ABSTRACTS OF PAPERS

SUBMITTED FOR PRESENTATION TO THE SOCIETY

The following papers have been submitted to the Secretary and the Associate Secretaries of the Society for presentation at meetings of the Society. They are numbered serially throughout this volume. Cross-references to them in the reports of the meetings will give the number of this volume, the number of this issue, and the serial number of the abstract.

318. Dr. J. S. Frame: *On the degrees of the irreducible components of simply transitive permutation groups.*

In a simply transitive permutation group G^* of order g and degree n, the subgroup leaving one symbol invariant will permute the others in λ sets of transitivity involving $k_1, k_2, \cdots, k_\lambda$ symbols respectively. If this group G^* be represented in the usual way by a group G of matrices of degree n, in which the matrix $(\delta_{\alpha'\beta})$, $\alpha, \beta = 1, \cdots, n$, corresponds to the permutation $\binom{\alpha}{\alpha'}$, the group will be a reducible linear group with $r = \lambda + 1$ irreducible components, having the respective degrees $n_0 = 1, n_1, n_2, \cdots, n_\lambda$. Let $N = n_1 n_2 \cdots n_\lambda$, and $K = k_1 k_2 \cdots k_\lambda$. The object of this paper is to show that $n^{\lambda-1}K/N$ is an integer, which is a perfect square when the numbers k_i are distinct. We prove this theorem for $\lambda \leqq 3$, and also for an infinite family of permutation groups (including all values of λ), which consist of the permutations induced by the symmetric group of degree ν on the $n = \binom{\nu}{\lambda}$ sets of ν symbols taken λ at a time. A somewhat complicated identity in binomial coefficients is proved as a lemma. (Received July 25, 1936.)

319. Professor W. L. Ayres: *Periodic transformations of sets.*

Periodic, pointwise periodic, and almost periodic transformations of sets are defined and shown to have unique inverses. A pointwise periodic continuous transformation of a graph is shown to be periodic. If the graph contains at least one branch point, pointwise periodic may be replaced by almost periodic. If any of these transformations on a tree leaves the end points invariant it leaves all points invariant. Either a pointwise or almost periodic continuous transformation of a tree is periodic if only a finite set of end points are non-invariant. (Received July 27, 1936.)

320. Dr. Reinhold Baer: *Abelian groups without elements of finite order.*

In this paper the properties of direct sums of rational groups and of separable groups are investigated (rational group = subgroup of the additive group of rational numbers; separable group = group whose finite subsets are contained in direct summands which are direct sums of rational groups). There exist

separable groups which are not direct sums of rational groups (e.g., the additive group of all the sequences of integers) but every countable separable group is a direct sum of rational groups. Further criteria for the existence of a decomposition into rational groups and for separability are given. Though every Abelian group without elements of finite order is contained in a direct sum of rational groups, it is possible to characterize a class of subgroups which are isomorphic with a direct summand and are themselves direct sums of rational groups. Finally the structures of direct sums of rational groups, the types of elements, and the characteristic subgroups of separable groups are enumerated. Only invariant properties are used in the mentioned criteria and enumerations. The applied method is essentially a calculus with multiplicities where the multiplicity of the element x in the group J is the l.c.m. of the positive integers n such that $x \equiv 0 \bmod nJ$. (Received July 22, 1936.)

321. Professor D. H. Ballou: *Functions representable by two Laplace integrals.*

Functions $f(z)$ are considered which are single-valued, meromorphic, and simply-periodic, and hence developable into series both of partial fractions and of exponentials. By considering the nature and position of the poles in a period strip classes of these functions are found which can be expressed as two different Laplace integrals. From the equality of these integrals there then follow as corollaries the equalities of the series of their integrands. The meromorphic trigonometric and hyperbolic functions are specific examples of this theory. (Received July 6, 1936.)

322. Dr. E. F. Beckenbach: *On a theorem of Fejér and Riesz.*

Let the analytic function $w=f(z)$ map $|z| \leqq 1$ conformally on a region R. The map on R of a diameter of $|z| \leqq 1$ will be called a conformal diameter of R. According to a theorem of Fejér and Riesz, a conformal diameter of R can be at most half as long as the perimeter of R. The result now is extended, by use of subharmonic functions, to conformal diameters of regions on surfaces of negative Gauss curvature. The average of the lengths of the conformal diameters of such regions also is discussed. (Received July 27, 1936.)

323. Mr. Garrett Birkhoff: *Linear representability of nilpotent Lie algebras.*

It is shown that every Lie algebra can be realized by infinite matrices with coefficients in the same abstract field, and that every nilpotent Lie algebra can be realized by finite triangular matrices. It follows that every nilpotent Lie algebra with real or complex coefficients is the infinitesimal algebra of a simply connected group of matrices. (Received July 23, 1936.)

324. Professor Richard Brauer: *Hypercomplex numbers and factor sets.*

A new method is developed for deriving the properties of factor sets. The basis for the considerations is formed by Wedderburn's theorems. Thus a new approach to some parts of the theory of algebras is obtained. (Received July 24, 1936.)

325. Professor Richard Brauer: *On algebras which are connected with the simple continuous groups.*

Let G be a group of linear transformations in an n-dimensional vector space R_n. The tensors of given rank f undergo a homomorphic group of linear transformations when the transformations of G are performed in R_n. The matrices of these tensor transformations generate an algebra A_f the study of which is of great importance for the theory of representations of G. It will be shown that the construction of a basis of A_f and the proof of the fundamental theorem of invariant theory imply each other. If G is the full linear group, A_f can be easily investigated in a direct way and thus we obtain a proof of the fundamental theorem of invariant theory. In case G is the orthogonal group or the complex group we go the opposite way. We thus obtain the algebra A_f. In case G is the complex group this algebra has been constructed by Weyl (Mathematische Zeitschrift, vol. 35) by using a quite different method. (Received July 24, 1936.)

326. Professor Richard Brauer: *On algebraic equations of the fifth and sixth degrees.*

The solution of equations of the fifth and sixth degrees is equivalent to the solution of a form problem for the icosahedron and the Valentiner group, as was shown by Klein, Gordan, Coble, and Fricke. The same question is here dealt with again by using a general theory for form problems already given in a previous paper (Mathematische Annalen, vol. 110). (Received July 24, 1936.)

327. Professor R. S. Burington: *R-matrices and equivalent networks.*

In recent papers the author has developed a theory relating to m-affine congruent matrices and their application to the study of networks. If B is a matrix of a linear m-terminal pair network and T is any real m-affine non-singular matrix, then $A = T'BT$ corresponds to a network (electrically) equivalent to B. The range V of T for which networks corresponding to A are realizable without ideal transformers is desired. In this paper a theory of R-matrices is developed, a real matrix $G \equiv (g_{rs})$ being termed an R-matrix if and only if $g_{ss} \geqq 0$ and $\sum_{r=1}^{n} |g_{rs}| \leqq 2g_{ss}$, $s = 1, \cdots, n$. Necessary and sufficient conditions are given that $C = P'DP$ be an R-matrix, where P and D are real and P is non-singular. With each D there exist n sets of functions $X^{(i)}$, each set forming a group of order 2^{n-1}. The $n \cdot 2^{n-1}$ quadric surfaces $X^{(i)} = 0$ bound a region in the space of P for which C, positive definite, is an R-matrix. Application is made to network theory, necessary and sufficient conditions being found, together with V, for realizability without ideal transformers of networks corresponding to A. (Received July 23, 1936.)

