BAUER ON THEORY OF GROUPS

Introduction à la Théorie des Groupes et à ses Applications à la Physique Quantique. Extrait des Annales de l'Institut Henri Poincaré. By E. Bauer. Paris, Presses Universitaires de France, 1933. 170 pp.

The concept of group is one of those very few primitive and fundamental ideas which play the part of an ordering, suggestive, and guiding principle in all branches of mathematics and wherever mathematics is applied. It is therefore a great satisfaction to the mathematician that not only the concept and some obvious statements about it, but also the deepest and most interesting part of group theory, dealing with the representation of groups by linear transformations, prove to be of paramount import to quantum mechanics. This close relationship is a subject so attractive from all points of view—mathematical, physical, philosophical, and even aesthetic—that it has allured writer after writer. Group theory accounts for the kinematical or combinatorial aspect of quantum mechanics, in particular for the schemes of atomic and molecular spectra, leaving aside the manner in which dynamics vests this skeleton with flesh and blood. Two groups stand in the foreground: first, the group P of permutations (of the electrons in a given atomic structure)—because of the essential likeness of all electrons; second, the group R of rotations around the origin O in 3-dimensional euclidean space—because the kinematic and dynamic constitution of an atom with the fixed nucleus in O is spherically symmetric.

After the reviewer's treatise on group theory and quantum mechanics, E. Wigner, who first discovered and explored the whole domain, gave his competent account, and was followed by van der Waerden's comprehensive and perspicuous presentation. The book under review, addressing the French scientist, cannot claim the same originality as Wigner's or van der Waerden's treatments. Nor does it cover the whole ground; it omits the more difficult parts: the general theory of the exchange phenomenon (group of permutations) and its applications to the classification of spectral lines and to chemical binding, quantum theory of radiation and quantization of the material and electromagnetic field equations, finally all things that are linked up with relativity including Dirac's dynamics of the spinning electron. The book intends to give an introduction only, by developing the fundamental ideas and illustrating them by the easiest acceptable physical instances. Within these limits it is a thoroughly readable, clear, and reliable narrative of the mating between groups and quanta.

Chapters I (Vector space unitary geometry), II (Principles of quantum mechanics) and III (Group theory), are arranged very similarly to the same chapters of the reviewer's book mentioned above. When defining the moment of momentum in Chapter II, the author grazes its group theoretical significance at once: namely, that it consists of the operators corresponding to the infinitesimal rotations of the functional space of wave functions. The theory of perturbation is given only as an answer to the question how the energy terms shift and split under the influence of the perturbing forces; quasi degeneracy (energy levels differ by quantities of the same order as the perturbing energy) is discussed along with actual degeneracy. The other problem, how the probability of a quantum state ascertaining a definite value to the unperturbed energy varies in time under the influence of perturbation, is not discussed. The examples in this part are scarce. The reader looks in vain for treatment of the electron moving in a Coulomb field, for any Stoss-problems. Heisenberg's general scheme corresponding to Hamilton's canonical equations in classical mechanics, with its commutation rules and canonical transformations, is likewise omitted. The quantum theory of interaction between matter and radiation lies beyond the scope of the present work. Zeeman and Stark effects are taken up in the last chapter of the book in connection with the group of rotations.

Chapter III makes use of the transcendental methods of group theory and the calculus of characters. In an appendix, however, all irreducible representations of the group R of rotations are derived by Cartan's infinitesimal, purely algebraic, and hence "elementary" approach. (The last point at which one still needed recourse to a transcendental consideration in the quantum applications of group theory—full reducibility of the representations of R—has recently been settled by Casimir and Pauli; more generally for all semi-simple infinitesimal groups by van der Waerden.)

Chapter IV is dedicated to "General applications to quantum mechanics." It centers around Wigner's theorem: a transformation in the configuration space leaving the Hamiltonian invariant sends every quantum state into a quantum state of the same energy level. The little that is said about permutations finds its place here.

Chapter V (Rotations in space) deals in detail with the quantum mechanical consequences of the rotational symmetry of space: inner quantum number j, rules of selection and intensity. The Clebsch-Gordan decomposition of the Kronecker product of two irreducible representations D_j and $D_{j'}$ of R:

$$D_j \times D_{j'} = D_{j+j'} + D_{j+j'-1} + \cdots + D_{|j-j'}$$

shows how the moment of momentum behaves when two kinematically independent systems are added to form a single system. The spin phenomenon is predicted in a general way from the double-valued representations of R (half integral j's) only after this has been discussed in detail from the mathematical standpoint. Pauli's idea of two-component wave function turns up and accounts for the distinction between azimuthal and inner quantum number (orbital, spin, and total momentum). Here the narrative breaks off just before the next event of the drama, after Schroedinger's great discovery: Dirac's relativistically invariant wave equation.

The book will prove useful for both mathematical and physical readers.

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