SHORTER NOTICES

The Nature of Mathematics. By Max Black. New York, Harcourt-Brace,
    1934. ix+219 pp.

This book contains expository accounts of the three most important schools
of mathematical philosophy: the logistic, the formalist, and the intuitionist.
The author calls his work a critical survey, and such it is; for the text is inter-
woven throughout with a web of philosophical criticism, frequently subtle and
penetrating, and always interesting. The technical weaknesses of the various
schools of thought are, presumably, fairly well known. But the true philosopher
is concerned with much broader problems: questions of form, structure, mean-
ing, the interrelations between different philosophies, the position of mathe-
matics in human knowledge. Although the author treats none of these questions
systematically (since his book is but an introduction), nevertheless he skill-
fully carries out his analysis from the viewpoint which they imply, and does so
in an engaging and personal style. In this type of criticism lies his real contribu-
tion.

The most extensive account is that of the logistic thesis, that is, the ag-
gregate of theories which assert that mathematics is a branch of logic. Here we
find a simplified but sufficiently technical description of the methods of Russell
and Whitehead's *Principia Mathematica*, followed by an account of the various
attempts to remedy its worst defects. The difficulties which seem to be inherent
to the logistic scheme can be judged by the rather desperate devices with which
they are met, devices which the author ironically suggests, really belong to the
technique of theology. "God arrives to solve the difficulties of Berkeleyan
idealism or Earl Russell in less ambitious times invokes the axiom of reduci-
bility." On a more technical plane the author concludes that the imperfections
of the logistic scheme "can be traced back to insufficiently precise technique
in manipulating systems of symbols," a situation more formidable than it first
appears since it reaches back into the structure of language itself. This constant
preoccupation with the analysis of systems of symbols (including languages)
and with the problems of correct manipulation constitutes a tenuous thread of
criticism running through the book. Mathematics may in a sense be regarded
as "the grammar of all symbolic systems . . . ," or again, as "the crystallized
syntax of all systems of interrelated objects."

The supplementary portions of the book include many interesting topics.
In connection with formalism there is a section on the development of geom-
etry and a note on Gödel's theorem,—a remarkable result which makes it
doubtful that the formalist program can be carried through. In connection
with intuitionism there is a section on the opinions of Kronecker and Weier-
strass, a sketch of the theory of point sets, a summary of the *polémiques sur le
transfini* of the contemporary Paris school, and finally, short accounts of the
intuitionist calculus of propositions and the intuitionist theory of cardinal
numbers.

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