8.

1910. 1. *Projective differential geometry*, The New Haven Mathematical Colloquium, 1906, pp. 151–72.

2. Schlesinger, *Vorlesungen über lineare Differentialgleichungen*, this Bulletin, vol. 16, pp. 483–9.

1911. 1. *One-parameter families and nets of plane curves*, Transactions of this Society, vol. 12, pp. 473–510.

2. *Sur la théorie générale des congruences*, Mémoires publiés par la Classe des Sciences de l'Académie Royale de Belgique, (2), vol. 3.

1912. 1. *A forgotten theorem of Newton's on planetary motion and an instrumental solution of Kepler's problem*, An. J., vol. 27, pp. 155–6.

1913. 1. *Ricerche geometriche intorno al problema dei tre corpi*, Ann. di Mat., (3), vol. 21, pp. 1–21.

2. *On a certain class of self-projective surfaces*, Transactions of this Society, vol. 14, pp. 421–43.

3. *Some general aspects of modern geometry*, this Bulletin, vol. 19, pp. 331–42.

1914. 1. *Plane Trigonometry and Applications*, Allyn and Bacon.

2. Darboux, *Leçons sur les Systèmes orthogonaux et les Coordonées curvilignes*. Deuxième édition, complétée, this Bulletin, vol. 20, pp. 247–53.

3. *Über Flächen mit unbestimmten Direktrixkurven*, Math. Ann., vol. 76, pp. 129–60.

4. *On a certain completely integrable system of partial differential equations*, Am. J., vol. 36, pp. 231–60.

5. Fischer, *Koordinatensysteme*, this Bulletin, vol. 20, pp. 212–3.

1915. 1. *The general theory of congruences*, Transactions of this Society, vol. 16, pp. 311–27.

2. *Conjugate systems of space curves with equal Laplace-Darboux invariants*, Wash. Nat. Ac. Sc. Proc., vol. 1, pp. 59–61.

1916. 1. *College Algebra with Applications*, Allyn and Bacon.

2. *Some remarks on the historical development and future prospects of the differential geometry of plane curves*, this Bulletin, vol. 22, pp. 317–29.

3. *Interpretation of the simplest integral invariant of projective geometry*, Wash. Nat. Ac. Sc. Proc., vol. 2, pp. 248–52.

1917. 1. *Integral invariants in projective geometry*, Pal. Circ. Mat., vol. 42, pp. 128–37.

1918. 1. *Invariants and canonical forms*, Wash. Nat. Ac. Sc. Proc., vol. 4, pp. 300–5.

1919. 1. *On the form of the power series for an algebraic function*, American Mathematical Monthly, vol. 26, pp. 9–12.

2. *Line-geometric representation of a function of a complex variable*, Transactions of this Society, vol. 20, pp. 271–98.

3. *In memory of Gabriel Marcus Green*, this Bulletin, vol. 26, pp. 1–13.

1920. 1. *Geometrical significance of isothermal conjugacy of a net of curves*, Am. J., vol. 42, pp. 211–21.

2. *One-parameter families and nets of ruled surfaces and a new theory of congruences*, Transactions of this Society, vol. 21, pp. 157–206.

3. *A set of properties characteristic of a class of congruences connected with the theory of functions*, Transactions of this Society, vol. 21, pp. 409–45.

1922. 1. *Charakteristische Eigenschaften der isotherm-konjugierten Kurvennetze*, Math. Ann., vol. 85, pp. 208–12.

2. *Some generalizations of geodesics*, Transactions of this Society, vol. 23, pp. 223–39.

3. *Differential properties of functions of a complex variable which are invariant under linear transformations*, Part I, Journal de Mathématiques, (9), vol. 1, pp. 293–435.

1923. 1. *Differential properties of functions of a complex variable which are invariant under linear transformations*, Part II, Journal de Mathématiques, (9), vol. 2, pp. 1–51.

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