SHORTER NOTICES

Sur les Espaces à Structure Uniforme et sur la Topologie Générale. By André Weil. Paris, Hermann, 1937. 39 pp.

A space E with uniform structure is a topological space restricted as follows. There exists on E a system of neighborhoods $V_\alpha(p)$, one for each point p of E and each α in a nonvacuous set of values. Four conditions are fulfilled: (1) For every (p, α), p is on $V_\alpha(p)$. (2) For any (p, q) distinct in E, there exists an α such that q is not on $V_\alpha(p)$. (3) For any two indices (α, β), there exists an index γ such that, for every p, $V_\gamma(p)$ is on the common part of $V_\alpha(p)$ and $V_\beta(p)$. (4) For every α there is a β such that if (p, q) are both on $V_\beta(r)$, then q is on $V_\alpha(p)$. The author of the book formulates these conditions as three axioms, combining (1) and (2). Any system $V'_\lambda(p)$ determines the same uniform structure in E provided each $V_\alpha(p)$ contains a $V'_\lambda(p)$ and vice versa. The spaces with uniform structure are those in which uniform continuity has meaning. They are completely regular and they include all metric spaces and all topologic groups, whether metrisable or not. The article under consideration contains proofs for uniform spaces of a number of results previously established only with the aid of a metric. The source of these results is seen to inhere in the possibility of certain comparisons of neighborhoods in different parts of a uniform space. The work includes an extension theorem for uniformly continuous functions, a discussion of compact and locally compact spaces with regard to their uniform structure, and a number of group theoretic applications. There is also, at the start, a severe criticism of the role often played by the hypothesis that a space be separable. The author characterizes this hypothesis as an evil parasite which infests many works, lessening their scope and their clarity. The book terminates with a few interesting "observations on topologic axioms," in which an attempt is made to distinguish between those axioms having only a historic interest and those truly important in the development of topology.

S. S. Cairns

Introduction Mathématique aux Théories Quantiques. Part 2. By Gaston Julia. (Cahiers Scientifiques, vol. 19.) Paris, Gauthier-Villars, 1938. 6+220 pp.

The present volume is the second of a series in which Julia develops the theory of unitary spaces and Hilbert space. It is based on a sequence of lectures begun in 1935. After dealing with unitary spaces in his first volume, Julia turns here to the study of Hilbert space. He divides the discussion into five chapters, as follows: Section I. Hilbert space: Chapter 1, Vectorial analytic representation of Hilbert space. Geometrical study. Chapter 2, Functional analytic representation of Hilbert space. Chapter 3, Axiomatic study of Hilbert space. Axiomatic definition of Hilbert space. Section II. Linear transformations or operators in Hilbert space: Chapter 4, Linear operators. General properties. Representation and algebraic calculus of operators. Chapter 5, Inversion of bounded linear operators. Resolution of infinite systems of linear equations in Hilbert space.

The separate study of the spaces $\mathfrak{H}_0$ and $\mathfrak{L}_2$ on their own merits in the first two chapters is followed by the unification of their properties under the abstract or axiomatic point of view in the third chapter. The fundamental definitions and theorems of the theory of operators, including their reduction by linear manifolds, are found in Chapter 4. The fifth and last chapter deals with the problem of finding the inverses, if any, of a bounded linear operator A—that is, the operators X such that XA or AX is the identity. In both Chapters 4 and 5 special attention is devoted to examining the relations between operators, matrices, and infinite systems of linear equations. According to the author's preface, the spectral theory for *bounded* operators will be reached in the succeeding volume.

The topics included in the book are presented from a purely mathematical point of view in a clear and lively style. The applications to the theory of matrices and equations, which are largely implicit in certain of the more abstract treatments, are elaborated here with a wealth of detail which renders them unusually accessible to the student. The author's approach to the modern theory of operators is obviously a cautious one, presumably because of his desire to keep the reader on ground which shall appear as nearly familiar as possible at every stage. In the absence of any indication of the methods which Professor Julia proposes to use in the treatment of the spectral theory or of unbounded operators, the reviewer cannot judge whether such caution is excessive or not. Nevertheless, there would be obvious and important advantages in a more rapid approach to these central topics, beyond which lie the really difficult parts of operator-theory.

The reviewer has noted very few misprints. On page 72, lines 11–14, $\leqq$ should be replaced by $<$; and on page 115, line 11, $<$ by $\leqq$. The attribution to A. Weil of the theorem stated and proved on pages 23–25 is incorrect; in the cases of $\mathfrak{H}_0$ and $\mathfrak{L}_2$ the theorem has long been known; in the abstract form it is a special case of a theorem of Banach (*Théorie des Opérations Linéaires*, page 80, Theorem 5); and the proof given here is essentially that published by von Neumann in Mathematische Annalen, vol. 102 (1929), page 380, footnote.

M. H. STONE

Nouveaux Éléments d'Analyse. Calcul Infinitésimal. Géométrie. Physique Théorique. Vol. 2: *Variables Complexes.* By A. Buhl. Paris, Gauthier-Villars, 1938. 214 pp.

This book is an ably written volume with a view towards a synthesis of the fields indicated. The main purpose of the work is to give young minds not possessed of an extensive mathematical knowledge a quick access to certain higher fields. Throughout the book pertinent references to significant papers and books are given, thus enabling the reader to study in greater detail any field in which he may become interested.

In Chapter 1 the author points out the importance of the equation $\phi(ax)=b\phi(x)$ (É. Borel) for the consideration of periodic, analytic, and nonanalytic functions. Emphasis is placed on the fact that it is possible to pass from the simplest cyclic functions to periodic functions. Certain quantitative generalizations of cyclic functions are given, together with the indication that generalizations of such type for the simplest Schrödinger equations lead to the study of quantitative configurations of the atom. The author then introduces analytic functions and indicates that trigonometric series may be connected in a profound manner with analytic considerations. The chapter concludes with the study of Abel's functional equation, Julia's iterations of rational fractions, automorphic functions (Henri Poincaré), and the homographic group. Among the problems reference is made to the Gibbs phenomenon.

In Chapter 2 the point of view is mainly that involved in the treatises of Picard and Goursat. An analytic function is defined by the property that (under suitable conditions) its integral along an arc AB depends only on the extremities A, B. The author states that a definition of such type is in harmony with modern science;