Constitution of Atomic Nuclei and Radioactivity. By G. Gamow. Oxford, Clarendon Press, 1931. viii+114 pp.

This book is one of the International Series of Monographs on Physics and gives an account of the knowledge of atomic nuclei at the time it was written (May 1931). Except in Chapter IV, where an interesting account of the theory of inelastic collisions, according to wave mechanics, is given, very little mathematics is used. The following chapter headings give an idea of the contents: The constituent parts and energy of nuclei. Spontaneous disintegration of nuclei. Excited states and electro-magnetic radiation of nuclei. Artificial transformation of nuclei.

F. D. MURNAGHAN

Vorlesungen ueber Funktionentheorie. Part 2. By Alfred Pringsheim. Leipzig, Teubner, 1932. xiii+498 pp.

The volume under review is the last part of the author's *Vorlesungen ueber Funktionentheorie*, and the final section of the two volume *Vorlesungen ueber Zahlen- und Funktionenlehre*,* and is devoted to a discussion of the properties of single-valued analytic functions. Coming as it does as the second section of a work on infinite series, it is quite natural that this treatise on analytic functions should be dominated by the Weierstrass angle of approach, namely, that of power series. In fact the author set himself the goal of presenting this theory exclusively from this angle, so far as possible. As a consequence the choice of material presented is dictated by the method employed. However, one cannot help but be impressed with the results obtainable, and the power of the method.

To give a brief summary of the contents, one finds in the first chapter a treatment of infinite products of functions, especially of the linear variety, the Weierstrass product theorem as applied to entire functions, and its application to the factorial and gamma functions, winding up with the extension of the Weierstrass product theorem to the case in which the zeros of the function are isolated, but may have continuous arcs as derived set.

The second chapter gives a systematic treatment of entire functions of a finite order, and derives beautiful results giving the relationship between the order of the function, the behavior of the coefficients of the power series, and the convergence properties of the reciprocals of the zeros of the function. It is gratifying to find these interrelations collected in one place, and so clearly presented. The chapter also contains a proof of the Picard theorem for entire functions of finite order, based on the result that any change in a finite number of coefficients of an entire function of finite order with a finite number of zeros makes it an entire function with an infinite number of zeros, and emphasizing the unusualness of the existence of the exceptional values mentioned in the theorem.

The third chapter treats of fractional transcendental functions, giving the Mittag-Leffler theorem for the case of simple poles, and leading up to an extensive treatment of the properties of doubly periodic functions. The last part of the chapter discusses the conditions that a function have on its circle of

* Reviewed in this Bulletin: Part I, 1: vol. 25 (1919), p. 470; part I, 2 and 3: vol. 28 (1923), pp. 63–65; part II, 1: vol. 32 (1926), pp. 551–554.

convergence a single pole of the first order and applies the results to the Heine series. The chapter concludes with the use of continued fractions of the form

$$\frac{a_0}{1+}\,\frac{a_1 z}{1+}\,\frac{a_2 z}{1+}$$

for the representation of analytic functions.

The fourth chapter is devoted to single-valued functions with arbitrary singularities, giving the Mittag-Leffler theorem, and extending it to the case in which the singular parts are known at an isolated set of points, whose derived set may be continuous arcs. Special consideration is given to functions expressed in the forms

$$\sum c_n \left(\frac{1}{x - a_n} - \frac{1}{x - a_n'} \right) \quad \text{and} \quad \sum \frac{c_n}{x - a_n} \cdot$$

The last part of the chapter is concerned with the conditions that certain points on the circle of convergence be singular points of a certain type, indicates the nature of series for which the circle of convergence is a singular line, and also gives a discussion of the Taylor expansion for one real variable.

The final chapter is concerned with complex integration. Up to this point the author has limited himself to a kind of integration, a mean value, defined by the values of a function on a circle of radius r, namely,

$$\lim_n \frac{1}{2^n} \sum_{m=1}^{2^n} f(c_n^m r)$$

where c_n is the fundamental root of $x^{2^n} = 1$. The second chapter uses an extension of this mean value, which is something of the nature of an integral over an arc of a circle. But complex integration as such has been avoided and barred. So this last chapter is concerned with this subject, and some of its uses. But even here, power series are dominant, and used in the proof of the Cauchy theorem. The author applies complex integration in three projects, the uniform approximation of single-valued analytic functions by polynomials in star shaped regions, the relation of the singularities of $\sum a_n b_n x^n$ to those of $\sum a_n x^n$ and $\sum b_n x^n$, and the discussion of the function represented in the form

$$\int_0^\infty \tau^{-1/2}(\tau + 1)^{-1/2}(\tau + x)^{-1/2} d\tau,$$

leading up to the proof of the Picard theorem for any transcendental entire function. The closing paragraph gives another proof of the Cauchy-Goursat theorem, and indicates that it may be made the basis of the theory of analytic functions.

The book is a model of clear, accurate, and careful exposition. Matters are taken up in logical order, and there is a definite appreciation of the possibilities of building up to climaxes. In his desire to be absolutely clear, the writer sometimes sins on the side of too much detail. But the result is a volume which can be read and understood by any one knowing the elements of the theory of analytic functions, and which gives a wealth of information on many important and interesting parts of the theory.

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