irreducible factors of $f(\lambda)=|\mathcal{A}-\lambda I|$ in R are likewise the distinct irreducible factors of $g(\lambda)$, the minimum function of $\mathcal{A}$ in R. Thus the characteristic divisors of $(\mathcal{A}-\lambda I)$ and A are the same and hence their invariant factors are the same. The same is true of $(\mathcal{B}-\lambda I)$ and B because $\mathcal{B}\sim B$. The following theorem has thus been established.

THEOREM 10. *A is similar to B if and only if the invariant factors of A are the same as the invariant factors of B.*

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THE THEOREM "$p \prec q. = .pq=p$" AND HUNTINGTON'S RELATION BETWEEN LEWIS'S STRICT IMPLICATION AND BOOLEAN ALGEBRA

BY TANG TSAO-CHEN

In this Bulletin, vol. 40 (1934), p. 729, E. V. Huntington pointed out that the relation called "strict implication" in C. I. Lewis's system of logic can be shown to be substantially equivalent to the relation called subsumption in ordinary Boolean algebra. His main result is as follows:

Whenever we find the formula "$p \prec q$" asserted, we may thereupon write down the formula "$p=pq$"; and conversely, whenever we find the formula "$p=pq$" established, we may write down that the formula "$p \prec q$" is asserted.

That is, Huntington's relation is

$$(\text{“}p \prec q\text{” is asserted}) \rightleftarrows (\text{“}pq=p\text{” is established}).$$

This relation is not the same as the following theorem:

$$p \prec q. = .pq = p,$$

where "$=$" takes the meaning of logical equivalence given in Lewis's *Symbolic Logic*.

This theorem being not explicitly mentioned in Lewis's *Symbolic Logic*, I shall prove it here.

Throughout this paper we shall follow Lewis's practice of ignoring the distinction, which is characteristic of Huntington's paper, between "p" and "p is asserted." Our Theorems 1, 2, 3, 4 correspond to Huntington's 15, 16, 17, 21; and our i corresponds to Huntington's Z^*.

Starting from Lewis's system of postulates, we can establish the following theorems (the references are to *Symbolic Logic* by Lewis and Langford and to theorems here):

1. $p \sim p = q \sim q$

$$[11.02] \quad p \sim p \prec q \sim q. \;=\; . \sim \Diamond\, [(p \sim p) \sim (q \sim q)] \qquad (1)$$

$$[12.11] \quad \sim \Diamond\, [(p \sim p) \sim (q \sim q)] = \sim \Diamond\, [(p \sim p) \sim (q \sim q)] \qquad (2)$$

$$[19.57, 12.15] \quad [(p \sim p) \sim (q \sim q)] = (p \sim p) \qquad (3)$$

$$[(1), (2), (3)] \quad p \sim p \prec q \sim q. \;=\; . \sim \Diamond\, (p \sim p) \qquad (4)$$

$$[11.02] \quad p \prec p. \;=\; . \sim \Diamond\, (p \sim p) \qquad (5)$$

$$[(4), (5)] \quad p \prec p. \;=\; .p \sim p \prec q \sim q \qquad (6)$$

$$[11.03, 11.2, (6)] \quad p \prec p. \prec .p \sim p \prec q \sim q \qquad (7)$$

$$[12.1, (7)] \quad p \sim p \prec q \sim q \qquad (8)$$

$$[(8)] \quad q \sim q \prec p \sim p \qquad (9)$$

$$[(8), (9), 11.03] \quad p \sim p = q \sim q.$$

From this last theorem, if we define 0 by

$$0 = q \sim q,$$

where q is any particular proposition, we obtain the following general theorems containing 0:

2. $p \sim p = 0$.

3. $p0 = 0$

$$[19.57, \text{Def. of } 0] \quad p0 = 0.$$

Let us now define i by the relation*

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* This definition of i may be replaced by $i = .0 = 0$, and therefore i means simply the identity $0 = 0$. But, by 5, it may easily be proved that $i = .p = p$.

$$i = \sim \Diamond 0.$$

Then a group of theorems containing i is obtained:

4. $pq \prec p. = i$

$[11.02]$ $pq \prec p. = . \sim \Diamond [(pq) \sim p]$ (1)

$[12.5]$ $(pq) \sim p = (qp) \sim p = q(p \sim p)$ (2)

$[2., 3., (2)]$ $(pq) \sim p = 0$ (3)

$[(1), (3)]$ $pq \prec p. = . \sim \Diamond 0$ (4)

$[(4)$, Def. of $i]$ $pq \prec p. = i.$

5. $p \prec p. = i$

$[4.]$ $pp \prec p. = i$ (1)

$[12.7, (1)]$ $p \prec p. = i.$

6. $p \prec q. \prec i$

$[19.6]$ $p \prec q. \prec .p0 \prec q0$ (1)

$[3., (1)]$ $p \prec q. \prec .0 \prec 0$ (2)

$[5.]$ $0 \prec 0. = i$ (3)

$[(2), (3)]$ $p \prec q. \prec i.$

7. $p \prec q. = :i.p \prec q$

$[19.6: |(p \prec q)|p, i|q, (p \prec q)|r]$

$p \prec q. \prec i: \prec :.p \prec q.p \prec q: \prec :i.p \prec q$ (1)

$[6., (1)]$ $p \prec q.p \prec q: \prec :i.p \prec q$ (2)

$[12.7]$ $p \prec q. = :p \prec q.p \prec q$ (3)

$[(2), (3)]$ $p \prec q. \prec :i.p \prec q$ (4)

$[12.17]$ $i.p \prec q: \prec .p \prec q$ (5)

$[(4), (5), 11.03]$ $p \prec q. = :i.p \prec q.$

Lastly, we obtain the required theorem:

8. $p \prec q. = .pq = p$

[19.63]	$p \prec pq. = .p \prec p.p \prec q$	(1)
[(1), 5.]	$p \prec pq. = .i.p \prec q$	(2)
[(2), 7.]	$p \prec pq. = .p \prec q$	(3)
[11.03]	$pq = p. = :pq \prec p.p \prec pq$	(4)
[(4), 4.]	$pq = p. = :i.p \prec pq$	(5)
[(3), (5)]	$pq = p. = :i.p \prec q$	(6)
[(6), 7.]	$pq = p. = .p \prec q.$	(7)
[12.11]	$p \prec q. = .p \prec q$	(8)
[(7), (8)]	$p \prec q. = .pq = p.$	

Huntington's relation may also be deduced from the above theorem. For, from 8., we have

$$p \prec q. \prec .pq = p \tag{a}$$

$$pq = p. \prec .p \prec q \tag{b}$$

If $p \prec q$ is asserted, then by (a) and the Rule of Inference, $pq = p$ may be asserted. That is,

$$(p \prec q) \text{ is asserted} \rightarrow (pq = p) \text{ is asserted.} \tag{c}$$

Again, if $pq = q$ is asserted, then by (b) and the Rule of Inference, $p \prec q$ may be asserted. That is,

$$(pq = p) \text{ is asserted} \rightarrow (p \prec q) \text{ is asserted.} \tag{d}$$

By (11) on page 730 and (a) and (b) on page 731 of Huntington's paper, our "$(pq = p)$ is asserted" is equivalent to his "$(pq \sim p)$ is asserted," which in turn is equivalent to his "$(pq = p)$ is established." So (c) and (d) together give Huntington's relation,

$$(p \prec q) \text{ is asserted} \rightleftarrows (pq = p) \text{ is established.}$$

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