

ABSTRACTS OF PAPERS

SUBMITTED FOR PRESENTATION TO THE SOCIETY

The following papers have been submitted to the Secretary and the Associate Secretaries of the Society for presentation at meetings of the Society. They are numbered serially throughout this volume. Cross-references to them in the reports of the meetings will give the number of this volume, the number of this issue, and the serial number of the abstract.

246. Professor L. E. Dickson: *First theory of homogeneous diophantine equations of degree two.*

The methods of Gauss and others yield only rational solutions and not exclusively integral solutions as claimed. When one solution in integers is given, the equation may be reduced to $zw = Q(x_1, \dots, x_n)$. When $n = 2$ the complete solution in integers is obtained by as many independent sets of formulas as there are classes of binary quadratic forms f having the same discriminant as Q . The generalized composition of forms f , and the resulting group of $h+h'$ classes if $h' = h$, where h is the number of all properly primitive classes and h' is the number of all improperly primitive classes of the same determinant, are considered for the cases $n = 4, 3$, and 8 . (Received March 8, 1937.)

247. Mr. Harold Chatland: *The asymptotic Waring problem for homogeneous polynomial summands.*

The asymptotic theory for homogeneous polynomial summands is developed in a manner somewhat similar to that given by L. E. Dickson (*Annals of Mathematics*, vol. 37 (1936), pp. 293–316). Modifications of some methods used in his paper are necessary to overcome difficulties not arising in the problem for n th powers. In this new problem the number of summands required is less than the number required when only n th powers are taken as summands. (Received March 8, 1937.)

248. Dr. J. L. Doob: *Stochastic processes depending on a discrete-valued parameter.*

The purpose of this paper is to set up the measure relations for the most general discrete stochastic process (that is one depending on a parameter running through integral values) and to show the significance of the conditional probability functions. The derivation of the conditional probability functions from the measure relations of the process, the reverse of the usual treatment, allows full generality, but introduces new difficulties. Applications are made to special cases, and new results are obtained for Markoff chains. (Received March 8, 1937.)

249. Professor Fritz John: *Representation of Stieltjes integrals by infinite series. II.*

Various infinite series representing integrals of arbitrary functions have been given by the author (Mathematische Annalen, vol. 110 (1935), p. 718, and a paper appearing in the American Journal of Mathematics) and by H. Rademacher (American Journal of Mathematics, vol. 58 (1936), p. 169). The present paper gives formulas of a more general kind. Let $f(x)$ and $g(x)$ be such that the Riemann-Stieltjes integral $I = \int_a^b f(x)dg(x)$ exists. Let there be given a sequence of subdivisions containing one another, the n th subdivision being $a = c_0^n < c_1^n < \dots < c_{r_n}^n = b$, and let $c_i^{n-1} \leq \eta_i^n \leq c_i^n$. Then I can be represented by a convergent series of the form $\sum [g(c_r^n) - g(c_{r-1}^n)] [f(\eta_r^n) - f(\eta_{r-1}^n)]$. As a special case the formula $\int_1^2 f(x)dg(x) = \sum_{\mu=1}^r [f(2\mu+1) - f(2\mu)] [g(2\mu+2) - g(2\mu+1)]$ is derived for every f and g for which the integral exists and $g(2) = g(1)$. (Received March 8, 1937.)

250. Professor Fritz John: *Normal forms of convex regions under affine transformations.*

The classes of plane convex regions which are equivalent under affine transformations are considered. A rule by which, from every such class, an element is chosen in a unique manner (except for similarity transformations) is called a normalization, and a region is called normal if it is the chosen representative of its class. As normal elements may be taken, for example, the regions for which the central ellipse of inertia is a circle (see the author's abstract 41-11-402), or those for which the ratio of the greatest and least distance between two parallel lines of support is least (see the paper of F. Behrend, Mathematische Annalen, vol. 114 (1937), p. 713). The normalizations of the two latter examples are continuous in a certain sense. It is proved that if to every direction in space a convex region corresponds in a continuous manner, then among those regions is at least one normal one, given any continuous normalization. Various applications of this theorem are discussed. (Received March 8, 1937.)

