A PARADOX OF LEWIS'S STRICT IMPLICATION

BY TANG TSAO-CHEN

The postulates for Lewis's strict implication are nine in number,* namely,

[11.1] $pq \prec qp$

[11.2] $pq \prec p$

[11.3] $p \prec pp$

[11.4] $(pq)r \prec p(qr)$

[11.5] $p \prec \sim(\sim p)$

[11.6] $p \prec q . q \prec r : \prec . p \prec r$

[11.7] $p . p \prec q : \prec . q$

[19.01] $\Diamond pq \prec \Diamond p$

[20.01] $(\exists p, q) : \sim(p \prec q) . \sim(p \prec \sim q)$.

By the operations of substitution, adjunction, and inference, a body of theorems is obtained. But the following theorem, which is a paradox of the strict implication, is not explicitly mentioned in Lewis's book.

Any two of the first eight postulates are such that each is deducible from the other, if $p \prec q$ be interpreted as 'p is deducible from q.'

In order to prove this theorem we assume the following eight theorems.†

1. $p \sim p = q \sim q$

Def. $0 = q \sim q$

* The references are to *Symbolic Logic*, by Lewis and Langford, 1932.

† For the proof of these theorems see the paper, *The theorem "$p \prec q \cdot = \cdot pq = p$" and Huntington's relation between Lewis's strict implication and Boolean algebra*, by Tang Tsao-Chen in this Bulletin, vol. 42 (1936), pp. 743–746.

2. $p \sim p = 0$

3. $p0 = 0$

Def. $i = \sim \Diamond 0$

4. $pq \prec p . = . i$

5. $p \prec p . = . i$

6. $p \prec q . \prec . i$

7. $p \prec q . = : i . p \prec q$

8. $p \prec q . = . pq = p.$

Note that the Theorems 4 and 5 are particular cases of the following theorem.

9. *If* $p \prec q$ *is asserted, then* $p \prec q \cdot = \cdot i.$

[Hyp.] $p \prec q$ (1)

[(1), 8.] $pq = p$ (2)

[12.11] $pq = p . = . pq = p$ (3)

[(2), (3)] $pq = p . = . p = p$ (4)

[11.03, 12.7] $p = p . = . p \prec p$ (5)

[(4), (5), 5.] $pq = p . = . i$ (6)

[(6), 8.] $p \prec q . = . i$

From the above theorem it is very easy to prove the following theorem.

10. *If* $p \prec q$ *and* $r \prec s$ *are both asserted, then*

$$p \prec q . \prec . r \prec s \quad (1)$$

and

$$r \prec s . \prec . p \prec q. \quad (2)$$

[Hyp.] $p \prec q$ (3)

[(3), 9.] $p \prec q . = . i$ (4)

[Hyp.] $r \prec s$ (5)

[(5), 9.]	$r \mathrel{-\!\!\!3} s. = .i$	(6)
[(4), (6)]	$p \mathrel{-\!\!\!3} q. = .r \mathrel{-\!\!\!3} s$	(7)
[11.03]	$(7) = (1)(2)$	(8)
[(7), (8)]	$(1)(2)$	(9)
[11.2]	$(1)(2) \mathrel{-\!\!\!3} (1)$	(10)
[12.17]	$(1)(2) \mathrel{-\!\!\!3} (2)$	(11)
[(9), (10)]	(1)	
[(9), (11)]	(2) .	

The paradox stated above is a particular case of Theorem 10, and therefore requires no further proof.

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THE BETTI NUMBERS OF CYCLIC PRODUCTS

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1. *Introduction.* In a recent paper† M. Richardson has discussed the symmetric product of a simplicial complex and has obtained explicit formulas for the Betti numbers of the two- and three-fold products. Acting on a suggestion of Lefschetz, we define a more general type of topological product and apply Richardson's methods to compute the Betti numbers of a certain one of these, the "cyclic" product.

2. *Basis for m-Cycles of General Products.* Let S be a topological space and G a group of permutations on the numbers $1, \cdots, n$. The *product of S with respect to G*, $G(S)$, is the set of all n-tuples $(P_1, \cdots, P_n)$ of points of S, where $(P_{i_1}, \cdots, P_{i_n})$ is to be regarded as identical with $(P_1, \cdots, P_n)$ if and only if the permutation $\binom{1 \cdots n}{i_1 \cdots i_n}$ is an element of G. A neighborhood of $(P_1, \cdots, P_n)$ is the set of all points $(Q_1, \cdots, Q_n)$ for which Q_i belongs to a fixed neighborhood of P_i. It is not difficult to verify that the

† M. Richardson, *On the homology characters of symmetric products*, Duke Mathematical Journal, vol. 1 (1935), pp. 50–69. We shall refer to this paper as R.