328. Professor L. P. Eisenhart and Professor M. S. Knebelman: *Invariant theory of homogeneous contact transformations.*

We define *a configuration* as an entity whose components undergo a certain linear homogeneous transformation when the element (x, p) is subjected to a homogeneous contact transformation. We introduce into our space a *contact*

frame $\Gamma_{\alpha\beta}(x, p)$ by means of which a configuration is given a unique set of tensor components. We then introduce a linear connection—subject to a set of symmetry conditions—by means of which covariant differentiation is defined. It is by this means that we can attack the problem of equivalence of configurations under contact transformations. We apply these ideas to the concepts of parallelism, geodesics, and curvature. (Received July 29, 1936.)

329. Mr. R. M. Foster: *A simplified set of postulates for a group.*

A simplified set of postulates for a finite group has been given by Garver (this Bulletin, vol. 42 (1936), pp. 125–129). It is shown in the present note that Garver's simplification is not essentially limited to the special case of finite groups, that the same simplification can be effected in the set of postulates for a general group. (Received August 1, 1936.)

330. Professor L. M. Graves: *On the uniform boundedness of sets of functional transformations between topological spaces.*

In this paper we consider certain generalizations of well known theorems (see Banach, *Opérations Linéaires*, p. 19, Theorem 11; Hildebrandt, this Bulletin, vol. 29 (1923), p. 309) to the case when the transformations involved have for domain a topological space and for range a linear topological space (see von Neumann, Transactions of this Society, vol. 37 (1935), p. 4). (Received July 27, 1936.)

331. Mr. O. H. Hamilton: *A property of hereditarily continuous curves.*

It is shown that, in any locally compact space satisfying Axioms 0, 1, 2, 3, and 4 of R. L. Moore's *Foundations of Point Set Theory*, if H and K are mutually exclusive closed point sets, there does not exist a compact hereditarily continuous curve M (that is, a continuous curve, every subcontinuum of which is a continuous curve) which contains an infinite number of mutually exclusive continua, each containing a point of H and a point of K. (Received July 16, 1936.)

332. Professor G. A. Hedlund: *Fuchsian groups and transitive horocycles.*

The region under consideration is the interior, Ψ, of the unit circle, U, provided with the Poincaré metric. The horocycles are curves of constant geodesic curvature unity and are euclidean circles internally tangent to U. Denoting by F a Fuchsian group with principal circle, U, and considering points congruent under F identical, we obtain a two-dimensional manifold, M, of constant negative curvature, its combinatorial topological properties being determined by F. In this paper it is shown that if F is of the first kind (nowhere properly discontinuous on U) there exist transitive horocycles; that if F has a fundamental region lying, together with its boundary, interior to U, all horocycles are transitive; that if F is of the first kind and has a finite set of generators the horocycles are transitive except for a denumerable set. These results

make it possible to show that the flow defined by the geodesics on M is asymptotically transitive and permit applications to automorphic functions. This paper will appear in the Duke Mathematical Journal. (Received July 20, 1936.)

333. Professor G. A. Hedlund: *Two-dimensional manifolds and transitivity.*

Morse has shown (Journal de Mathématiques, vol. 14 (1935), pp. 49–71) that if the geodesics on a closed orientable surface, S, of genus greater than one are uniformly unstable, there exists a non-denumerable infinity of transitive geodesic rays passing through any point of the surface. By making use of results known in the case of constant negative curvature the fundamental result of Morse can be extended in two senses: (1) the hypothesis of uniform instability can be slightly weakened and (2) the surfaces under consideration can include cases of infinite genus and surfaces with singularities. This paper will appear in the Annals of Mathematics. (Received July 20, 1936.)

334. Dr. Charles Hopkins: *Concerning uniqueness-bases of finite groups with applications to regular p-groups of class* 2.

An ordered set of elements $P_1, P_2, \cdots, P_r$ is said to constitute a uniqueness-basis for a finite group G if every element of G can be expressed uniquely in the form $P_1^{x_1}P_2^{x_2}\cdots P_r^{x_r}$, $0 \leqq x_i < g_i$, where g_i is the order of P_i. Several sufficient conditions are established for the existence of a uniqueness-basis in an arbitrary finite group; and it is proved that every regular p-group G' has an ω-normal uniqueness-basis: that is, a uniqueness-basis $P_1, P_2, \cdots, P_r$, the order of P_i being p^{δ_i}, such that (1) $\delta_1 \geqq \delta_2 \geqq \cdots \geqq \delta_r$; (2) in the commutator $(P_j, P_i) = P_1^{\alpha_{ji1}} \cdots P_r^{\alpha_{jir}}$, $i<j$, each exponent α_{jik} is divisible by p for $k \leqq j$. Under the assumption that G' is of class 2 a method is developed for obtaining a true representation of G' by $(r+1)$-rowed square matrices. (Received July 22, 1936.)

335. Professor Dunham Jackson: *Orthogonal polynomials on a plane curve.*

In a recent paper (abstract 42-1-18) the writer has given an exposition of some of the fundamental properties of polynomials in two real variables orthogonal over a region of the (x, y)-plane. The present paper is concerned with relations of correspondingly fundamental character for polynomials orthogonal on a curve in the plane. The distinction between algebraic and non-algebraic curves gives rise to considerations of some interest which are not encountered in the case of a two-dimensional region. (Received July 24, 1936.)

336. Professor B. W. Jones: *On the reduction of positive quaternary quadratic forms.* Preliminary report.

If the coefficients of a positive quaternary quadratic form b are a_{ij}, $i, j=1$, 2, 3, 4, and $a_{i5} = -(a_{i1}+a_{i2}+a_{i3}+a_{i4})$ it has been shown that f is equivalent to a form of one of three types: all $a_{ij} \leqq 0$, $a_{ij}>0$ for just one pair ij, $a_{ij}>0$ for just two pairs which do not have a subscript in common (where in each of the

types $i, j=1, 2, 3, 4, 5, i\neq j$). Conditions are found which, for forms of the first type, define a *unique* reduced form. These conditions are of two kinds: first, inequalities like $a_{11}\geqq a_{22}$ which are not symmetric; second, conditions like: if $a_{ip}=a_{ij}=0$, then $|a_{in}|\alpha|a_{im}|$ and $|a_{pn}+a_{jn}|\beta|a_{pm}+a_{jm}|$, where one of α and β is $=$, both are $<$, or both are $>$. The second kind of condition is symmetric to the extent that it is independent of any permutations of the subscripts. (Received July 25, 1936.)

337. Dr. P. W. Ketchum: *On certain infinite systems of linear equations and an expansion problem of Pincherle.*

Expansions in series of the form $\sum c_n F_n(z)$, where $F_n(z)$ is a given analytic function of the form $g_n(z)+\sum_{i=1}^{\infty}\alpha_{n,i}g_{n+i}(z)$ have been studied extensively for for the case $g_n(z)=z^n$ by Pincherle and numerous later writers. We show that these expansion theorems possess analogues in the theory of the solution of certain systems of linear equations in an infinity of unknowns. Conversely, these results are used to obtain generalizations of the above expansions to cases where $g_n(z)$ no longer has the special form z^n, but may, for instance, have n zeros at a certain set of points $\beta_{n,i}$, $i=1, 2, \cdots, n$. (Received July 27, 1936.)