251. Professor E. J. McShane: *Existence theorems for single integral problems in the calculus of variations.*

The problem first considered is that of minimizing an integral $\int g(x, y, \dot{x}, \dot{y})d^s$ (where $y = (y_1, \dots, y_n)$) in a class of curves $x = x(t)$, $y = y(t)$ with $\dot{x}(t) \geq 0$. The existence of a minimizing curve is shown for problems in which g is lower semi-continuous as a function of (x, y, \dot{x}, \dot{y}) , is positively homogeneous of degree 1 and convex in (\dot{x}, \dot{y}) , and for each (x, y) exceeds a linear function of (\dot{x}, \dot{y}) . If g is independent of x and \dot{x} , this gives a generalization of the Hahn-Tonelli existence theorem for parametric problems. If $\dot{x}(t) \equiv 1$, the problem reduces to an ordinary problem. The minimizing curve can be represented parametrically with $\dot{x}(t) \equiv 1$, if g satisfies the supplementary hypothesis that $g(x, y, 0, \dot{y}) = \infty$ except for (x, y) in an exceptional set of a type here defined and studied. From the existence theorem thus derived it is easy to obtain a corollary (equivalent to the existence theorem) applying to problems involving higher derivatives. (Received March 10, 1937.)

252. Professor P. G. Hoel: *A significance test for component analysis.*

By making certain normality assumptions and treating a set of statistical variables from the component analysis point of view, a significance test is derived from the rank of a correlation matrix. The test is based upon an approximation to the sampling distribution of the generalized variance. (Received March 11, 1937.)

253. Professor W. H. Roever: *Meaning and function of a picture.*

In this paper a picture of a space object is defined as a resembling representation of this object on a plane surface. A criterion of resemblance is then stated and it is shown that this criterion is satisfied by projection. A representation or correspondence between space and the plane is defined and it is shown that at least two projections are required for its satisfaction. A stereographic picture is cited as an example of adequate pictorial representation but it is stated that such a representation could also be attained, for example, in a photograph in which the shadow of an object, as well as the object itself, is shown. In particular, the source of light casting the shadow might be in the zenith and thus, if the object were a house or some more or less rectangular object of technology, the shadow might coincide with the base of this object on the ground. Consequently one might lose sight of the fact that the picture is a double projection. Nevertheless, such a picture as, for example, the isometric projection of a piece of machinery, while giving the impression of being a single projection, possesses the property of representing each point of space by two simply related points of the picture plane. The relation between these points is shown. (Received April 7, 1937.)

254. Professor L. M. Blumenthal: *Note on the characterization of pseudo r -spheric (S_n) sets.* Preliminary report.

The problem (fundamental in the metric characterization of the sphere) of characterizing those semi-metric spaces that are pseudo r -spheric (S_n) has been completely solved by the writer for $n=1$ (American Journal of Mathematics, vol. 56 (1934), pp. 225-232). This note is concerned with the much more complicated case $n=2$. It is shown that if a semi-metric space (1) contains more than five points and (2) each quintuple of the space is pseudo r -spheric (S_2), then for each pair p, q of points of the space, the distance $pq = r \cos^{-1}(\pm \frac{1}{3})$, with the negative sign holding for a certain minimum number $\mu > 0$ of the distances. There is reason to believe that the conclusion of the theorem might be valid under a considerable weakening of the hypothesis (2) such as, for example, merely supposing that the pseudo r -spheric space does not contain a pair of diametral points. This possibility, together with the developing of methods to treat the general case S_n , is the object of the investigations now in progress. (Received April 9, 1937.)

255. Dr. H. H. Goldstine: *The theorem of Hildebrandt.*

In this paper a new proof is given of the well known theorem of Hildebrandt (this Bulletin, vol. 29 (1923), p. 309) which plays a fundamental role in the

theory of linear operators. Not only is this proof much shorter than the one ordinarily given, but also it enables one to dispense with some of Hildebrandt's hypotheses. The proof follows immediately from a generalization of a theorem given by Banach in his monograph on linear operators (p. 19). This generalization is stated and proved in this article. (Received April 9, 1937.)

256. Dr. W. T. Reid: *On Klein's oscillation theorem for boundary problems of the calculus of variations.*

This paper contains an extension of Klein's oscillation theorem to the general differential and integro-differential boundary value problems associated with the second variation of simple integral problems of the calculus of variations. The treatment is dependent upon the previous results of the author concerning such systems involving a single characteristic parameter (see abstracts 41-1-44 and 41-5-232). (Received April 9, 1937.)

