

The Theory of Functions. By E. C. Titchmarsh. Oxford University Press, 1932. x+454 pp.

Assuming a foundation of elementary analysis, as provided by Hardy's *Course in Pure Mathematics* (Cambridge, Fifth Edition, 1928), this volume introduces its readers to a number of branches of the theory of functions. A comprehensive first chapter on infinite processes is followed by eight chapters on the theory of functions of a single complex variable and four quite independent chapters on functions of a real variable, which are included because of the conviction of the author that their contents form a necessary part of the equipment of a student of modern analysis. The exercises include many useful theorems and form an attractive feature of the work. An extensive bibliography is an aid to further study.

The discussion throughout reveals the author's wide acquaintance with recent literature and contains many improvements and refinements of exposition derived from that source. The treatment of analytic functions is based on the Cauchy integral theorem. An analytic function is any differentiable function. The term region is left undefined, except for the remark that the interior of a square or circle is a region, and the theory of Jordan curves and boundaries of regions is left aside with the remark that the subject belongs to analysis *situs*. The chapters on analytic functions, residues, and analytic continuation are followed by a chapter on the maximum-modulus theorem containing results credited to Schwarz, Vitalli, Montel, Hadamard, Carathéodory, and Phragmen-Lindelöf. Chapter 6 on conformal representation contains an attractive discussion of simple functions and their application to the problem of the representation of a region on a circle. The next three chapters discuss the properties of functions defined by power series and Dirichlet series. They contain an exposition of the beautiful and powerful theory of over-convergence and carry the student to the frontier of the subject of integral functions.

The chapters on real functions consider the measure of linear point sets, Lebesgue integrals, derivatives, functions of bounded variation, change of variable in a definite integral, convergence in mean, and repeated integrals. The final chapter, on Fourier series, includes the Riesz-Fischer theorem together with discussions relating to the uniqueness of trigonometrical series and Fourier integrals.

In evaluating a work of this extent it is important to consider the omissions. The geometrical side of analytic function theory receives scant consideration. There is little on algebraic functions, no mention of groups of linear transformations and the classes of functions defined by differential equations. The chapters on real functions contain very little on the descriptive properties of sets of points. The topics transfinite arithmetic, classification of functions, Baire and Suslin sets, Borel measure, metric density, and the well known Weierstrassian result regarding the representation of a continuous function by a uniformly convergent sequence of polynomials are excluded. The author has failed to utilize the opportunity to apply Lebesgue integration to contour integrals and the summation of series to power series. The discussion of summation of series is very brief and does not suggest its importance to the student of analysis.

Thus the student is led deeply into some parts of the theory of functions and left ignorant of others of comparable importance. The concentration on functions of a single variable is a further disadvantage. Works of this type, valuable as they are, show by their condensations, omissions, and inevitable lack of balanced emphasis, the importance of collections of monographs each devoted to the exposition of a single field.

The author and publishers are to be congratulated on the presentation of a work so useful and attractive. The typography is unusually clear and restful to the eye. As a whole the book is timely and accurate. It is convenient to have in one compact volume demonstrations of so many fundamental propositions with indications of many more in exercises and citations. A brilliant scholarship and much teaching experience have been combined in an interesting and reliable introduction to a large part of the theory of functions.

E. W. CHITTENDEN

La Relativité: 1. *Cinématique de la Relativité*. By E. Bauer. 2. *La Dynamique Relativiste et l'Inertie de l'Énergie*. By F. Perrin. 3. *Conséquences de la Relativité dans le Développement de la Mécanique Ondulatoire*. By L. de Broglie. 4. *La Théorie Einsteinienne de la Gravitation; les Vérifications Expérimentales*. By G. Darmois. 5. *Le Parallélisme Absolu et la Théorie Unitaire du Champ*. By E. Cartan. 6. *La Relativité; Conclusion Generale*. By P. Langevin. Paris, Hermann, 1932.

This is a series of six monographs, issued separately as parts of the series Actualités Scientifiques et Industrielles, and forming a symposium on Relativity under the auspices of the Centre International de Synthèse and directed by P. Langevin. The treatment is quite elementary and in general non-mathematical. M. Bauer's part deduces in some 23 pages the Lorentz transformation and its elementary consequences. The deduction is unsatisfactory because it is assumed, p. 18, that because when $x^2 + y^2 + z^2 - c^2t^2 = 0$, then $x'^2 + y'^2 + z'^2 - c^2t'^2 = 0$, it follows that $x^2 + y^2 + z^2 - c^2t^2 = x'^2 + y'^2 + z'^2 - c^2t'^2$. M. Perrin's part (19 pages) gives the elementary dynamical consequences of the Lorentz formulas (transverse and longitudinal mass, variation of mass with velocity, etc.). In the third part (14 pages) L. de Broglie shows the connection between the Lorentz formulas and the fundamental quantum relation $E = h\nu$, and how the relativity theory resolves the conflict between the wave and corpuscular theories of light (pressure on a reflecting mirror, relation between Fermat's principle and the principle of Maupertuis). Part 4 (30 pages), by G. Darmois, discusses Schwarzschild's centro-symmetric solution of the equations of general relativity in free space and its application to the classical verifications of the theory (secular advance of perihelion of the planets, deflection of a ray of light passing near the sun, spectrum shift to the red). Part 5, by E. Cartan, is to the reviewer the most interesting of all. In less than twenty pages, and without mathematical symbols, is developed a readable and thought provoking account of the theory of distant parallelism and of the significance of torsion as opposed to curvature of space. In part 6, Langevin gives, in some 15 pages, a general summary.

Taken as a whole the set of six monographs forms a useful résumé of the elementary parts of relativity theory.

F. D. MURNAGHAN