ENCYKLOPÄDIE—III 2, 7–13

Encyklopädie der mathematischen Wissenschaften mit Anschluss ihrer Anwendungen. Band III 2, Hefte 7–13. Teubner, Berlin and Leipzig.

The early parts on algebraic geometry appeared more than twenty years ago. The manuscripts of some of the other parts were also prepared, but various circumstances prevented their publication at that time. Most of these have now been brought down to date, but in two cases the deaths of their respective authors have made this impossible.*

Mehrdimensionale Räume. By C. Segre. Band III 2 A, Heft 7. 1920. Pages 769–972.

Corrado Segre (1864–1924) had been interested chiefly in the geometry of hyperspace prior to and a few years after the beginning of the present century. Because of the variety and value of his contributions to this field, it was natural for him to be chosen to write the monograph on hyperspace for the Encyklopädie; indeed, as Loria says in his biography of Segre,† the plan of a great encyclopedia of mathematics could scarcely be thought of without designating him to discuss the geometry of n dimensions.

In the monograph, a bibliography is given first with abbreviated titles for future reference, followed by a short table of frequently used symbols. The historical introduction covers sixteen pages. It contains numerous references and copious footnotes. Here and all through the monograph, the footnotes contain discussions of the references which are very valuable.

Following the historical introduction, the monograph is divided into ten large subdivisions, which we shall call chapters. The chapters are further subdivided into sections, of which there are forty-seven, numbered continuously throughout the monograph.

In the first two chapters, the elementary properties of hyperspace and the generalized principles of projective geometry applied to n dimensions are developed.

The third chapter deals with hyperquadrics—their properties, collineations under which a manifold of the second order is invariant, and systems of hyperquadrics. This is followed by a short chapter on null systems and the linear line complex.

In Chapter 5, hyperspace curves are discussed in detail, first as to their characteristics and their generation by intersecting manifolds. The formulas of Veronese are then derived, followed by a treatment of normal curves and the postulation of curves on surfaces. The last two sections are devoted to rational and elliptic curves.

Similarly, in Chapter 6, surfaces in hyperspace are discussed. The generation of surfaces by intersecting manifolds and the tangent spaces of surfaces

* In what follows, the separate parts are reviewed by different persons, under the general direction of Virgil Snyder.—THE EDITORS.

† Annali di Matematica, (4), vol. 2 (1924–25), p. 9

are treated. Then follows a discussion of ruled surfaces, rational surfaces, Veronese's quartic and its generation in S_n, and finally surfaces whose plane sections have a given genus.

The next chapter deals with manifolds of dimension greater than two, the discussion being similar to that for surfaces. Hypersurfaces are studied in detail, including their characteristics, systems, postulation, intersections with manifolds. Seven pages are given to cubic hypersurfaces. Manifolds generated by a single infinity of lines and other special manifolds are treated in the latter part of the chapter.

In Chapter 8 there are developed geometries in which any linear S_k is the element. Chapter 9 contains a very brief discussion of the principle of correspondence in S_n.

In Chapter 10, the concept of hyperalgebraic geometry is presented. The manifolds of hyperalgebraic geometry have equations in the coordinates of conjugate imaginary elements. The study of linear transformations connecting hyperalgebraic manifolds leads to antiprojectivities and their special cases. An antiprojectivity is a linear transformation in which the anharmonic ratios of four elements and their four corresponding elements have conjugate imaginary values. These ideas were developed by Segre in 1890–91. He seems not to have known that they had been treated previously in a thesis by a Danish geometer, C. S. Juel, under the name of "symmetralities." This thesis was published in Copenhagen in 1885 and later in the Acta Mathematica (1890). No reference to Juel is given in this monograph.

There is no separate index since the volume is indexed as a whole. The brief but pithy contents, however, present an excellent outline of the subject matter.

There is evidence that Segre did not particularly enjoy writing this monograph—in the decade or more preceding, he had become interested in other phases of mathematics. He did the work, however, with an exactitude, finesse, and comprehension that have rarely been equalled. One who studies it will readily agree with Loria* that the difficult task of writing a digest of the geometry of n dimensions was performed with such great care and insight that this article deserves to serve as a model for future similar works.

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Algebraische Liniengeometrie. By Konrad Zindler. Band III 2, Heft 8. December, 1922. Pages 973–1228.

The author of this report had three decided advantages: the subject was comparatively new and sharply defined, he had contributed a considerable part of the literature himself, and was still in the prime of productivity when the report was written. The results of these conditions are everywhere apparent in a comprehensive and well-rounded product.

It is curious that a subject that was practically unknown three quarters of a century ago is now an indispensable tool in algebraic and projective geometry besides furnishing a vast laboratory in the foundations of mathematics.

The first section, about 100 pages, is devoted to the elementary concepts,

* G. Loria, loc. cit.