Carrés Magiques au Degré n. By Général E. Cazalas. Paris, Hermann, 1934. 191 pp.

It is a relief to find a book or memoir on the subject of magic squares which is not a mere collection of specimens with no attempt at unification and analysis. This work by General Cazalas is an attempt, and a very successful one, to bring these interesting and baffling forms into some sort of unified theory.

The author has undertaken to put on a solid analytical basis the results announced in 1906 by G. Tarry who, by the use of what he called "numeral series," was able to arrive at results of astonishing generality in constructing squares characterized not only by the fact that the sum of the elements in any row or column or in the two main diagonals should be constant, but also by the further property that the sum of the squares and the sum of the cubes, etc., of the elements in any of these lines should also be constant. Thus, for example, the square of order 9 which follows is not only magic in the ordinary sense, but the sum of the squares in any of the lines is also constant.

0	64	47	14	75	31	25	62	42
34	17	78	36	19	56	50	3	67
59	39	22	70	53	6	72	28	11
69	52	8	74	27	10	58	41	21
13	77	30	24	61	44	2	63	46
38	18	55	49	5	66	33	16	80
48	4	68	35	15	79	37	20	54
73	29	9	57	40	23	71	51	7
26	60	43	1	65	45	12	76	32

To illustrate the method of "numeral series" we give a square of order 5 constructed with the "keys" (11) and (32).

00	11	22	33	44
32	43	04	10	21
14	20	31	42	03
41	02	13	24	30
23	34	40	01	12

The symbol ab stands for $5a+b$, a and b taking values from 0 to 4. In ordinary notation the square would read:

0	6	12	18	24
17	23	4	5	11
9	10	16	22	3
21	2	8	14	15
13	19	20	1	7

This is a magic square of the ordinary sort, using, however, the numbers from 0 to $n-1$ instead of the numbers from 1 to n as is customary.

In the construction it is seen that the elements in any horizontal row are obtained from the one on the left by the addition of the numbers of the "key," multiples of 5 being thrown out as they arise. Similarly the elements in any column are obtained from the one above by addition of the numbers of the second "key," multiples of 5 again being thrown out as they arise. This method, applicable to any square of prime order, will, however, not always work when the order of the square is composite. The squares obtained by it are more easily obtained by the "uniform step" method, a complete theory of which has been worked out in a paper apparently not known to the author published in the Transactions of this Society in 1930 (vol. 31, No. 3, pp. 529–551).

Although the method of "numeral series" is of no great importance for the problem of ordinary squares, it yields extraordinary results when applied to "hypermagic squares," that is, squares which are magic in the squares of the elements or in the cubes of the elements. Thus for squares of order p^2, squares magic in the squares of the elements are obtained, and trimagic squares are also obtained, that is, magic in the cubes of the elements. The smallest bimagic square given is of order 8 and the smallest trimagic square of order 128. The method of construction does not seem to indicate whether bimagic squares of order less than eight are possible. It is not difficult to prove that squares of order less that 7 can not be bimagic. Whether there are bimagic squares of order 7 or not is in doubt, and the reviewer hopes to answer that question before long.

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Einführung in die Differentialrechnung und Integralrechnung. By Edmund Landau. Groningen-Batavia, P. Noordhoff N. V., 1934. 368 pp.

The reviewer's task of reporting on this book is considerably simplified by the fact that it has already been reviewed in the American Mathematical Monthly (Nov., 1934). Presumably Professor Ritt's review has been seen by most of the readers of the Bulletin, but the novelty of this work seems to justify some further comment.

The book is confined solely to the analytical parts of calculus; neither are there any problems for the student to work, nor are there the familiar applications to geometry and mechanics. The author's aim is to present the theory "in exacter und lückenloser Weise." Your reviewer will not appraise the author's success in this endeavor by searching for slips in rigor, but will confine his remarks to points likely to be of interest to the teacher of calculus.

A knowledge of the fundamental rules of algebra is presupposed. These are set forth in the introduction, which also contains the notion of the Dedekind cut, the necessary theorems on finite and infinite sets of numbers, and a proof of the existence of $a^{1/n}$ for $a \geqq 0$ and n integral. This last, however, is used only for $n=2$.

Chapter 1 contains the usual definitions and theorems on the limit of a sequence. Since the student is supposed to know merely the meaning of a^n for n a positive or negative integer or zero and the existence of $a^{1/n}$ for $a \geqq 0$, it is necessary in Chapter 2 to develop the whole theory of logarithms and exponentials. We find $\log x$ defined as $\lim_{n\to\infty} k(x^{1/k}-1)$, where $k=2^n$. The existence and usual properties of logarithms are then deduced. The constant e is now defined as the solution of $\log y=1$, e^x as the solution of $\log y=x$, and $a^x=e^{x\log a}$ for $a>0$. The usual theory of exponentials readily follows with unusual brevity.

Chapters 3, 4, and 8 deal with functions, continuity, and limits of functions.