257. Mr. R. W. Wagner: *Multiple-valued functions in matrix space.*

Cipolla has shown how to extend analytic functions to matrices in a natural way. This paper investigates the new properties of the extended functions geometrically. If one is given any value of a function of a matrix A , there exist, in the subspace of matrices whose elementary divisors are of the same degrees as those of A , closed paths such that by taking A around these paths one obtains all values of the function. If the paths are taken in the subspace in which not only the elementary divisors but also the reduced characteristic function are of the same degrees, one obtains, in general, from a given value, not all values, but a class of values. The classification of values thus obtained is shown to agree with that published by Schwerdtfeger and obtained by a different method. Degenerate matrices are shown to be singular points of these functions. When A approaches a degenerate point, some of the values become infinite or depend upon the path of approach, according as the degree of the reduced characteristic function of the degenerate matrix is n or less. (Received April 9, 1937.)

258. Mr. H. A. Arnold: *A set of independent postulates for a ring with equality undefined.*

A set of ten independent postulates for a non-commutative ring is given, involving equality as an undefined notion. The properties of symmetry and transitivity are postulated for the equality relation, as well as the following: if $a = b$, then $ca = cb$, $ac = bc$ and $a + c = b + c$. The existence of at least one solution of the equation $a + x = b$ is postulated. A method is given of adding a neighborhood topology in the form of four independent postulates. A similar treatment of non-commutative fields and of linear algebras with finite bases will follow shortly. These of course will include modular fields and modular arithmetics as special cases. (Received March 27, 1937.)

259. Professor Felix Bernstein: *Gaussian and Laplacian statistics*.

Gaussian statistics are those based on the square of the deviation, Laplacian statistics, those based on the absolute value of the deviation. It can be shown that the maximum amount of information from the data will not be generally gathered by the use of the one or the other type of statistics. This situation leads to a reconsideration of the classical problems of mathematical statistics. New methods are given for determining regression and correlation. (Received March 11, 1937.)

260. Mr. J. J. DeCicco: *The differential geometry of series of lineal elements*.

The author gives some results additional to those given in a paper, *The group of turns and slides and the geometry of turbines*, by Kasner, published in the American Journal of Mathematics, vol. 33 (1911), and a paper, *The geometry of turbines and flat fields*, by Kasner and the author, which is being published in the American Journal of Mathematics. Theorem 1: Necessary and sufficient conditions that ∞^1 circular series be an osculating set are that they and their central turbines be each a set of enveloping series in such a way that corresponding elements on the two envelopes are parallel but have opposite orientations. Theorem 2: Two series for which curvature and torsion are the same functions of the angle u are equivalent under the whirl-rigid motion group. Theorem 3: If α is the angle between a supplementary section and a section with the same tangent at an element E of a field whose curvatures at E are $1/\gamma_s$ and $1/\gamma$ respectively, then $\gamma_s = \gamma \cos \alpha/2$. Theorem 4: Let $1/\gamma_0$ be the extremal supplementary curvature and λ the curvature of the point curve of the equi-parallel series at E . If δ is the distance between the centers of the tangent turbines of the extremal supplementary section and any supplementary section whose curvature is $1/\gamma_s$, then $\gamma_s = \gamma_0 + \lambda\delta^2$. (Received March 12, 1937.)

261. Professor W. C. Graustein and Mr. S. B. Jackson: *The four-vertex theorem for a certain type of space curve*.

This paper presents a generalization to certain twisted closed curves of results obtained by Graustein for plane ovals (Monatshefte für Mathematik und Physik, vol. 43). The vertices of a curve are classified into primary and secondary vertices. A primary vertex is a point of the curve at which the curvature has a relative maximum (minimum) which is greater (less) than the average curvature of the curve. Any other extremum of the curvature is called a secondary vertex. The general curve considered is a closed curve of class C'' with positive, non-constant curvature, which satisfies the condition that the orthogonal projection of the tangent indicatrix on a suitably chosen plane is an oval. It is shown that for such a curve the number of primary vertices, if finite, exceeds the number of secondary vertices by at least four, and is infinite if the number of secondary vertices is infinite. (Received March 8, 1937.)

262. Professor Karl Menger: *A new proof of the Euler-Lagrange equation.*

Each curve passing through a given linear element $L_0 = \{x_0, y_0, \phi_0\}$ and minimizing an integral must have the curvature of the circle which passes through L_0 and minimizes the integral in the neighborhood of the point x_0, y_0 with respect to all circles passing through L_0 . It is easy to see that the latter curvature is the number which according to Euler's equation is the curvature of the extremal passing through L_0 . This simple proof of the equation may also be formulated by considering for each point a single two-parametric family of curves approaching the minimizing curve while Lagrange's proof operates with a family depending on a continuous function of one-parametric families of curves. (Received March 8, 1937.)

