SHORTER NOTICES

Subharmonic Functions. By T. Radó. Berlin, Springer, 1937. iv+56 pp.

In concluding his review of the *Theorie der konvexen Körper* by Bonnesen and Fenchel (this Bulletin, vol. 41 (1935), pp. 613–614) the reviewer expressed his hope that a monograph on convex functions as such "will appear soon in the Ergebnisse series, and that it will prove just as exciting as the monograph by Bonnesen and Fenchel." The present monograph represents a realization of this hope at least in so far as the theory of subharmonic functions is concerned. It is the first number of the fifth volume of the eminently useful series, *Ergebnisse der Mathematik und ihrer Grenzgebiete*, published by the Zentralblatt der Mathematik. A detailed treatment of the theory of convex functions which are a special case of subharmonic functions is promised in a subsequent monograph by W. Fenchel.

According to the author's statement the purpose of the present monograph "is first to give a detailed account of those facts which seem to constitute the general theory of subharmonic functions, and second to present a selected group of facts which seem to be well adapted to illustrate the relationships between subharmonic functions and other theories." The following list of contents gives some, though a very inadequate, idea of the material contained in the book. Chapter 1. Definitions and preliminary discussion, pp. 1–7. Chapter 2. Integral means, pp. 7–12. Chapter 3. Criterions and constructions, pp. 12–22. Chapter 4. Examples, pp. 22–31. Chapter 5. Harmonic majorants, pp. 31–40. Chapter 6. Representation in terms of potentials, pp. 40–46. Chapter 7. Analogies between harmonic and subharmonic functions, pp. 46–53. References, pp. 54–56.

A unified exposition of the general theory of subharmonic functions was badly needed owing to the fact that various authors used various apparently different (at least in form) definitions of subharmonic functions. The author's contributions toward this goal will be welcomed by general readers and will be particularly valuable for specialists who have to use subharmonic functions as a tool of research. The applications treated in the book are numerous and the topics are wisely selected. The reviewer feels however that the number of pages devoted to this purpose might have been gainfully increased. The book contains a considerable amount of new material (both new results and new and better proofs of older results) which never has been published before and which is due to several mathematicians (mainly to Evans, Saks, and the author himself).

The exposition is elegant and clear throughout the book, although of necessity condensed in some places, the amount of hints and leading remarks being always sufficient, however, to allow an attentive reader to reconstruct a complete proof. The only exception to the above which came to the reviewer's attention is the proof of the theorem in 2.16 (p. 10). The proof of this theorem is based on a statement to the effect that a sufficient condition for $\log f(\rho)$ to be convex in $\log \rho$, $\rho_1 < \rho < \rho_2$, is that $\rho^\alpha f(\rho)$ be convex in $\log \rho$ for each $\alpha > 0$, with reference to a paper by F. Riesz. In this form the statement is not correct, as can be shown by simple examples such as $f(\rho) = \log \rho$. The statement becomes correct if α is assumed to be any real number, and this assumption is explicitly used in F. Riesz' argument. Incidentally there is an obvious misprint in the page reference here and at the end of the book, there being no paper of F. Riesz on the allotted pages 3–8! On the whole the book contains remarkably few misprints, and the list of references is very inclusive. In fact the only omission that the reviewer was able to discover is an extensive memoir by N. Günther, *Sur les intégrales de Stieltjes et leur applications aux problèmes de la physique mathématique*, Travaux de l'Institut Physico-Mathématique Stekloff, vol. 1 (1932), Ch. 8, which contains, among many other things, a discussion of F. Riesz' theorem concerning the representation of superharmonic functions in terms of potentials of positive mass distributions and of harmonic majorants.

The author and the Ergebnisse series should be congratulated upon publishing this exceedingly interesting and valuable monograph, which will prove indispensable for anyone who works in the field of the theory and applications of subharmonic functions.

J. D. TAMARKIN

Einführung in die analytische Geometrie und Algebra. Volume 2. By O. Schreier and V. Sperner. Leipzig and Berlin, Teubner, 1935. 308 pp.

The first volume of this work, bearing the same title, was published in 1931 and reviewed in this Bulletin (vol. 38 (1932), p. 622). The main feature (complete fusion of the foundations of geometry and algebra) and the excellent qualities of the book, were sufficiently pointed out in the review of the first volume, so that at present we restrict ourselves merely to giving a short list of contents. Chapter 1 (Elements of the theory of groups) includes Abelian groups. Chapter 2 (Linear transformations, matrices) deals primarily with the theory of elementary divisors, orthogonal transformations, symmetric and Hermitian matrices and the like. Chapter 3 (Projective geometry) contains an excellent exposition of the foundations of projective geometry in n-dimensional spaces. After a perusal of this second volume the reviewer feels confirmed in his previous opinion that the book could be successfully used as a text or reference book in a graduate course in this country.

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