second "key," multiples of 5 again being thrown out as they arise. This method, applicable to any square of prime order, will, however, not always work when the order of the square is composite. The squares obtained by it are more easily obtained by the "uniform step" method, a complete theory of which has been worked out in a paper apparently not known to the author published in the Transactions of this Society in 1930 (vol. 31, No. 3, pp. 529–551).

Although the method of "numeral series" is of no great importance for the problem of ordinary squares, it yields extraordinary results when applied to "hypermagic squares," that is, squares which are magic in the squares of the elements or in the cubes of the elements. Thus for squares of order p^2, squares magic in the squares of the elements are obtained, and trimagic squares are also obtained, that is, magic in the cubes of the elements. The smallest bimagic square given is of order 8 and the smallest trimagic square of order 128. The method of construction does not seem to indicate whether bimagic squares of order less than eight are possible. It is not difficult to prove that squares of order less that 7 can not be bimagic. Whether there are bimagic squares of order 7 or not is in doubt, and the reviewer hopes to answer that question before long.

D. N. LEHMER

Einführung in die Differentialrechnung und Integralrechnung. By Edmund Landau. Groningen-Batavia, P. Noordhoff N. V., 1934. 368 pp.

The reviewer's task of reporting on this book is considerably simplified by the fact that it has already been reviewed in the American Mathematical Monthly (Nov., 1934). Presumably Professor Ritt's review has been seen by most of the readers of the Bulletin, but the novelty of this work seems to justify some further comment.

The book is confined solely to the analytical parts of calculus; neither are there any problems for the student to work, nor are there the familiar applications to geometry and mechanics. The author's aim is to present the theory "in exacter und lückenloser Weise." Your reviewer will not appraise the author's success in this endeavor by searching for slips in rigor, but will confine his remarks to points likely to be of interest to the teacher of calculus.

A knowledge of the fundamental rules of algebra is presupposed. These are set forth in the introduction, which also contains the notion of the Dedekind cut, the necessary theorems on finite and infinite sets of numbers, and a proof of the existence of $a^{1/n}$ for $a \geqq 0$ and n integral. This last, however, is used only for $n=2$.

Chapter 1 contains the usual definitions and theorems on the limit of a sequence. Since the student is supposed to know merely the meaning of a^n for n a positive or negative integer or zero and the existence of $a^{1/n}$ for $a \geqq 0$, it is necessary in Chapter 2 to develop the whole theory of logarithms and exponentials. We find log x defined as $\lim_{n\to\infty} k(x^{1/k}-1)$, where $k=2^n$. The existence and usual properties of logarithms are then deduced. The constant e is now defined as the solution of $\log y=1$, e^x as the solution of $\log y=x$, and $a^x=e^{x\log a}$ for $a>0$. The usual theory of exponentials readily follows with unusual brevity.

Chapters 3, 4, and 8 deal with functions, continuity, and limits of functions.

Continuity of $f(x)$ at $x=a$ is defined in the usual ϵ, δ fashion. The concept $\lim_{x\to a}f(x)$ is defined by first proving that there is at most one value η, such that the function $g(x)$ is continuous at a, where $g(x)=f(x)$ for $x\neq a$ and $g(x)=\eta$ for $x=a$. If such an η exists, then $\eta=\lim_{x\to a}f(x)$.

Differentiation for the functions so far defined is discussed in the next three chapters. The formula $dy/dx=(dy/du)(du/dx)$, almost always incorrectly proved, here has a correct proof. The troublesome theorem on the limit of a function of a function is avoided by introducing the auxiliary function $g(h)=[f(a+h)-f(a)]/h$ for $h\neq 0$, $g(h)=0$ for $h=0$, which is continuous at $h=0$ and simplifies this and other proofs. The terms maximum and minimum, and increasing and decreasing at a point are defined and the few theorems possible at this point are proved.

