

of line elements (éléments d'appui) as the geometry of surface elements. The whole seems to be very closely related to the study of conservative dynamical systems as treated by contact transformations.

The pamphlet as a whole is extremely well done. The formulas are usually given geometrical content—an attribute often lacking in works on tensor analysis—and the proofs are clear and not too formal. Above all, brief as this pamphlet is, it contains many interesting ideas that seem to be worth elaborating.

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*Gesammelte Abhandlungen.* By David Hilbert. Zweiter Band. *Algebra, Invariantentheorie, Geometrie.* Berlin, Julius Springer, 1933. 453+8 pp.

Turning over the leaves of the new volume of Hilbert's mathematical papers and comparing these theories more than forty years old with our present point of view brings again a vivid realization of the tremendous influence of Hilbert on the trends of mathematical thought. The investigations on invariant theory and algebraic systems form the main part of the second volume leading from his thesis (1885) to the two fundamental papers *Über die Theorie der algebraischen Formen*, *Mathematische Annalen* vol. 36 (1890) and *Über die vollen Invariantensysteme*, *ibid.*, vol. 42 (1893). In a short expository article van der Waerden gives an account of the continuation of Hilbert's work on algebraic systems through Lasker and Macaulay up to the abstract formulation by E. Noether's ideal theory in commutative rings.

Among other papers of great importance one should mention paper No. 18 containing Hilbert's irreducibility theorem and the investigations on equations with a prescribed Galois group and also Nos. 10 and 20 dealing with the representation of definite forms as the sum of squares, a problem which was first completely solved by Artin. Paper 26 contains the results on the still unsolved problem of representing the solution of a general algebraic equation by means of functions of a minimal number of variables.

The geometric part of the works is incompletely representative since the well known book *Grundlagen der Geometrie* could not be included. This lacuna is partly filled by an interesting article by Arnold Schmidt on the recent contributions to the axiomatics of euclidean geometry.

OYSTEIN ORE

*Grundbegriffe der Wahrscheinlichkeitsrechnung.* By A. Kolmogoroff. Berlin, Julius Springer, 1933. 62 pp.

It is the purpose of this monograph to develop probability theory from a postulational standpoint. For this purpose a probability field is defined as an assemblage with a definite ordering of numbers that satisfy the system of axioms. A brief exposition is given of the construction of such fields and of the manner in which the framework of the postulational system can be related to the applications to phenomena. The addition and multiplication theorems follow at once. Moreover, the theorem of Bayes, concerning whose validity there have been many controversies, is also an almost immediate consequence of the system of postulates, but the reviewer does not think this derivation of

the theorem of Bayes settles the old contention relative to the validity of inferring the characteristics of a statistical population from a sample by means of the theorem of Bayes.

Use is made of the Lebesgue theories of measure and of integration. Indeed, it is held that it seemed almost hopeless to deal with the logical foundations of probability without these theories.

The development includes infinite probability fields by means of an additional axiom, distribution functions in space of many dimensions, differentiation and integration of mathematical expectations, and the law of large numbers. This little book seems to the reviewer to be an important contribution directed towards securing a more logical development of probability theory.

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