338. Professor Otto Laporte: *Stereographic parameters and pseudo-minimal hypersurfaces.* II.

In a previous paper it was shown that to every pseudo-minimal hypersurface there belongs a solution of a certain simple linear elliptic partial differential equation. Such solutions were called pseudo-minimal potentials and an infinity of special potentials having singularities at either the point zero or the point infinity were obtained. In this paper displaced solutions, having singularities at arbitrarily given points, are constructed through an application of the group of the differential equation. These solutions are symmetric in the coordinates of the singularity and of the point under consideration. The second part of the paper is concerned with the possibility of constructing one-sided pseudo-minimal hypersurfaces in generalization of the classical researches of Lie, Henneberg, and others. It is seen that for a pseudo-minimal hypersurface to be one-sided the potential has to fulfill a certain functional equation. Linear combinations of the particular solutions given in the previous paper are easily constructed which satisfy the functional equation. The above developments hold in spaces of any number of dimensions. (Received July 29, 1936.)

339. Dr. Walter Leighton: *On the convergence of continued fractions.*

Let $1+K_1^{\infty}[b_n/1]$ be a continued fraction in which the b_n are complex numbers $\neq 0$. Further, let $m_0, m_1, m_2, \cdots$ be any sequence of positive numbers such that $m_0=2$, $m_{n+1}-m_n\geqq 2$ $(n=0, 1, 2, \cdots)$. The numbers $|b_{m_0}|$, $|b_{m_1}|$, $|b_{m_2}|, \cdots$ can be chosen in such a fashion that excluding at most one value in the complex plane from each of the other b_n's, the continued fraction will converge. (Received July 18, 1936.)

340. Dr. Walter Leighton: *A test-ratio test for continued fractions.*

This paper presents what is believed to be the first test-ratio test for continued fractions. Let $1+K_1^{\infty}[b_i/1]$ be a continued fraction in which the b_i are complex numbers $\neq 0$. If the ratio $|b_{i+1}/b_i| \leqq k<1$ for i sufficiently large, the continued fraction converges at least conditionally. It diverges, if ultimately $|b_{i+1}/b_i| \geqq 1/k>1$. If $\lim_{i=\infty}|b_{i+1}/b_i| = 1$, the question of convergence is indeterminate. (Received July 24, 1936.)

341. Dr. Saunders MacLane: *When can a graph be mapped on a torus?*

This paper gives a strictly combinatorial condition that a given combinatorial graph be mappable on a torus by generalizing a method previously used for planar graphs (abstract 42-5-195-*t*). The method consists essentially in combinatorially characterizing the boundaries of complementary regions of a graph on a torus. First, a non-separable graph G of nullity n can be mapped on a torus if it contains a set S of $n-1$ oriented circuits such that: (1) each directed arc of G is contained in just one circuit of S; (2) any two arcs a and b both ending on a vertex q are connected by a chain of arcs $a=a_1, a_2, \cdots, a_m=b$, such that each a_i is "adjacent" to the succeeding arc a_{i+1}. Here two arcs a_i and a_{i+1} are "adjacent" if they belong to one of the circuits of S. In the second place, a non-separable graph can also be mapped on the torus if it is a planar graph or can be made planar by "splitting" one of its vertices in two. Conversely, it is then shown that any non-separable graph on a torus must satisfy one of the two combinatorial conditions stated above. This necessary and sufficient condition is extended to separable graphs. (Received July 25, 1936.)

342. Dr. H. M. MacNeille: *A normal form for the extension from a distributive lattice to a Boolean ring.*

The elements of the extension, L, to a Boolean ring of a distributive lattice, K, can be represented by finite descending sequences of elements of K, whether the extension be made by means of the symmetric difference or by Boolean addition. If K has completely distributive products, a property which is extensionally obtainable from any distributive lattice, then K is shown to be a homomorph of L under a correspondence which preserves the ordering relation and Boolean sums of L but not, in general, products or complements. By means of this homomorphism, a representative is selected from each class of equal elements in L without employing the axiom of choice. The set of these representative elements is a normal form for the extension of K to a Boolean ring. (Refer to the Proceedings of the National Academy of Sciences, vol. 22 (1936), pp. 45–50.) (Received July 28, 1936.)

343. Professor Morris Marden: *On Kakeya's problem.* Preliminary report.

This is the problem of determining the least $R>0$ such that, for every polynomial of degree n having p $(2 \leqq p \leqq n)$ zeros in or on the unit circle, the deriv-

ative has at least $p-1$ zeros in or on the circle $|z|=R$. The number R has been found for $p=n$ by Gauss and Lucas, for $p=2$ by Alexander, Kakeya, and Szegö, and for $p=n-1$ by Biernacki. In the present note it is proved that $R \leqq \csc [\pi/2(n-p+1)]$ by using a lemma that if $\alpha_1, \alpha_2, \cdots, \alpha_p$ are any $p(2 \leqq p \leqq n)$ distinct zeros of a polynomial $f(z)$ of degree n and if $\zeta_1, \zeta_2, \cdots, \zeta_q$ ($\zeta_j \neq \alpha_k$, all j and k) are any $q=n-p+1$ distinct zeros of $f'(z)$, then $\sum A^{-1}_{k_1 k_2 \cdots k_q}(\zeta_1-\alpha_{k_1})^{-1}(\zeta_2-\alpha_{k_2})^{-1}\cdots(\zeta_q-\alpha_{k_q})^{-1}=0$, the sum being taken for $k_1, k_2, \cdots, k_q = 1, 2, \cdots, p$ and $A_{k_1 k_2 \cdots k_q}$ being equal to the number of different permutations of the q numbers $k_1, k_2, \cdots, k_q$ taken q at a time. The author hopes both to sharpen the limits given and to extend the results to higher derivatives. (Received July 26, 1936.)

344. Professor W. H. McEwen: *An extension of Bernstein's theorem associated with general boundary value problems.*

This paper deals with sums $S_n(x)=\sum_{k=1}^{n}(a_{1k}u_{1k}(x)+a_{2k}u_{2k}(x))$, in which the u_{ik}'s are characteristic solutions of $L(u)+\lambda u=0$, $U_j(u)=0$ $(j=1, 2, \cdots, m)$, a given mth order linear differential system with regular boundary conditions on $a \leqq x \leqq b$, and the a_{ik}'s are arbitrary constants. If the maximum of $|S_n(x)|$ on (a, b) be denoted by L, it is shown that (1) $|S_n'(x)| \leqq qn^2L$ on $a \leqq x \leqq b$, and (2) $|S_n'(x)| \leqq QnL$ on $a+\epsilon \leqq x \leqq b-\epsilon$, q and Q being constants independent of n, the latter depending on ϵ. Applications of these results are made to certain least rth power problems of best approximation. (Received July 9, 1936.)

345. Professor E. J. McShane: *Semi-continuity of integrals in the calculus of variations.*

A theorem on the semi-continuity of single integrals of the calculus of variations is established under hypotheses weak enough so that the ordinary problem, the parametric problem, the Lagrange problem in ordinary and in parametric form, and the parametric problem associated with an ordinary problem all are contained as special cases. In addition, this theorem enables us to establish the existence of minimizing curves for several problems (for example, the Zermelo navigation problem) not covered by theorems in the literature. (Received August 3, 1936.)

346. Professor A. D. Michal: *The tensor calculus in a general Riemannian differential geometry.*

In this paper the author continues his investigations on general Riemannian geometry with Banach coordinates. The laws of transformation of the abstract Christoffel symbol and its adjoint are derived and the resulting absolute differential calculus is then studied with the aid of the author's more general absolute calculus in non-Riemannian geometry. (Received July 27, 1936.)