263. Mr. A. N. Milgram: *Decompositions and dimension of closed sets in R^n .*

In this paper the author establishes a new characterization of dimension of closed sets in R^n . This may be stated as follows: A necessary and sufficient condition that a closed subset F of R^n be of dimension r (in the sense of Urysohn-Menger) is that for any sufficiently small $\epsilon > 0$, F may be decomposed into the sum of a countable infinity of closed sets $F_1, F_2, \dots, F_s, \dots$, each of diameter less than ϵ , such that $\dim F_i \cdot F_j \leq r-1$ for any pair of integers $i \neq j$; but that for any such decomposition there exists a pair of integers $m \neq n$ such that $\dim F_m \cdot F_n = r-1$. This theorem is a consequence of the following: If F is a closed subset of R^n ; F_1, F_2, \dots a decomposition of F into closed sets; Z^{n-r-1} a cycle in $S(p, \epsilon)$ —the interior of a sphere of radius ϵ and center p , a point of F —which does not bound in $S(p, \epsilon) - F$ but does bound in $S(p, \epsilon) - F_i$ ($i=1, 2, \dots$); then there exists a pair of numbers $m \neq n$ such that $\dim F_m \cdot F_n \geq r-1$. Another interesting consequence of this theorem is a generalization of a theorem due to Miss Mullikin: The sum of a countable number of closed sets, no one of which separates R^n and the dimension of whose products taken pairwise is at most $n-3$, cannot separate R^n . (Received March 12, 1937.)

264. Mr. W. H. Ingram: *Initial motion of a forced dissipative dynamical system under a sudden change in the force regime.*

The author considers a system for which $\partial L / \partial q = 0$ for one or more coordinates, and it is found that the disturbed motion is initially the same as for a system in which such coordinates can be ignored. The theory has application to electrical machinery; an inspection of the equations of motion derived from Routh's function reveals, in the case of the alternator, simple algebraic relations between the numerous "transient" reactances on the one hand and the fundamental constants (inductances) appearing in T on the other. For example, $l'_d = l_d - 3M_0^2 / 2L$ (Philosophical Magazine, vol. 21 (1936), p. 302) where the prime follows the notation of Park and Robertson. (Received March 20, 1937.)

265. Professor L. V. Ahlfors: *An extension of Schwarz's lemma.*

In differential form Schwarz's lemma asserts that the non-euclidean element

of length decreases when the unit circle is transformed by a bounded analytic function, $|f(z)| \leq 1$. The hyperbolic metric has the constant negative curvature -4 , and the classical theorem refers to a case where the curvature is preserved. In the extension the hyperbolic metric is still considered in the z -plane, but an arbitrary conformal metric with curvature ≤ -4 is introduced in the function plane. It is proved that Schwarz's lemma still holds under these weaker conditions. Applications are given which yield a numerical inequality for the Picard-Schottky theorem and a new lower bound for Bloch's constant. In the applications it is important that the metric is allowed to have singularities of a certain kind, corresponding to a negatively infinite curvature. (Received April 19, 1937.)

266. Dr. M. T. Bird: *On a classification of integral functions by means of certain invariant point properties*. A supplement.

In a recent paper of Carmichael, Martin, and Bird (Transactions of this Society, vol. 40 (1936), pp. 462-473) α -sequences were defined but were not given further consideration because the authors were unable to prove their existence or non-existence. This supplement establishes the impossibility of α -sequences. Hence the point properties which the authors of the previous paper studied are seen to be significant mainly, if not entirely, for integral functions. (Received April 19, 1937.)

267. Professor Alonzo Church: *Combinatory logic as a semigroup*. Preliminary report.

By a *semigroup* is meant a set in which the product of any two elements is a unique element of the set, the multiplication being associative but not necessarily obeying a law of cancellation. Consider the system of combinators, in the sense of Rosser (Duke Mathematical Journal, vol. 1 (1935), p. 336), allowing as equivalence operations r -conversions, p -conversions, and also the operations (allowed by Curry) of replacing BI by I and inversely. This system is a semigroup, with identity element I , if we take as multiplication the operation (introduced by Curry) which is denoted by Rosser as \times . From the relations $ab = Tb \times Ta \times B \times T$ and $T(ab) = Tb \times Ta \times B$ it follows that every element is expressible as a product formed out of the four particular elements TI, TJ, B, T . The semigroup thus has a finite set of generators, although the set of generating relations must apparently be infinite. There is, however, an effective process for writing out the series of generating relations to as many terms as desired; also an effective means of distinguishing generating relations from others. From the results of the author (American Journal of Mathematics, vol. 58 (1936), pp. 345-363), it follows that the *word problem* of this semigroup is unsolvable. (Received April 14, 1937.)

