SHORTER NOTICES

Die Liesche Theorie der partiellen Differentialgleichungen erster Ordnung. Vorlesungen von Friedrich Engel, Professor in Giessen. Bearbeitet von Dr. Karl Faber. Leipzig und Berlin, Teubner, 1932. xi+367 pp.

The theory of partial differential equations of the first order has been attacked from various angles by several of the foremost mathematicians of the last century, and the comprehensive theories of Lie stand only at the end of the historical development. In the text-books on this subject the theories originating before Lie are usually represented in different chapters, one after another, and almost without connection. At the end of such a text-book there are to be sure some chapters on the theory of Lie, and the reader can more or less see that the preceding theories have to be subordinated to the points of view of Lie, but an intimate fusion of all these theories with that of Lie is not established. Thus F. Engel, first his pupil, then collaborator, and finally editor of the collected works of Lie, has fulfilled, by the publication of this volume, an old obligation.

In this book the notion of the infinitesimal transformation is the basis for all considerations. The whole theory of the partial differential equations of the first order is presented as built up on the conception of the continuous group, on that of the group of infinitesimal transformations of systems in involution, and finally on the notion of the contact transformation. In this general connection the different methods of integration due to Cauchy, Jacobi, Mayer, and Lie are systematically developed. Even such introductory notions as complete and total differential systems, or the elements of the theory of the Pfaffians, are not treated in an isolated manner, but in connection with the general points of view of Lie. The theory of invariants of the finite ("integrated") transformations of contact is based on an elegant presentation of the theory of function groups. In an analogous manner the theory of the homogeneous transformations of contact is also treated. One of the central problems of the book is the problem of Lie requiring a utilization of known infinitesimal transformation groups for the problem of integration. The theory of the Poisson-Jacobi bracket expressions especially is extensively developed.

There are not many applications given, and even the theory of the canonical equations (that is, of the equations of perturbation of the astronomers) deduced by Lie from his theory of contact transformations is not treated. At the end of the book the elegant treatment of the Galilei group, first published by Engel at the request of F. Klein in the Göttinger Nachrichten (1916), is given. The integration problem of the n-body problem is also completely discussed.

The book comprises lectures which Engel frequently gave at the University of Giessen. His co-author K. Faber attended these lectures and revised them for presentation in book form. Whether he has succeeded in all details throughout the entire book is a matter for question. The student who was forced to acquire the technique of the ϵ's in the first semesters may well be astonished if one or two years later he hears in a lecture, say on algebraic geometry, or even still more analytic subjects, an utterly pre-Weierstrassian language associated with a corresponding manner of thinking. In reality, the student can only be convinced of the necessity of the Weierstrassian exactness, which he unwillingly adopted in the first semesters, in connection with advanced subjects. Now the present book is not written in the language of Weierstrass. For example the fundamental definition of the infinitesimal transformations or the use of the symbol ∞^n could not have been more primitive even fifty years ago. The failure of an exact definition for such notions makes itself felt, for instance, in connection with the processes of elimination, always appearing in the theory of Lie. This lack of exactness is supposed to have been removed by the restriction of analyticity. Not only is such a restriction in the present book almost everywhere superfluous but it fails to compensate for the lack of exactness. Thus the very first proof (pp. 2–3) is no proof at all, notwithstanding the restriction of analyticity. Fortunately these insufficiencies of the book are not so severe that they could not usually be completed without too much trouble. It is, however, a pity that the removal of such insufficiencies is left, even nowadays, to the reader.

Of course it is clear that Engel intended to publish a book not in the spirit of Weierstrass but in the spirit of Lie. If Lie had lived for a longer time and had summarized his ideas on partial differential equations of the first order in the form of a text-book, the book would be about the same as these lectures of his collaborator. Thus Engel has certainly been guided also by historical points of view. This is perhaps also the reason that the formalism of the tensor analysis, which would have furnished some abridgment in the proofs, does not attain its full value, although it is implicitly applied. Yet only one who knows the manner of writing in the difficult original papers of Lie can appreciate how much has been done in these lectures by the uniformization and simplification of the proofs. The book also contains some interesting, hitherto unpublished, investigations of Engel.

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Vorlesungen über Fouriersche Integrale. By S. Bochner. (Mathematik und ihre Anwendungen, Band 12.) Leipzig, Akademische Verlagsgesellschaft, 1932. 8+229 pp.

This text is concerned with the theory of the Fourier transform, and transforms allied to it. It provides a readable account of those parts of the subject useful for applications to problems of mathematical physics or pure analysis.

The author has given in detail such of the results of the theory of functions required as are not included in the standard treatises. He has also preferred simple, useful forms of theorems to complicated statements which aim at the utmost generality. With the exception of one short chapter, which deals with convergence in the mean, the results relate to ordinary convergence. The application of Fourier analysis to various types of linear equations is given in some detail. Numerous other applications, for example to almost periodic functions, to Bessel functions, and to harmonic functions, are sketched.

The book should prove useful to students wishing an introduction to this branch of analysis, to which so many recent contributions have been made.

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