347. Professor A. D. Michal, Dr. I. E. Highberg, and Dr. A. E. Taylor: *Abstract euclidean spaces.* V: *Rotations in spaces with indefinite geometrical metric.*

The work of Michal and Elconin on abstract transformation groups with abstract parameters is used to characterize rotation groups in several func-

tional spaces with an indefinite geometrical metric by means of completely integrable differential equations in Fréchet differentials. Tonelli's recent work on non-linear functional equations is employed in the proof of the existence of non-linear rotations in a complete functional space with an indefinite geometrical metric. (Received July 27, 1936.)

348. Professor A. D. Michal and Mr. D. H. Hyers: *Theory and applications of abstract normal coordinates in a general differential geometry.*

One of the authors has initiated the study of general differential geometry with a linear connection in which the customary arithmetic coordinates of n-dimensional geometry are replaced by elements of a Banach space (A. D. Michal, Proceedings of the National Academy of Sciences, vol. 21 (1935), pp. 526–529, and several forthcoming papers in Acta Mathematica, Annali di Matematica, this Bulletin, and Compositio Mathematico). In this paper we define abstract normal coordinates and prove their existence with the aid of a theorem on second-order differential equations in Banach spaces. This theory does not make use of any abstract power series expansions so that many of the recent results of Veblen, Whitehead, and T. Y. Thomas on normal coordinates in n-dimensional geometry may be obtained as special cases of our theorems. Theories of abstract "normal tensors" and of extensions of abstract tensor forms are developed. A theory of adjoints and their differentials, although trivial in the n-dimensional case, plays an important role here and in all infinitely-dimensional instances. In case the Banach space is a space of continuous functions, this paper gives new results in a Michal functional geometry. (Received July 27, 1936.)

349. Professor C. N. Moore: *On convergence factors in restrictedly convergent double series.*

The origin and subsequent development of the notions of restricted convergence and restricted summability of multiple series have been outlined in the abstract of a paper presented at the last annual meeting in St. Louis (abstract 42-1-34). In the present paper necessary and sufficient conditions are found for convergence factors in restrictedly convergent double series. For convergence factors of type I (Transactions of this Society, vol. 29 (1927), p. 227) no modifications are needed in the necessary and sufficient conditions for the case of series convergent in the general sense. For convergence factors of type II (l.c.) two supplementary conditions are required. In the first of these conditions it is stipulated that the convergence factors $f_{ij}(\alpha)$, defined over a set $E(\alpha)$ with a limit point α_0, not of the set, be such that the upper limit, as α approaches α_0, of $\sum_{j=0}^{\infty}\sum_{i=0}^{[\theta j]}|\Delta_{11}f_{ij}(\alpha)|$ tend to zero as θ approaches zero. The other condition is the analogue of this, with the roles of i and j interchanged. The results indicated include as special cases two theorems due to Lösch (Mathematische Annalen, vol. 110 (1934), pp. 33–53; Theorems I and II). (Received July 14, 1936.)

350. Professor F. D. Murnaghan: *A symmetric reduction of the planar three body problem.*

If a, b, c denote the sides of the triangle formed by the three bodies, of masses m_1, m_2, m_3, and θ_1, θ_2, θ_3 the inclinations of the sides to a fixed line in the plane of the three bodies, the dynamical system has four degrees of freedom when the center of mass is taken as the origin of an inertial reference frame and a, b, c, and $\phi=\frac{1}{3}(\theta_1+\theta_2+\theta_3)$ may be taken as the coordinates of the system; ϕ is ignorable and if k is the corresponding momentum (angular) constant, a simple calculation yields the following expression for the modified (Routh) Hamiltonian function H^*: $H^*=\sum(1/(2m_1))[\omega_2^2+\omega_3^2+2(\omega_2\omega_3+k^2/(bc))\cos A+2k(\omega_2/b-\omega_3/c)\sin A+k^2(1/b^2+1/c^2)]-U$, where ω_1, ω_2, ω_3 are the momenta corresponding to the coordinates a, b, c and $U=-r[(m_2m_3)/a+(m_3m_1)/b+(m_1m_2)/c]$ is the force function. (Received July 24, 1936.)

351. Professor F. D. Murnaghan: *On universally valid stress-strain relations for an elastic solid possessing an energy of deformation.*

The general relations connecting the stress and strain tensors are derived without making the hypothesis that the strain is infinitesimal. The classical relation $P^{rs}=\partial\phi/\partial\epsilon_{rs}$, where P is the stress and ϵ the strain tensor, is shown to hold only for infinitesimal strains; the generally valid formula which reduces to the above classical relation when the strain is infinitesimal is given. For an isotropic medium it is $P^{rs}=\partial\phi/\partial\epsilon_{rs}-2\epsilon_\alpha^r(\partial\phi/\partial\epsilon_{rs})$. A numerical application to the theory of metals under high hydrostatic pressures and the theory of Young's modulus for large extensions is given. (Received July 24, 1936.)

352. Mr. F. J. Murray: *Complementary manifolds in L_p and l_p, $1<p\neq 2$.*

If Λ is a Banach space, $\mathfrak{M}$ a closed linear manifold in Λ, a closed linear manifold $\mathfrak{N}$ such that every $f\epsilon\Lambda$ is uniquely expressible in the form $g+h$, $g\epsilon\mathfrak{M}$, $h\epsilon\mathfrak{N}$, is called a complementary manifold to $\mathfrak{M}$. In his treatise on *Linear Operations*, Banach presents the following problems. (a) To every $\mathfrak{M}$ in L_p does there exist a complementary manifold? (b) To every $\mathfrak{M}$ in l_p does there exist a complementary manifold? In this paper it is shown that the answer to both questions is "No." (Received July 24, 1936.)

353. Professor John von Neumann and Dr. I. J. Schoenberg: *Fourier integrals and metric geometry.*

Let $\phi(t)$ $(-\infty<t<\infty)$ be a continuous function with functional values in a metric space γ with the distance function $d(\phi,\psi)$. If $d(\phi(t),\phi(t'))=\sigma(t-t')$ is a function of the difference $t-t'$ only, we call the curve Γ: $\phi(t)$ a screw curve (s.c.) of γ and $\sigma(t)$ a screw function (s.f.) of γ, since Γ admits the one-parameter group $t\to t+\tau$ of rigid displacements within itself. W. A. Wilson (American Journal of Mathematics, vol. 57 (1935), pp. 63–64) has shown that $\sigma(t)=|t|^{1/2}$ is a s.f. of Hilbert space $\mathfrak{Z}$. We prove: (i) The most general s.f. of $\mathfrak{Z}$ is given

by the equation $\sigma^2(t)=\int_0^\infty(\sin tu/u)^2 d\gamma(u)$, where $\gamma(u)(u\geqq 0)$ is non-decreasing and such that $\int_1^\infty u^{-2}d\gamma(u)$ exists. (ii) The most general s.f. of the unit-sphere of $\mathfrak{S}$ is given by the equations $\cos\sigma(t)=g(t)$, $0\leqq\sigma(t)\leqq\pi$, where $g(t)=\int_0^\infty \cos tud\alpha(u)$, $\alpha(u)$, non-decreasing, is a real positive definitive function with $g(0)=1$. The s.f. of euclidean spaces and finite-dimensional spheres are obtained, if $\gamma(u)$ and $\alpha(u)$ have a finite number of points of increase only. (iii) Obviously (ii) must be a special case of (i). It is characterized by the finiteness of $\int_0^\infty u^{-2}d\gamma(u)$, or, equivalently, by the boundedness of $\sigma(\tau)$. (ii) may readily be derived from known results; (i) and (iii) necessitate an investigation of the continuous one-parameter linear groups of rigid displacement of $\mathfrak{S}$ within itself. (Received July 20, 1936.)