268. Reverend W. C. Doyle: *A further generalization of Lambert series*.

The series here considered $\sum (a_n b_n z^{\lambda n}) / (1 - a_n z^{\mu n})$ includes those of Lambert, Weierstrass, Hansen, and Garvin (American Journal of Mathematics, vol. 58 (1936), p. 507). The regions of convergence for integral values of λ and μ are

studied; and uniform convergence is established within the regions of convergence. Every power series can be expressed as a series of this generalized type; the recursion formula contains a new generalization of Möbius coefficients. (Received April 6, 1937.)

269. Dr. Norman Levinson: *On the growth of analytic functions.*

Simple function theoretic proofs are given of theorems originally due to Vladimir Bernstein on the determination of the growth of analytic functions from their growth on a set of points. A set of new and more precise theorems depending on the author's theory of non-vanishing functions are also given. (Received April 15, 1937.)

270. Professor R. L. Wilder: *Decompositions of compact metric spaces.*

Let C be a certain class of compact metric spaces, and ψ a local topological property. Then ψ is called *expansive relative to C* if, given any $M \in C$, the set S , closure of the set of non- ψ points of M , if non-vacuous, contains a continuum (compare Whyburn, this Bulletin, vol. 41 (1935), p. 95). The set of components of S together with the points of $M - S$ form the points of M' , the ψ -prime part decomposition of M (see Hausdorff, *Mengenlehre*, 1935, pp. 223-224). If ψ is expansive relative to a class C of compact metric spaces, and $M \in C$, then if $M' \in C$, M' has no non- ψ points. Let C_k^m be the class of compact metric spaces that are l . i -c. for $0 \leq i \leq k$ ($k = -1$ to indicate no l . i -c. properties imposed) and whose Betti numbers p^i are finite for $0 \leq i \leq m$. Then the property of being l . $(n+1)$ -c. is expansive relative to C_n^{n+1} and the property of being l . i -c. for $0 \leq i \leq n$ is expansive relative to C_{n-1}^n . These theorems yield prime part decompositions of various spaces, as well as certain theorems concerning the decompositions of complicated domain boundaries in n -space. (Received April 15, 1937.)

271. Professor R. L. Wilder: *Property S_n .*

By a *pair* (U, V) the author means a pair of sets U and V such that $U \supset V$. The maximum number of n -cycles of V independent with respect to homologies on U is denoted by $h^n(U, V)$. By the *diameter* of a pair (U, V) is meant the diameter of U . A set M is called the *sum* of pairs (U_k, V_k) if $M = \sum U_k = \sum V_k$. A metric space M is said to have *Property S_n* if for every $\epsilon > 0$, M is the sum of a finite number of pairs (U_k, V_k) of diameter $< \epsilon$ such that all $h^n(U_{n_1} + \dots + U_{n_m}, V_{n_1} + \dots + V_{n_m})$ are finite. Property S_0 is equivalent to the Sierpinski-Moore "Property S ." Property S_n for $n > 0$ yields the "justification theorems" that one would expect of a generalization of Property S . The original Sierpinski characterization of Jordan continua extends in terms of S_n for spaces l . i -c. for $0 \leq i \leq k$; the relations of various types of locally connected sets to their complements in euclidean spaces are characterized by means of S_n ; and the relations to uniform local i -connectedness and local i -connectedness established. (Received April 15, 1937.)

272. Professor R. L. Wilder: *Neighborhoods in locally connected spaces.*

Let M denote a locally compact, locally connected, metric space. Then for every point P of M and $\epsilon > 0$, there exists a uniformly locally connected open neighborhood of P of diameter less than ϵ . Certain well-known theorems on spaces M are obvious corollaries of this theorem. Furthermore, the theorem itself is true for much more general spaces than the locally compact spaces. (Received April 15, 1937.)

