

*Geometrical Optics, An Introduction to Hamilton's Method.* By J. L. Synge. Cambridge University Press, 1937. 9+110 pp.

The book of Synge gives an introduction into the application of Hamilton's original methods. The author avoids the abstract treatment of Hamilton and some modern textbooks by wisely restricting himself to the problems of homogeneous isotropic media.

This book develops from Fermat's principle the laws of refraction and reflection and the law of Malus. It introduces in its second chapter the three characteristic functions of Hamilton,  $V$ ,  $T$ , and  $W$ , and shows how the knowledge of these functions permits the calculation of the image ray to each object ray. It gives the explicit form of  $V$  for a plane mirror and a triple plane mirror, the explicit form of  $T$  for a sphere and a paraboloid of revolution.

Chapter 3 contains the first order laws, not in modern form, but in the form Hamilton has given. Chapter 4 gives the Gaussian theory and the image error theory of Seidel for a system of revolution. At the end of the chapter Abbé's sine condition is demonstrated and the development of the characteristic function of  $T$  up to fourth order members is given for a thin system, that is, for an optical system where the thicknesses of the single lenses and their distances are neglected. The last chapter demonstrates that most of the laws found remain true if we assume the media to be heterogeneous, but isotropic.

The book is clearly written and can be highly recommended from a didactic standpoint. It is, however, regrettable that the author completely omits everything that has been done since Hamilton's time; for example, the fundamental work of Allvar Gullstrand. It is true that Hamilton's ideas should form the fundamentals of every scientific textbook of optics, but new methods of investigation have been found and problems studied which would be difficult to attack with Hamilton's *methods*. I will give one example.

Instead of Hamilton's characteristic function  $V$  defined as the light path,  $\int_{P_1}^{P_2} \mu ds$ , between two points  $P_1$  and  $P_2$ , we ordinarily use a modification by Bruns, which does not allow  $P_1$  and  $P_2$  to be arbitrary in space, but restricts them to two surfaces such that through every point of the first surface there goes one and only one ray to every point of the second surface. It is always possible to find for each regular surface a second regular surface fulfilling this condition. This function characterizes the optical image formation as completely as does Hamilton's function. Moreover, it is free from singularities, whereas Hamilton's function has indefinite derivatives as soon as  $P_2$  becomes a point conjugate to  $P_1$ . A great part of contemporary work in optics is devoted to the problem of finding Bruns' modification of Hamilton's characteristic for certain image formations, or, vice versa, to find the geometric qualities of the image formation if Bruns' function has a special form.

These remarks are not intended to disparage Synge's book; they are meant only to show its limitations.

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