354. Professor Gordon Pall: *Applications of the rational automorphs of* $x^2+y^2+z^2$. Preliminary report.

The theory of certain relations among the solutions of $n=x^2+y^2+z^2$ is studied. The known formula for $r_3(nm^2)/r_3(n)$ is derived by a simple method. New properties of integral quaternions are obtained. (Received July 25, 1936.)

355. Mr. E. W. Paxson: *The integral in abstract linear topological spaces*. Preliminary report.

This paper continues the discussion of the analysis of linear non-metric topological spaces instituted with studies of the differential by A. D. Michal and the author. (See Comptes Rendus, vol. 202 (1936), pp. 1741–1743, and abstract 42-5-137 and a paper offered to the Fundamenta Mathematicae.) The "Riemann" integral is defined and developed for functions on the real numbers to the space L. The usual classical theorems of manipulation are verified. Some important topological facts for the linear space L are established with the aid of which the existence of a unique primitive solution (the indefinite integral) for the equation $F'(\mu)=f(\mu)$ is demonstrated. (Received July 27, 1936.)

356. Dr. Hillel Poritsky: *A problem of replacement.*

Suppose that a new installation requiring for its operation a constant number N_0 of units of a particular article—say thyratron tubes—is put into operation at the time $t=0$. The replacement problem leads to the integral equation $r(t)=N_0p(t)+\int_0^t r(s)p(t-s)ds$, where $r(t)$ is the probable rate of replacement, and $p(t)$ is the life expectancy of a unit, that is, the probability of a unit failing (a tube burning out) after it has been in operation a time t. More convenient than the classical Volterra solution is the operational method of solution which consists in replacing $r(t)$, $p(t)$ in the above equation by Laplace or Carson integrals. After solving the resulting equation for $\rho(p)$, we determine r by the Bromwich integral. Of practical interest is the case $p(t)=1$ for $0<t<h$ and $p(t)=0$ elsewhere. In this case the above integral equation may be replaced by a difference-differential equation. The operational method readily yields an expansion for $r(t)$ in terms of the characteristic functions of this equation. Similar solutions are obtainable for cases where $p(t)$ is a step function or is represented by polynomials in successive intervals. (Received July 25, 1936.)

357. Dr. Hillel Poritsky: *On finger print patterns and their singular points.*

This note contains a study of finger (also of palm) print patterns, particularly in regard to the singular points of these curve families and their indices. The index for each finger segment is zero. At the terminal segments there is one singular point with a positive index of either 1/2 or 1. In the former case there is also one singular point of index $-1/2$; in the latter case there are two such points. A speculation on the mode of origin of the patterns connecting them with lines of maximum curvature is discussed. (Received July 25, 1936.)

358. Dr. Hillel Poritsky: *Thermal stresses in cylindrical tubes.*

The problem considered is that of stresses due to a two-dimensional steady state temperature distribution. The elastic and thermal coefficients, as well as the coefficient of expansion k are assumed to be constant. After a uniform expansion and bending have been allowed for, a solution of the thermoelastic equations is given by $w=0$, $u+iv=\int\phi(x+iy)d(x+iy)$, where (u, v, w) are the displacement components in the x, y, z directions, and ϕ is the analytic function whose real part is kT, T being the temperature. For solid cylinders the above solution is satisfactory and results only in axial stresses. For pipes, however, owing to the multiple connectivity, the above displacement is multiple-valued and increases by a rigid rotation for a path enclosing the hole. The correcting state of stress, known as a "dislocation," is determined, and expressions are derived for the Airy's functions corresponding to these dislocations, in terms of analytic functions. (Received July 25, 1936.)

359. Professor Hans Rademacher: *Dedekind's ζ-function and Hecke's $\zeta(s, \lambda)$-functions in totally real algebraic fields.*

The sum $\Phi(a, s)=\sum_{\mu}N(a+i\mu)^{-s}$ for real $a_1, a_2, \cdots, a_n>0$, $R(s)>1$, where μ runs through all integers of a given totally real algebraic field, is treated in two different ways, one of which includes a Fourier development with respect to the point lattice of all integers of the field. Comparison of both results yields an identity which contains the functional equations of the Dedekind ζ-function and of all Hecke $\zeta(s, \lambda)$-functions simultaneously. (Received July 27, 1936.)

360. Professor Hans Rademacher: *On Waring's problem in algebraic fields.*

Hardy-Littlewood's method has already been applied to certain problems of additive number theory in algebraic fields. But Waring's problem in algebraic fields still offered a particular difficulty, arising from the "minor" Farey sections, since no analogue of Weyl's method of approximation was known. In the present paper this analogue is developed for totally real, in particular, real quadratic, fields. The generalized finite geometric series to which by Weyl's procedure a certain sum of roots of unity is reduced necessitates a Fourier expansion which can be avoided in the rational case. (Received July 27, 1936.)

361. Dr. W. T. Reid: *Sufficient conditions by expansion methods for the problem of Bolza in parametric form.*

In a recent paper (abstract 41-11-379) the author has obtained, using the expansion method introduced by E. E. Levi, sufficient conditions for a strong relative minimum in the non-parametric problem of Lagrange in the calculus of variations. The expansion method is used in the present paper to prove a sufficiency theorem for a strong relative minimum in the problem of Bolza in parametric form. The hypotheses of the theorem do not involve any assumptions of normality, and the results of this paper are essentially the same as those recently obtained by Hestenes [American Journal of Mathematics, vol. 58 (1936), pp. 391–406] using the classical field construction methods of the calculus of variations. (Received July 27, 1936.)

362. Mr. H. E. Robbins: *On a class of recurrent sequences.*

Recurrent sequences have been defined by Birkhoff (*Dynamical Systems*, p. 246) and Morse (Transactions of this Society, vol. 22 (1921), p. 95). The present paper contains a definition of a class of recurrent, non-periodic sequences which prove the existence of a r.n.p. sequence whose ergodic function is bounded above by a linear function, and of r.n.p. sequences whose ergodic functions increase arbitrarily rapidly. (Received July 27, 1936.)

363. Dr. R. M. Robinson: *The theory of classes: a modification of von Neumann's system.*

The theory of classes presented in this paper is a simplification of that presented by J. von Neumann in his paper *Die Axiomatisierung der Mengenlehre* (Mathematische Zeitschrift, vol. 27 (1928), pp. 669–752). As in his paper, *function* rather than *class* is taken as the fundamental idea; the terms *function* and *argument*, and the operator $[f, x]$ (value of the function f for the argument x), are primitive concepts of the system. In von Neumann's paper, the idea *ordered pair* was an additional primitive concept; here it is defined in terms of the others. Also, the present paper gives a much simpler proof of the well-ordering theorem based on von Neumann's *equivalence* axiom, which stated roughly is: *A function is an argument if and only if it is defined for fewer arguments than there are arguments in all.* The central idea of the proof is a simple characterization of ordinal numbers, which makes it unnecessary to develop a theory of ordering. (Received July 9, 1936.)

364. Professor Robin Robinson: *A condition in invariant form for a net without detours.*

The term *net without detours* is used here as a translation of the German *Kurvennetz ohne Umwege*. A necessary and sufficient condition for such a system of curves on a surface has been given by Rothe (Sitzungsberichte der Berliner Mathematischen Gesellschaft, 7, (1908), pp. 14–15). The condition given in this note is equivalent to that of Rothe, but it is written in invariant form, and is derived in an entirely different manner. (Received July 6, 1936.)