273. Professor R. P. Agnew and Mr. A. P. Morse: *Extensions of linear functionals, with applications to limits, integrals, measures, and densities.*

The main result of this paper is that existence of a linear functional $F(x)$ (dominated by $p(x)$ as in Banach, *Théorie des Opérations Linéaires*, p. 28) which is invariant under a group G of linear transformations of its domain under which $p(x)$ is invariant, and which is an extension of a linear functional $f(x)$ dominated by $p(x)$ and invariant under G , is implied by existence of a linear extension $F_1(x)$ of $f(x)$ which is dominated by $p(x)$ and invariant under some derived group G^* of G . Corollaries and applications of this result furnish functionals invariant under groups of transformations larger than those previously known. The resulting generalized limit has (in addition to those of Banach, p. 33) the property that $\lim_{s \rightarrow \infty} x(\mu s + \lambda) = \lim_{s \rightarrow \infty} x(s)$ when μ and λ are real with $\mu > 0$. The resulting generalized integrals have (in addition to those of Banach, p. 31) the property that $|\mu| \int_{-\infty}^{\infty} x(\mu s + \lambda) ds = \int_{-\infty}^{\infty} x(s) ds$ when μ and λ are real with $\mu \neq 0$. The resulting measures and densities have analogous properties. (Received April 29, 1937.)

274. Professor J. J. Gergen: *Some theorems related to number theory.*

In Part I, some theorems of the following character are obtained. Let $\phi(z)$ be an entire function of order ≤ 1 . Let $2\lambda(r)$ denote the number of zeros of ϕ in the circle of radius r about $z=0$. Suppose that $\lambda(r) = Ar \log r + Br + o(r)$, $r \rightarrow \infty$; that $0 \leq t$, and that $\phi(it) \neq 0$. Then a necessary and sufficient condition that all the zeros of ϕ lie in the strip $|I\{z\}| \leq t$ is found to be that $\int_0^r \log |\phi(z)\phi(\bar{z})/\phi^2(it)| dx/x^2 = -\pi^2(A \log r + B) - \pi I\{\phi'(it)/\phi(it)\} + o(1)$, where $z = x + it$. The case $t=0$, $A=0$ is due to Wiener and Paley. Applications of the results in this part are made to $\zeta(s)$. In Part II Wiener's method of proof of the prime number theorem, which depends on Lambert series, is extended to obtain the corresponding result for the number of primes in a progression. Part III is concerned with Jessen's theorem that, if $f(x) \in L$ on $(0, 1)$, and is of period 1, and if $f_k(x) = (1/n) \sum_{p=1}^{n-1} f(x+p/n)$, then $f_2^k(x) \rightarrow \int_0^1 f(t) dt$, $k \rightarrow \infty$, p.p. An example is given to show that this result is not always true if 2^k is replaced by k . Use is made of the classical theorem on the approximation to irrationals by rationals. (Received April 28, 1937.)

275. Mr. Philip Hall: *On primitive groups of linear substitutions.*

Let G be a primitive group of linear substitutions irreducible in the field of complex numbers, and let H be a normal Sylow-subgroup of G . According to Blichfeldt, the central of H is contained in that of G , and is therefore cyclic. *If H is not itself cyclic, it is of class 2, and its quotient-group is elementary abelian.* Thus H belongs to a well-known system of prime-power groups met with in other connections, for example, theta-characteristics, abelian collineation groups, Weyl-Brauer theory of spinors. There follows a theorem on solvable primitive linear groups analogous to the theorem of Galois on primitive solvable permutation groups: given the degree n , such a group is equivalent to a subgroup of a certain well-defined group Ω_n , in general insolvable. For $n = p^\alpha$, p prime, Ω_n may be described as the "linear holomorph" of an irreducible H , having central index $p^{2\alpha}$ and belonging to the system mentioned above. For m prime to N , $\Omega_{mn} = \Omega_m \times \Omega_n$. (Received April 26, 1937.)

276. Professor Dunham Jackson: *Orthogonal polynomials in two complex variables.*

Recent observations on the properties of orthogonal polynomials in two real variables (Duke Mathematical Journal, vol. 2 (1936), pp. 423-434; this Bulletin, abstract 42-9-335), taken in conjunction with the well-known work of Szegő, Walsh, and Carleman on orthogonal polynomials in a complex variable, point clearly to a corresponding theory for polynomials in two complex variables. This paper records the working out in detail of some of the more obvious features of the extension. The real orthogonal transformations which play a prominent part in the case of polynomials in two real variables are naturally replaced here by unitary transformations with complex coefficients. (Received April 26, 1937.)