Chapters 9–11 cover Rolle's theorem, the mean value theorem, and the extended mean value theorem, with applications to maximum and minimum values, indeterminate forms, and the theorem that, if $f(x)$ is continuous and $f'(x)=0$ in ab, then $f(x)=f(a)$. The next four chapters discuss series, uniform convergence, and power series, including the theorem on the termwise differentiation of power series. By means of this last and an ingenious use of the theorem mentioned before, the ordinary expansions of e^x and $(1+x)^\alpha$ are shown to be correct. The usual expansion by Taylor's series is not given except in the form of a theorem applicable only to functions defined as sums of power series.

In Chapter 16, $\sin x$ and $\cos x$ are defined as sums of power series, and $\tan x$ and $\operatorname{ctn} x$ as quotients of $\sin x$ and $\cos x$. The formulas for differentiating $\sin x$ and $\cos x$ come at once from the theorem on termwise differentiation of power series, without the use of $\lim_{x\to 0}(\sin x/x)$. By the mean-value theorem $\cos x$ is shown to have a least positive zero, which is defined as $\pi/2$. Then formulas for $\sin(x+y)$ and $\cos(x+y)$ are derived by means similar to those used in proving the expansion of e^x in power series. The remaining formulas follow at once. The remainder of Part I consists of chapters on partial derivatives, inverse and implicit functions, inverse trigonometric functions, and the resolution of rational functions into partial fractions.

In Part II, on integral calculus, there are fewer novelties and the student of real functions is on familiar ground. The first four chapters contain the definition of indefinite integrals, general theorems, and sufficient technique to show the integrability of rational functions and of some irrational functions. The next three chapters discuss definite integrals, based on the Riemann definition, and give the usual theorems, including termwise integration of series. Two chapters on improper integrals follow and the book closes with short chapters on the Gamma function and Fourier series. As in Part I there is an entire absence of drill work and of applications.

What shall we say is the chief value of this book? It is clear that it could not serve as a text for the American college student of the present time. Nevertheless it should exert a considerable influence on teachers of mathematics and it should be required reading for writers of textbooks in calculus. Nothing is more needed for the effective teaching of this subject than a textbook which is rigorous as well as simple, and such books are almost non-existent. The chief value of this work, to the writer's mind, is that it shows that rigorous proofs are simple proofs and that the theory is to be used, instead of being laid away on a shelf. Let this be a stimulus to the textbook writer, but let him beware of slavish copying. For we must build on what the student has already learned in the secondary school, and it is questionable whether the topsy-turvy treatment of logarithms, exponentials, and trigonometric functions here given could be "put across" to the average beginner. Finally it must not be forgotten that the most difficult parts of calculus to make rigorous and at the same time simple and concise are those applications to geometry and mechanics which Professor Landau eschews. There is still work to be done.

W. A. WILSON

Introduction to General Topology. By W. Sierpinski. Translated from the Polish by C. Cecilia Krieger. The University of Toronto Press, 1934. 235 pp.

Topology, long considered as more or less outside the pale of "normal" mathematics, is at last coming into its own. It has passed in the last few years from the pure "mémoire" to the book stage. The number of treatises on diverse phases of the subject is still restricted enough. We can only welcome therefore the present volume by the leader of the Polish school, and thank the translator for having made it available to the non-Polish mathematical public.

Professor Sierpinski's treatise is of modest proportions, and deals exclusively with the abstract space side of the question. It is the work of a "purist" and shows little trace of the steady drift towards combinatorial topology on the part of those particularly interested in the applications of the subject, or in its contacts with other branches of mathematical science. The postulates are carefully defined, their inter-relations investigated in detail and with thoroughness. The author has very wisely chosen the class of open sets as fundamental in defining topological spaces, a procedure amply justified by experience. Without indulging in a description of topics, we may mention that metrization is fully treated, but dimensionality barely touched upon at the end.

This is the second volume of a more extensive series probably projected by the eminent author, the first of which (translated into French) dealt with transfinite numbers. Are we to hope for a third volume, which, as the program would seem to indicate, would begin with the Urysohn-Menger theory?

SOLOMON LEFSCHETZ