DIRAC ON QUANTUM MECHANICS

The Principles of Quantum Mechanics. By P. A. M. Dirac. Second edition. Oxford, Clarendon Press, 1935. xii+300 pp.

The first edition of Dirac's *Principles of Quantum Mechanics* appeared in 1930.* The second edition is essentially the same book, but is in many respects clarified and recast in exposition, with a shift in emphasis to the non-relativistic conception of *state*, and with additional matter, in particular, a chapter on the quantization of the electromagnetic field.

In the introductory generalities, comprising the preface and first chapter, such matters are discussed as the inadequacy of the classical theory with its determinism and causality, and the notions of states, probability, and the principles of superposition and indeterminacy. The non-technical reader is apt to be misled by the form of the discussion, which is that of a logical analysis of experimental data with the drawing of necessary physical and philosophical conclusions. Yet we believe that this is not at all what the author is undertaking. One hardly needs to be a practiced epistemologist to be aware of the order of magnitude of philosophical analysis required to treat adequately such questions as those of causality and indeterminacy, of which the author disposes—too often with scarcely more than the conventional phrases. Nor is unusual penetration needed to perceive that the experimental evidence claimed as overthrowing classical physics and establishing the new theory, requires a far more penetrating critique. As a pure matter of logic, one is left with the impression that all might about as well have been argued in the opposite direction. We believe that what the author has attempted to do here is a task far more important for the needs of a student of contemporary physics. He is attempting to *prepare the intuition* of the reader, so that the old theory becomes replaced by the new by a sort of *intuitive* necessity, and so that the form and substance of the new theory may be apprehended not only as a logical abstraction but in terms of a new conceptualism—almost a new imagery. In this respect he has made a distinguished contribution. The present edition is far clearer at this point than the first.

The next section of the book, from the second to the fifth chapters, deals with mathematical preliminaries and their physical interpretation: the linear vector spaces and operators and their representations, the transformation theory, and the quantum conditions. The point of view is to regard the aggregate of states of a system somehow as forming a linear vector space, the elements of which both are and are not the states (there is an indeterminate phase factor!), and the linear transformations of which are *en rapport* with the observables of the system. The familiar spaces of the quantum theory, com-

* Reviewed by me in this Bulletin, vol. 37 (1931), p. 495. The observations in the review of the first edition apply to a considerable extent to the second; we take the occasion of the present review to treat a rather different side of the question.

posed of number-sequences or of wave functions, together with their matric, differential, and integral operators, appear a posteriori as *representatives* of the states and observables, the agency in this representation being that embodiment of mathematical faith, the expansion theorem. A notation is introduced for the representatives, and is used consistently throughout the non-relativistic portions of the book; it has the great advantage of emphasizing the role of the analytical constructs as representatives, but the disadvantage of insufficient explicitness and practical inconvenience.

This view of the states and observables as prior realities, and the mathematical constructs as but their representatives, is one of the most notable features of the whole book. It is a conception in the grand manner; and with its aid everything is seen in a new clarity. Unfortunately it involves some grave difficulties. As a matter of conceptual experimental physics, there is a vagueness exceedingly difficult to dispel about such definitions as those of states and superposition, as general ideas; and the precise sense in which the states go to make up the linear spaces is scarcely more than indicated. And on the mathematical side, there is the question of the validity of the expansion theorem, and the related difficulty of the δ-function. In the closed systems of the non-relativistic theory, to be sure, the states may be represented in Hilbert space: the expansion theorem applies, and the δ-function may be dispensed with. But the quantum theory deals with non-closed systems, and also, with both closed and non-closed relativistic systems, in none of which is the expansion theorem in its general form valid. It is greatly to the credit of our author that he has not sought to fit these systems to the Procrustean bed of Hilbert space.

A possible exit from both orders of difficulty, as well as an explanation of the practical successes which the theory has achieved in spite of them, may lie in the observation that *the quantum theory is not a general theory*: it deals with a small number of exceedingly special physical systems, in which the states and observables are of a highly specialized character. This is not merely a fact of contemporary physics,—it is inherent in the very notion of a theory of primary phenomena. The postulational generality is purely linguistic—in obvious analogy with the form of the macroscopic theories of classical physics, dealing as they do with a high order of infinity of essentially distinct conceptual systems. And now in the few special cases of our theory which correspond to its only reality, the meaning in terms of experiment of state, observable, and so on, may be made quite definite and clear, and the expansion theorem actually is observed to apply. Yet the generality of form in which the theory is cast has a significance which becomes clear the instant that it is envisioned not as a system of propositions, but as a body of principles of the intuition, a pattern and guide in the unknown.

Along the lines of this clarification by specialization to actual systems, the δ-function is quite simply treated: The reader will perceive—and at this point the author is far clearer than in the first edition—that the δ-function is not a function at all, but a symbol for a category of definitions. In what concerns harmonic analysis, certain kernels involving a large integer are called for. And in the collision problems, all is made clear and rigorous once the treatment is paraphrased in terms of discontinuous additive set-functions. While such a transcription places quite a task upon the reader, we are of the opinion that the present form of treatment has great value in showing the underlying unity behind the special mathematical situations into which the explicitly rigorous treatments tend to disintegrate.

The third part of the book, from the sixth to the tenth chapters, deals with the conventional non-relativistic theory: the equations of motion and their applications, including the hydrogen atom, angular momentum, and spin; perturbations and collisions; systems of similar particles. Here the author's conception of the mathematical objects as but representations of an underlying reality comes into its own. It receives an expression, for example, in the abstract algebra of observables, which forms so beautiful and characteristic a feature of the work. It is hardly a criticism to observe that this method could have been carried much further, and much that the author does with the aid of the representatives could have been done abstractly in a simpler manner, and one more in the spirit of the book.

The last three chapters deal with the relativistic equation of the electron and the electromagnetic field; they form the advanced portion of the work. Here the fundamental conceptions developed earlier have free play, and the result is a treatment which carries the ideas far deeper into the nature of the situation than do so many current discussions of the subject, where so much is buried under the analytical machinery. The relativistic conception of state is employed, with its wave-function defined throughout space-time. This is in sharp contrast with many of the treatments of the relativistic electron which, in order to confine the situation to Hilbert space, regard the wave-functions as representing a one-parameter family of states in 3-space, and are thus not relativistic in essence. It should be remarked that the author abandons his notation for the representatives, for no apparent reason, in the twelfth chapter. Our only criticisms are that we wish that the author had given a less abbreviated discussion of the positron, and had amplified that of the quantized electromagnetic field.

Aside from its function as a textbook on quantum theory,—a function which, as far as the uninitiated is concerned, it does not fulfill any too well—Dirac's *Quantum Mechanics* has a particular significance for the mathematician. Mathematics of the present day is resounding with the triumphs of the abstract method: we have abstract sets, abstract space, abstract algebra and arithmetic, abstract analysis, and so forth. And now here is a physicist who is approaching the highest department of modern physics with so completely abstract a point of view that he is regarded as a pure mathematician by many of his colleagues. Yet the general and abstract standpoint of our author is but the garb and vestment of a deeper thing, an inner principle, without which he has the vision to perceive that his abstract discipline would be but a lifeless form.

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