SHORTER NOTICES


A space $E$ with uniform structure is a topological space restricted as follows. There exists on $E$ a system of neighborhoods $V_\alpha(p)$, one for each point $p$ of $E$ and each $\alpha$ in a nonvacuous set of values. Four conditions are fulfilled: (1) For every $(p, \alpha)$, $p$ is on $V_\alpha(p)$. (2) For any $(p, q)$ distinct in $E$, there exists an $\alpha$ such that $q$ is not on $V_\alpha(p)$. (3) For any two indices $(\alpha, \beta)$, there exists an index $\gamma$ such that, for every $p$, $V_\gamma(p)$ is on the common part of $V_\alpha(p)$ and $V_\beta(p)$. (4) For every $\alpha$ there is a $\beta$ such that if $(p, q)$ are both on $V_\beta(r)$, then $q$ is on $V_\alpha(p)$. The author of the book formulates these conditions as three axioms, combining (1) and (2). Any system $V_i(p)$ determines the same uniform structure in $E$ provided each $V_\alpha(p)$ contains a $V_i(p)$ and vice versa. The spaces with uniform structure are those in which uniform continuity has meaning. They are completely regular and they include all metric spaces and all topologic groups, whether metrisable or not. The article under consideration contains proofs for uniform spaces of a number of results previously established only with the aid of a metric. The source of these results is seen to inhere in the possibility of certain comparisons of neighborhoods in different parts of a uniform space. The work includes an extension theorem for uniformly continuous functions, a discussion of compact and locally compact spaces with regard to their uniform structure, and a number of group theoretic applications. There is also, at the start, a severe criticism of the role often played by the hypothesis that a space be separable. The author characterizes this hypothesis as an evil parasite which infests many works, lessening their scope and their clarity. The book terminates with a few interesting “observations on topologic axioms,” in which an attempt is made to distinguish between those axioms having only a historic interest and those truly important in the development of topology.

S. S. CAIRNS


The present volume is the second of a series in which Julia develops the theory of unitary spaces and Hilbert space. It is based on a sequence of lectures begun in 1935. After dealing with unitary spaces in his first volume, Julia turns here to the study of Hilbert space. He divides the discussion into five chapters, as follows: Section I. Hilbert space: Chapter 1, Vectorial analytic representation of Hilbert space. Geometrical study. Chapter 2, Functional analytic representation of Hilbert space. Chapter 3, Axiomatic study of Hilbert space. Axiomatic definition of Hilbert space. Section II. Linear transformations or operators in Hilbert space: Chapter 4, Linear operators. General properties. Representation and algebraic calculus of operators. Chapter 5, Inversion of bounded linear operators. Resolution of infinite systems of linear equations in Hilbert space.

The separate study of the spaces $\mathcal{H}_0$ and $\mathcal{H}_1$ on their own merits in the first two chapters is followed by the unification of their properties under the abstract or axiomatic point of view in the third chapter. The fundamental definitions and