365. Professor I. M. Sheffer: *A simplified solution of equation* $\Delta y = F(x)$.

We have elsewhere considered, from the point of view of a local solution, the equation (1) $\Delta y = F(x)$ and certain related equations. It has now been found possible to simplify the form of the solution of (1) by altering the method of solution. This is the purpose of the present note. It is believed that the new method may be effective for other equations as well, although this has not as yet been carried out in detail. (Received July 25, 1936.)

366. Professor L. L. Silverman: *On products of Nörlund transformations.*

Inasmuch as the ordinary product of two Nörlund transformations is in general not of the Nörlund type of summability it has been found desirable to introduce a new definition of product. These two definitions of product are not in general equivalent. The object of the present paper is to study the relations of these two definitions of product of Nörlund transformations. (Received July 6, 1936.)

367. Professor A. E. Staniland: *Analytic affine transformations in euclidean space of* $2n$ *dimensions.*

A euclidean space, S_{2n}, is employed as a real representation of a complex linear n-space, Σ_n, the real point $(x_1, x_2, \cdots, x_{2n})$ corresponding to the complex point $(z_1, z_2, \cdots, z_n)$, with $z_k \equiv x_{2k-1} + ix_{2k}$. Subjecting the real coordinates to the general affine transformation and insisting that the generalized Cauchy-Riemann equations continue to hold for all complex functions initially analytic we obtain necessary and sufficient conditions that the transformation be analytic. The analytic affine transformations of S_{2n} correspond exactly to the complex affine transformations of Σ_n, $A: z_k = \sum A_{kj} z_j' + B_k$ and $\bar{A}: \bar{z}_k = \sum A_{kj} z_j' + B_k$, $|A_{kj}| \neq 0$. The unitary transformation U of type A reduces to $U = T^{-1} U_0 T$ where T, unitary of type A, is used to effect a change to new coordinate analytic planes, and $U_0 = \pi R_k$, $R_k: z_k = e^{i\phi_k} z_k'$, $z_h = z_h'$, $h \neq k$ $\bar{U}$ is obtained from U by replacing z_k by $\bar{z}_k$. The A_{kj} are exhibited as rational functions of the complex parameters, together with their conjugates, which determine the planes, and as transcendental functions of the ϕ_k. Moreover, ϕ = amplitude $|A_{kj}| = \sum \phi_k$. (Received July 24, 1936.)

368. Mr. Alvin Sugar: *Ideal Waring theorem for the polynomial* $m(x^3 - x)/6 - m(x^2 - x)/2 + x$.

As a consequence of a theorem obtained in a previous paper (to appear soon in the American Journal of Mathematics) by the author, we know that every positive integer is a sum of $m+3$ values of $G(x) = m(x^3 - x)/6 - m(x^2 - x)/2 + x$ for $m \geqq 16$. This result, however, is not ideal. In this paper we prove the ideal theorem for $G(x)$: *Every positive integer is a sum of* $[(m+1)/2] + 3$ *values of* $G(x)$ *for* $m \geqq 42$. (Received July 27, 1936.)

369. Dr. A. E. Taylor: *A generalization of the Cauchy-Riemann equations.*

In two previous reports on analytic functions in general analysis (abstracts 41-11-424 and 42-3-147) it is shown that a large portion of the Cauchy-Weierstrass theory of analytic functions may be generalized for functions on one complex vector space to another, the second space being complete. The foregoing results may then be applied to pairs of "conjugate" functions of two variables ranging over a *real* vector space, by forming a complex space of couples, in exactly the same way that the complex numbers are formed by pairs of reals. As a consequence we obtain necessary and sufficient conditions for pairs of conjugate functions in the form of generalized Cauchy-Riemann equations in Fréchet differentials. (Received July 20, 1936.)

370. Dr. C. C. Torrance: *On product-continuous sets.*

The object of this paper is to define, and to develop some of the properties of, product-continuous sets in topological spaces. (Received July 28, 1936.)

371. Professor W. J. Trjitzinsky: *Non-linear difference equations.*

In the present work an investigation is given of the analytic character of solutions (in the neighborhood of the singular point $x=\infty$) of the non-linear nth order difference equation $y(x+n)=x^{p/q}a(x, y(x), y(x+1), \cdots, y(x+n-1))$ (integers $p, q; q\geqq 1$), the function $a(x, z_0, z_1, \cdots, z_{n-1})$ being analytic in $z(=x^{1/q})$, $z_0, z_1, \cdots, z_{n-1}$ at $(z=\infty, z_0=z_1=\cdots=z_{n-1}=0)$ while $a(x, 0, 0, \cdots, 0)\equiv 0$. At the basis of this work lies a new method due to W. J. Trjitzinsky for treatment of non-linear problems (in the fields of difference and differential equations) and introduced by this writer for the first time in a memoir on differential equations which will appear in the Mémorial des Sciences Mathématiques. (Received July 23, 1936.)

372. Professor A. W. Tucker: *Mappings and products of cell spaces.*

Following a previous paper (Annals of Mathematics, vol. 37 (1936), pp. 92–100) which dealt with cell spaces and their closed and open mappings, this paper shows how these mappings may be characterized in terms of products. A closed mapping $S\rightarrow S'$ of one cell space on another is represented on the product $S^*\times S'$ (S^* = dual of S) by a sub-set whose left and right closures are identical. If S and S' are complexes, the algebraic counterpart of a mapping is represented by a chain on $S^*\times S'$; this chain is a cycle when the mapping is closed. If $S\rightarrow S'$ is a single-valued mapping and S' has closure uniformity, the common left and right closure mentioned above is an "expanded" image of S^*. S itself has as subdivided image a sort of intersection complex derived from $S\times S'^*$. Open mappings lead to dual considerations. (Received July 28, 1936.)

373. Professor H. S. Wall: *On hypergroups.*

The concept of hypergroup given in an earlier communication is enlarged. The product of two elements is a complex of n elements of the system. Here n

is a fixed integer $\geqq 1$. If $n=1$ the hypergroup is an ordinary group. The associative law holds; there is at least one element e such that ea and ae contain a (at least once) for every a; corresponding to an arbitrary element a there is at least one element a^{-1} such that aa^{-1} and $a^{-1}a$ contain e. Certain hypergroups are related to the Mischgruppe of Loewy and Baer. They include a large class of multiplicative systems derived from an ordinary group. About thirty-five theorems are given, many of which are analogs of theorems for ordinary groups. (Received July 21, 1936.)

374. Professor H. S. Wall: *On the nth derivative of $f(x)$.*

This note gives explicitly the coefficient of $ff_1^a f_2^b \cdots f_n^c, (a+2b+\cdots+nc=n)$, in the polynomial expression for $f^{(n)}(x)$ in terms of f and $f_1, f_2, \cdots, f_n$, the first n derivatives of $\log f(x)$. The coefficient is $n!/(a!b!\cdots c!(1!)^a(2!)^b\cdots(n!)^c)$. (Received July 21, 1936.)

