HILBERT AND ACKERMANN ON LOGIC

Grundzüge der theoretischen Logik. By D. Hilbert and W. Ackermann. 2d edition. Berlin, Springer, 1938. 8+133 pp.

This book gives an account of the symbolic treatment of certain topics of logic which have been particularly susceptible to that form of treatment. The first chapter treats of the propositional calculus. The second chapter gives a brief introduction to the calculus of classes, and shows how it can be made to give the traditional Aristotelian figures. The third chapter is devoted to the simple function calculus, that is, the calculus of classes of individuals and of dual and multiple relations between individuals.The fourth chapter elucidates the logical paradoxes and the use of a theory of types to avoid paradox.

The principal changes from the first edition are these: The nomenclature has been revised to agree with that of Hilbert-Bernays. The book has been brought up to date by rewriting the sections which deal with topics in which significant advances have been made in the last ten years. The deduction rules of the simple function calculus have been revised so as to eliminate certain inaccuracies occurring in the first edition.

This leaves the first and second chapters essentially unchanged. In the third chapter is added a proof of the independence of the axioms of the simple function calculus, and Gödel's proof that these same axioms are complete in one sense. The section on the decision problem of the simple function calculus is completely rewritten, reference being made to Church's result that any decision method that could be found for the simple function calculus would have to be non-constructive in a sense defined by Church and accepted by several other writers. The last chapter is considerably improved by the use of a simpler theory of types.

The first chapter opens with an intuitive discussion of the propositional calculus. This includes such topics as normal forms of a proposition, definition of universally true proposition, and principle of duality. The authors then introduce formal postulates for the propositional calculus and illustrate the use of the postulates by proving various of the intuitively obvious facts which they have been taking for granted in the previous intuitive discussion. This of course raises the question whether every universally true proposition is a consequence of the postulates, and also the question whether every consequence of the postulates is a universally true proposition. These are answered in the affirmative in the course of proving the completeness and consistency of the postulates. The consistency and independence of the postulates are proved by the familiar method of using numerical examples. Two kinds of completeness are proved, one kind which says that all universally true propositions are consequences of the postulates, and a sharper kind which says that if any proposition which is not a consequence of the postulates be adjoined as an additional postulate, then a contradiction will result. The decision problem for the propositional calculus is also solved. The decision problem is the problem of deciding whether a given proposition is universally true or not, and the solution formed part of the intuitive discussion which preceded the introduction of the postulates.

The results of the second chapter are not used elsewhere in the book, and so the second chapter would be irrelevant for anyone not acquainted with traditional logic. However, the chapter is short, and should interest enough readers to justify amply its inclusion.

The third chapter opens with a short intuitive discussion of the simple function calculus, including, among other things, a definition of universally true proposition of the simple function calculus. Then formal postulates for the simple function calculus are introduced. These postulates are applied to the proof of a considerable number of propositions, many of which would be trivially true from purely intuitive considerations and all of which are universally true. However, the authors do not prove that every consequence of the axioms is universally true. They point out this omission and state that such a proof, if possible, would be of the highest importance. They do, however, prove the consistency of the postulates in the sense that there is no formula A such that A and not-A are both consequences of the postulates. This and the independence of the postulates are both proved by the use of numerical examples. They reproduce Gödel's proof that all universally true propositions are consequences of the axioms, thus proving completeness in one of the two senses mentioned previously. They also exhibit a formula for which they prove that it is not a consequence of the axioms and that it can be added to the axioms without getting an inconsistency, thus disproving completeness in the other sense mentioned previously. The section on the decision problem for the simple function calculus contains a general discussion and a summary (with references) of known results.

The fourth chapter, on the extended function calculus, does not contain a set of formal postulates, but is simply an intuitive discussion throughout. The application of this calculus to cardinal numbers and point set theory is sketched briefly. The various logical paradoxes which can arise in this calculus are discussed. A theory of types which avoids the known paradoxes is then explained. To show that this theory of types does not weaken the extended function calculus too much for the purposes of mathematics, several problems in the foundation of the theory of real numbers are handled by means of it.

The revision has increased the recognized value of this book as a reference work without sacrificing the quality of easy readability which distinguished the first edition. In fact, for persons with no previous knowledge of formal logic but with a reading knowledge of German, it offers a very satisfactory introduction to the subject.

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