

SHORTER NOTICES

Moderna Teoria delle Funzioni di Variabile Reale. Parte Prima. By G. Vitali and G. Sansone. Monografie di Matematica Applicata per cura del Consiglio Nazionale delle Ricerche. Bologna, Zanichelli, 1935. viii+183 pp.

This excellent booklet is the first volume of a new series of monographs on applied mathematics published under the auspices of the Italian National Research Council. It is based upon a posthumous manuscript of Vitali, edited and completed by Sansone. It gives a remarkably clear and condensed introduction to the theory of functions of a real variable. In five chapters of about 33 pages each are presented the basic facts from the theories of sets and cardinal numbers, measure, analysis of functions [measurable functions, classes of Baire, bounded variation, absolute continuity], integration, and differentiation. As is to be expected in view of the prominent role played by Vitali in the development of modern analysis, the presentation is original in many places. The typographical makeup is pleasing and of high order. A minor point of notation could perhaps be mentioned. In a modern treatment one does expect that the variable of integration and the upper limit should not be denoted by the same letter; the authors fail to observe this desideratum.

Those preparing to give a course on real variables will find this book highly stimulating and useful. In the hope that the further publications in this series will come up to the high standard set by the first one the mathematical world will eagerly await successors to this monograph. A treatise by Sansone on expansions in orthogonal series seems to be the next in order.

EINAR HILLE

The Dilemma of Modern Physics. By D. E. Richmond. New York, G. P. Putnam's Sons, 1935. xiii+120 pp.

This small book attempts to give in a non-technical form some idea of the revolution which has taken place during the past quarter century in the domain of physics. The following chapter headings give a good idea of the matters treated: *The classical physics; The quantum theory; New developments; The dilemma* (that is, waves versus particles); *The principle of uncertainty; On meaning; Causality and probability.* In his preface the author explains that the book arose from a series of semi-popular lectures on the modern quantum theory. It is the present reviewer's opinion that the book itself is a good instance of what a semi-popular book should be.

F. D. MURNAGHAN

Aristotle's Theory of the Infinite. By Abraham Edel. Privately printed, New York, 1934. For sale at The Journal of Philosophy, Inc., 515 W. 116th Street, New York City. Price \$1.00. 102 pages.

Here we have a careful inquiry into Aristotle's theory of the infinite in which the principal emphasis lies in the attempt to understand how Aristotle

himself conceived the problem and what he says in his discussion of it. The author has brought a discriminating intelligence to bear upon his task and has presented an analysis which is worthy of the careful attention of students of Aristotle. The principal interest of the essay is for those whose attention is centered mainly upon the philosophical aspects of the infinite; consequently this is not the place for a detailed review of it. But a fact frequently overlooked should be pointed out here, namely, that in his discussion of the infinite by way of addition, Aristotle makes manifest a clear understanding on his part of the essential elements involved in the notion of convergence of an infinite geometric series whose ratio is less than unity. See *The Physics*, Book 3, Chapter 6 (206b 3–33).

R. D. CARMICHAEL

Théorie des Probabilités. By P. Van Deuren. Paris, Gauthier-Villars, 1934. xvii+546 pp.

This treatise on probability, written by Professor Van Deuren after teaching the subject for thirty years at l'École Militaire de Belgique, is very well organized. It constitutes "Tome Premier," introductory to a volume dealing with applications. The style is formal; indeed, the author intended to make it "didactic." The material throughout the book is in general very well arranged and well chosen for the purpose in view.

To avoid the much criticized use of "equally likely" events to define probability, the author adopts a quasi-axiomatic method. He states (p. 4): "La probabilité est un notion première intuitive. Elle apparaît comme une grandeur qui mesure à l'aide des données (4), la vraisemblance actuelle, ou apparence de vérité de l'arrivée d'un événement." This second statement virtually leaves the "intuitive" notion undefined, except that probability is required to be a magnitude, "grandeur." Later the author associates such a magnitude with a real number. Certain properties of probability are then set forth as principles or theorems—many of which might well be regarded as axioms—collectively building up the notion of probability. These properties include the additive law for incompatible events, and laws for composite events—such as the law that the probability of a composite event cannot exceed that of any constituent. When a set of incompatible events exhausts all the possibilities, and the probabilities therefor are all equal, these events are called "chances," and are said to arrive "ou hasard."

The book deals primarily with the classical theory of probability. There are very few references to other writers. Modern topics, such as small samples, are not included. Nevertheless, the size of a sample is a matter of great concern. The author determines the minimum size of a sample—the least number of trials or observations—to make valid various approximations. An event, with a probability of at least 0.99, he defines as "pratiquement certain"—following artillery usage. Using this measure of practical certainty, he determines practical domains of certainty, practical moments, practical coefficients of concentration, and so on. It is difficult in a few words to give an adequate idea of the thoroughness with which these approximations are introduced and controlled; this is one of the outstanding and unique features of the book.