Collected Papers of Charles Peirce. Vol. III, *Exact Logic*, xiv+433 pp., and Vol. IV, *The Simplest Mathematics*, x+601 pp. Edited by Charles Hartshorne and Paul Weiss. Harvard University Press. 1933.

Nearly all the papers in Volume III of this series have been previously published, and the most important of these have been summarized and evaluated by C. I. Lewis in his *Survey of Symbolic Logic*. The volume is devoted almost wholly to a development of the logic of relations and to a discussion of the bearing of this logic upon mathematics and upon the analysis of propositions of ordinary discourse. There are two papers in which Peirce discusses parts of Schröder's *Vorlesungen über die Algebra der Logik*.

Volume 4 comprises unpublished papers on logic and mathematics. Although of less intrinsic value than those of the preceding volume, they are perhaps of more interest now because of the wealth of brilliant suggestions they contain. The volume is divided into three parts. The first contains miscellaneous papers on logic, on the foundations of arithmetic, and on Cantor's transfinite numbers; the second is devoted to Peirce's theory of *graphs*; while the third contains an exposition of a number of mathematical curiosities.

The editors call special attention to an unpublished paper (vol. IV, pp. 13 ff.) written about 1880, which, as they point out, involves a clear anticipation of Sheffer's stroke-function. Peirce writes AB to mean "not-A and not-B." So AA means "not-A," and the other definitions follow as with Sheffer. Later on (p. 216) Peirce points out how the same definitions can be given if AB is read "not-A or not-B," which is the other interpretation Sheffer puts upon his stroke-function. Peirce says that the single function AB can replace the seven symbols $=$, $>$, $+$, $-$, $\times$, 0, 1, which are the constants of Boole's calculus of classes. It looks, however, as if this claim could not be allowed. If we give the symbols the propositional interpretation, then there will be nothing in the functions constructed on AB which represents a class; if we give the symbols the class interpretation, then, in order to carry through all the definitions as given by the editors in a note on page 18, we shall have to go over occasionally to the propositional interpretation. It is the propositional interpretation to which the scheme fully applies.

The second part of Volume IV contains, as we have said, an exposition of Peirce's system of graphs, which are to represent any propositions (after the fashion of the Euler diagrams) and to represent argument or proof by rules which govern transformations of these graphs. Peirce held that all formal proof (logical and mathematical) consists in reading off facts from a diagram of some form or other; such proof is pictorial or *Iconic* (see p. 430). He therefore places much emphasis upon formal algebras and graphical representation.

In the course of developing his graphs, Peirce tries in several places to supply graphical representations of modal propositions and analytical or necessary connections of propositions (see esp. pp. 398 ff.) such as those subsequently dealt with by C. I. Lewis in his *Strict Implication*. But Peirce's conception of modality differs in an important respect from that of Lewis: he makes possibility and necessity relative to states of information. For this reason the comparison which the editors draw in a footnote on page 403 between Peirce's diagrams and propositions of Lewis's system cannot be exact.

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