SHORTER NOTICES

Lehrbuch der Gruppentheorie. Vol. 1. By Hans Zassenhaus. (Hamburger Mathematische Einzelschriften, no. 21.) Leipzig and Berlin, Teubner, 1937. 6+151 pp.

This text, mostly on finite discrete groups, was suggested, the author states, by Artin's Hamburg lectures of 1933–1934. Its spirit is that of van der Waerden's *Moderne Algebra*, but of course this specialized book covers much more ground on groups than does that general treatise on modern algebra. Readers familiar with older books on finite groups, or with papers written in the classical manner, will recognize much of the material; the technique of the proofs is frequently simpler, less fortuitous, and more direct than the old. A consistent attempt is made to exploit the principle of homomorphic mapping to the limit. In addition to the practitioners of this technique cited by the author, particularly E. Noether and her many pupils in abstract algebra since about 1921, Gauss should be remembered as its originator. The abstract notions of congruence relations and equivalence relations are seen to have far wider scope than those for which they were devised in the theory of numbers. Dedekind should also be remembered in this connection.

The five chapters of the book present a wealth of material in compact form. The insistence is upon general theorems; the special properties of substitution groups, for instance, receive only passing mention, and linear groups are not discussed. A considerable amount of quite recent material is included; thus there is some account of P. Hall's work on groups whose order is a power of a prime, and Schreier's refinement of the Jordan-Hölder theorem is proved. Specific citations to the literature are likewise recent, and contain references to Ore's work on structures and group theory and G. Birkhoff's on transitive groups. In the historical references, G. A. Miller's reports on certain aspects of group theory and the account in his *Collected Works* might have been included with advantage. As would be expected, most of the references are to modern German work.

As the exposition itself is condensed, we can give only a brief indication of the contents. After a somewhat crowded first chapter of 27 pages dealing with fundamental notions, including the algebra of complexes, the book gets into its stride in the second chapter, devoted to homomorphism (as applied to groups), and groups with operators. This chapter of 45 pages includes, incidentally, the automorphisms of a group, commutative groups, and normal series, also a brief exposition of the groups occurring in algebra. Ideals are here introduced in connection with the subgroups of a skew ring.

One of the author's aims is the construction in a finite number of non-tentative steps of the several algebraic objects of the theory. Accordingly, Chapter 3, of 23 pages, is concerned with the construction of composite groups. The basis theorem for Abelian groups is quickly derived after a brief discussion of direct products (following Fitting, 1934) and some necessary preliminaries on matrices. The chapter concludes with an account of Schreier's Erweiterungstheorie and applications.

The fourth chapter presents Sylow's theorem, its extensions and consequences; and p-groups (groups of prime-power order). It includes a short discussion of Hamiltonian groups. Among the examples is P. Hall's generalized Sylow theorem for finite solvable groups. The concluding chapter, of 15 pages, presents, among other things, monomial representations, the theorems of Grün (1935), groups with all Sylow subgroups cyclic, and the principal-ideal theorem of class-field theory transposed to group theory. Throughout the book there are short sets of approachable exercises.

The treatment here presented achieves a certain unity which the classical presentation lacked. It also exhibits the theory of abstract finite groups (infinite abstract groups are not discussed in much detail) against the background of modern algebra as that subject has developed in the past seventeen years. In the reviewer's opinion, this method of approach is likely to appear more coherent than the former to students approaching groups in detail for the first time. A considerable number of new technical terms must be kept in mind, but with either van der Waerden's book or Albert's as a preliminary course, the terminology and the ideas behind it will offer no serious difficulty.

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Axiomatische Untersuchungen zur projektiven, affinen und metrischen Geometrie. By Eugen Roth. (Forschungen zur Logik und zur Grundlagen der exacten Wissenschaften, new series, no. 2.) Leipzig, Hirzel, 1937. 58 pp.

This work, incorporating the material of a doctoral dissertation, falls into three main subdivisions: (1) an axiomatic development of projective, affine, and euclidean geometry; (2) concerning the axiomatizing of geometries; (3) the monomorphism of the postulate systems of projective, affine, and euclidean geometry. The author uses formal logical symbols explained in a glossary, whose introduction renders the reading difficult without considerable study. The emphasis here is upon the logical structure, and because of this point of view certain problems are brought out which were largely ignored by earlier writers on postulates. The specializations required to obtain affine geometry from projective, and euclidean from affine, are shown to be postulational, not definitional (American readers may recall R. L. Moore's critical remarks on this score in reviewing Veblen and Young's *Projective Geometry*). The "simplifications" familiar in reducing postulates for projective geometry, by prescribing lines and planes to be sets of points, rather than wholly undefined symbols, do violence to the principle of duality, as this writer points out effectively. The author remarks that the complete postulates for projective geometry require continuity of points on a line, and hence the "incidence geometries" illustrated by finite geometries are not "projective." Critical remarks upon Hilbert's "completeness postulate" for euclidean geometry (as given prior to the 1930 edition, see 4th to 6th editions) incorporate much that is found in recent German literature on the subject. The author is apparently unaware of the work of H. G. Forder, *The Foundations of Euclidean Geometry.* He is aiming at certain logically vulnerable spots in modern axiomatic treatments, rather than giving a survey of the subject as a whole.

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