375. Professor Morgan Ward: *Arithmetical functions on rings.*

Let $\mathfrak{o}$ be a ring, Σ a structure (lattice), and ϕ a function on $\mathfrak{o}$ to Σ. ϕ is arithmetical if ϕ_a divides ϕ_b in Σ whenever a divides b in $\mathfrak{o}$. This paper presents a general investigation of the properties of various types of arithmetical functions which clarifies and extends earlier work of the author and others on linear recurring series and divisibility sequences. As a sample result, let S be any element of Σ, and assume that the aggregate of elements s of $\mathfrak{o}$ such that S divides ϕ_s form an ideal $\mathfrak{s}=\rho_s$ of $\mathfrak{o}$, "the rank of apparition of S in ϕ". Then ρ is an arithmetical function on Σ to $\mathfrak{o}$. If furthermore A, B are any two divisors of ϕ with ranks of apparition $\mathfrak{a}$, $\mathfrak{b}$, and if the cross-cut M of A, B is also a divisor of ϕ, then the cross-cut of $\mathfrak{a}$, $\mathfrak{b}$ is ρ_M. (Received July 25, 1936.)

376. Dr. J. F. Wardwell: *Concerning separating transformations which are entirely complete.*

A continuous transformation $T(A)=B$ which is not (1-1) will be called separating provided that every set $T^{-1}(b)$, $b\epsilon B$, separates A. Such a transformation T will be called completely separating if every $T^{-1}(b)$, $b\epsilon B$, separates every $T^{-1}(p)$, for $p\epsilon B-b$, in A. Finally, any completely separating transformation T will be called entirely complete provided that, for any possible separation $A-T^{-1}(b)=A_1+A_2$, $T^{-1}(p)\cdot A_1\neq 0$ and $T^{-1}(p)\cdot A_2\neq 0$, for every $p\epsilon B-b$. This paper deals mainly with the latter type of transformation. If A is a compact continuum and $T(A)=B$ is continuous, some necessary and sufficient conditions in order that T be entirely complete are obtained. Furthermore if T is entirely complete, it is shown that B will contain no cut points. Hence, if A is locally connected, B will be cyclicly connected. It is also shown that, in this case, A will contain at least one true cyclic element and, under certain conditions, will contain some true cyclic element E_a such that $T(E_a)=B$. Among various other results in this paper are those obtained by the application of such transformations to some particular curves and surfaces. (Received July 27, 1936.)

377. Mr. A. H. Wheeler: *A general formula for polyhedra with applications to higher dodecahedra and icosahedra.*

In a previous paper (see Proceedings of the International Mathematical Congress, Toronto, 1924, vol. I, pp. 701–708) the author defined and classified high icosahedra by means of lines bounding the faces of the solids. The purpose of the present paper is to provide a general formula which will completely define all polyhedra independently of relations of symmetry, as in the previous paper. Applications are readily made to higher forms of solids, including those having two or more kinds of faces as in the case of the Archimedean solids. (Received July 22, 1936.)

378. Professor Hassler Whitney: *On regular closed curves in the plane.*

Let C be a closed curve with continuously turning tangent, with any singularities. Let γ be the total angle through which the tangent turns while traversing the curve. It is shown (theorem and a different proof suggested by W. C. Graustein) that two curves with the same γ may be deformed into each other. If one follows the curve, starting from an "outside point," and counts the numbers N^+ and N^- of times that one crosses a part of the curve already traced in the "positive" and "negative" sense, then $\gamma = 2\pi[\mu + (N^+ - N^-)]$, where $\mu = \pm 1$ according to the orientation of the curve. (Received July 20, 1936.)

379. Mr. Reinhard Korgen: *An extension of the concept called language.*

This paper distinguishes between "articulate" and "inarticulate" *languages*, and describes as an example of an inarticulate language a "language of materials" which does not require man as an agent for its use. It is used by man *in combination* with articulate languages to form a "language of thinking," and is thus also used by man to communicate ideas. Implications bearing on the foundations of mathematics will be considered. (Received July 27, 1936.)

380. Professor Alonzo Church and Dr. S. C. Kleene: *Constructions of formal definitions of functions of ordinal numbers.*

In an article entitled *Formal definitions in the theory of ordinal numbers* (forthcoming in the Fundamenta Mathematicae) the authors have set up definitions extending the theory of definition to transfinite ordinals and have constructed λ-definitions of some ordinals and functions of ordinals. In the present paper it is shown how the construction can be carried out under more general circumstances. In particular it is proved that Veblen's generalizations of the Cantor ϵ-numbers are λ-defineable (a result previously stated without proof). Further an explicit proof is given that the equality of ordinals is not λ-defineable; also that no function is λ-defineable which associates with every λ-defineable ordinal a unique enumeration of the class of lesser ordinals. A definition of *recursive ordinal*, suggested by Gödel, is introduced, but, probably because of the difficulty of finding a definition of *recursive function of ordinals*, the notion of recursiveness fails to play the role that it does in connection with λ-definition in the theory of positive integers. (Received July 22, 1936.)

381. Mr. E. C. Berkeley: *Two practical problems requiring techniques from symbolic logic.*

This paper outlines two problems arising in group insurance and in beneficiary settlements where methods from symbolic logic would prove of considerable use. (Received July 22, 1936.)

382. Professor Paul Weiss: *The self-contradictory.*

The supposition that the self-contradictory is a proposition, obtainable by substitution and by negating a necessary truth is beset by six difficulties. 1. Necessary truths cannot, by definition, be denied. 2. There are many necessary truths and only one self-contradictory, though each proposition has its own denial. 3. Since $p \cdot q$ might be true and $p \cdot -p$ cannot be true, to substitute $-p$ for q is to obtain a necessary falsehood from a possible truth by a legitimate act of substitution. 4. The disjunction in $p \vee -p$ is different from the disjunction in $p \vee q$, and the conjunction in $p \cdot v - p$ must be different from the conjunction $p \cdot q$. Yet there is only one kind of "and" in extensional logic. 5. The self-contradictory negates and yet entails the necessary. But a conclusion must be consistent with its premiss. 6. The antecedent contains the meaning of its consequent. But if a self-contradictory proposition entails a necessary proposition, a part will be incompatible with its whole. A solution is offered in the shape of the contention that negated elements are values of the terms of a necessary truth, and that a so-called self-contradictory has an indeterminate subject and is not a proposition at all. This entails the denial of the generality of the law of transposition, but not of De Morgan's theorem or of the *reductio ad absurdum.* (Received July 22, 1936.)

383. Dr. A. F. Emch: *The calculus of logical implication.*

It is the purpose of this paper to show that "the calculus of logical implication" proposed in the first two issues of The Journal of Symbolic Logic is such that (a) its properties coincide with the properties of deducibility, (b) the paradoxical theorems of strict as well as of material implication are explained, (c) logical and mathematical propositions can be both consistent and independent, and (d) the entire calculi of strict and material implications can be derived from it. The paper is developed under the following rubrics: *Introduction*; §1. *The primitive ideas, definitions, postulates and rules of operation*; §2. *C properties* (theorems which do not include either explicitly or by definition the primitive idea of "possibility"); §3. *The calculus of material implication*; §4. *S properties and the calculus of strict implication* (theorems which do not include, either explicitly or by definition, the primitive idea of "logical consistency" but do include, either explicitly or by definition, the primitive idea of "possibility"); §5. *L properties, consistency and the modal functions* (theorems which include, either explicitly or by definition, both the primitive ideas "logical consistency" and "possibility"); §6. *Existence principles*; and §7. *Conclusion* (in which four optional postulates are suggested for further consideration). (Received July 22, 1936.)

384. Dr. L. O. Kattsoff: *Group theory for truth values.*

Aristotelian logic is now seen to be a logic of two truth values as is also the Russell-Whitehead logic. Lukasiewicz in conjunction with Tarski published in 1930 a set of postulates for a logic of four truth values. Becker has also developed postulates for logic for six and twelve truth values. The indications today are towards an ever-increasing number of truth values. This paper makes a first attempt to develop not the properties of a logic having a given number of truth values, but the properties of the truth values themselves. A truth value is treated as the result of an operation upon a group of symbols. Hence, it is possible to speak either of the operator or of the result of the operation. The concepts of group theory are applied to these operations and certain theorems are proved which indicate two important results. (1) Every logic based on a set of truth values not forming a group is bound to break down. (2) The two-value logic has been the most successful because its truth values do form a group and there are indications that they are a subgroup of a larger group. (Received July 22, 1936.)

385. Professor Paul Henle: *The algebras of concentric sectors.*

Boolean algebra is intimately connected with the number two. It contains 2 unique elements; it has a law of double negation; and the number of its elements, in finite cases, is a power of 2. This paper presents a generalization of these characteristics, yielding a calculus containing n unique elements, a law of n-tuple negation, and containing, in finite cases, a number of elements which is a power of n. When $n=2$, Boolean algebra results; but for each $n\neq 2$, there is a new algebra. All these algebras may be developed in terms of two binary operators, "$+$" and "$\times$". Any law of Boolean algebra composed only of the symbols, "$+$", "$\times$", "$=$", and real variables holds for all of them. They are capable of a spatial interpretation, their elements being represented as sectors or combinations of sectors of concentric circles; the number of concentric circles being determined by n. In every algebra, $a+b$ may be represented by the area included either in a or b and $a\times b$ may be represented by their common area. Each algebra contains a unary function, definable in terms of $+$ and $\times$, corresponding to the negation of Boolean algebra, and it is chiefly in respect to laws involving this function that the algebras differ. (Received July 22, 1936.)

386. Dr. F. B. Fitch: *A formal theory of types.*

Instead of introducing types into a system of formal logic by way of informal conventions or special rules of procedure, a method of introduction is proposed which consists of the mere addition of a finite number of formal axioms. The formal system to which these axioms are added employs material implication, propositional negation, universal, quantification, and operators analogous to the combinatory operators I, B, and C; but it differs from combinatory logic in not employing an analog to the operator W. The effect of the additional axioms is to introduce an infinite sequence of analogs to the W-operator, W^1, W^2, W^3, $\cdots$, such that a function not involving W^m for any $m\geqq n$ would be "of nth type," and such that if θ is of nth type, then

$W^n(\phi, \theta)$ and $\phi(\theta, \theta)$ would be materially equivalent propositions in the logic. The Russell paradox is still avoided; because although a ψ can be found such that, for all θ of nth type, $\sim\theta(\theta)$ would be materially equivalent to $\psi(\theta)$, yet ψ would have to be of higher than nth type. The other paradoxes of Mengenlehre may be analogously treated. Much of number theory seems to be deducible from the resulting formal system. (Received July 22, 1936.)

387. Professor Rudolf Carnap: *Truth in mathematics and logic.*

First problem. What is the nature of the truth of mathematical and logical theorems? We have to distinguish between factual truth and formal truth. A sentence of empirical science is true or false according to whether the (possible) fact asserted by it subsists or not. A sentence of the language of mathematics and logic does not assert a fact. Such a sentence is true or false according as the rules of the language-system state it to be true (analytic) or false (contradictory). *Second problem.* How to construct a system of mathematics, that is, how to define the concept "true mathematical sentence"? (a) The customary *method of demonstration* consists in laying down some primitive sentences (postulates) and some rules of inference (with a finite number of premises). Example: The system of Principia Mathematica. But, according to Gödel's results, such a system is always incomplete. (b) Therefore we are compelled to apply *rules of a new kind* referring to infinite classes of premises. By rules of this kind the concept of the truth of universal sentences concerning integers can easily be defined; but for sentences concerning real numbers, that is, classes of integers, certain difficulties arise. (Received July 22, 1936.)

338. Professor Richard Courant: *Conformal mapping and Plateau's problem.*

The connection between Plateau's problem and the problem of conformal mapping has been emphasized already by Radó and Douglas, the latter of whom has stressed the point that the solution of Plateau's problem should be considered as a natural way to solve problems of conformal mapping.

The present paper intends to show that, by using freely the classical theory of conformal mapping and a certain generalisation of it, one obtains a very simple solution not only of the problem of Plateau, but also of the widest generalisation of it by Douglas and also of the more general problems in which parts of the contours are free to move on prescribed manifolds. The details of this theory (see two Notes in the Proceedings of the National Academy of Sciences for June, 1936) will be published in the Annals of Mathematics together with another solution of the problems of Plateau, independent of conformal mapping. (Received August 24, 1936.)

389. Mr. Philip Hall: *Solvable groups.*

A group of order g is solvable if and only if it has at least one subgroup of order m, for every divisor m of g which is prime to g/m. Let G be solvable of order $p_1^{\alpha_1} \cdots p_r^{\alpha_r}$, ($p_1, \cdots, p_r$ being distinct primes). Then G possesses a "Sylow system" of subgroups $S_1, \cdots, S_r$ with the properties (*i*) S_i is of order

$p_i^{\alpha_i}$, and (*ii*) $S_iS_j=S_jS_i(i, j=1, \cdots, r)$. Any two Sylow systems are conjugate, and the Sylow systems of any subgroup H of G can all be obtained as meets of H with suitably chosen Sylow systems of G. Let $G=G_0>G_1> \cdots >G_k=1$ be any chief series of G. Call G_i/G_{i+1} *central* or *eccentric* according as it does or does not belong to the central of G/G_{i+1}. Then the number of distinct Sylow systems in G is the product of the orders of the eccentric factors G_i/G_{i+1}. More precisely: Let N be the normalizer of any Sylow system $S_1, \cdots, S_r$ of G (that is, the meet of the normalizers of the S_i). Then, if G_i/G_{i+1} is central, the join of N with G_{i+1} contains G_i; while if G_i/G_{i+1} is eccentric, the meet of N with G_i lies in G_{i+1}. If the S_i are all abelian, the meet of N and the derived group G' of G is 1, and their join is G. (Received August 11, 1936.)

390. Dr. A. N. Lowan: *On some two-dimensional problems in heat conduction.*

This paper deals with the problem of heat conduction in an infinite parallelopiped bounded by the planes $y=0$ and $y=b$ which are assumed to be impervious to heat and by the planes $x=0$ and $x=a$ for which the expressions $\alpha\partial T/\partial x+\beta T$ are prescribed functions of y and t, the initial temperature distribution being $f(x, y)$, the solid containing a continuous distribution of heat generating sources described by the function $\Psi(x, y, t)$. The method employed is the two-dimensional extension of that presented in a number of earlier papers. (A. N. Lowen, *Note on the cooling of a radioactive sphere*, American Journal of Mathematics, vol. 56 (1934), No. 2; *Heat conduction in a semi-infinite solid*, American Journal of Mathematics, vol. 56 (1934).) With the aid of the Laplace transform the problem is reduced to the integration of certain ordinary differential equations and the final solution is obtained by the inversion of the Laplace transform, frequent use being made of Borel's theorem and of other theorems in the operational calculus. From the solutions obtained, the corresponding solutions are derived for the cases $a=\infty$ and $b=\infty$, the passage to the limit being accomplished with the aid of the known solutions of some suitably chosen auxiliary problems in heat conduction. (Received August 11, 1936.)