## JEFFREYS ON SCIENTIFIC INFERENCE

*Scientific Inference.* By Harold Jeffreys. Cambridge, University Press, 1937. 6+272 pp.

This book was first published in 1931, and is now reissued, with addenda. Its beginnings were in a series of papers published by the author and Dorothy Wrinch in the "Philosophical Magazine" and in "Nature" during the years 1919 to 1923. Professor Jeffreys, of St. John's College, Cambridge, ventures here into the nature of scientific inquiry, and especially into what he considers the chief guiding principle of both scientific and everyday knowledge: that it is possible to learn from experience and to make inferences from it beyond the data directly known by sensation. These inferences themselves, however, are subjected to change. Fundamental scientific laws have been found inaccurate. For twenty years physical science has been modifying and reconstructing its laws as a result of new knowledge. The reconstruction has followed the old method, but will this always be possible? Have recent developments shown that scientific method itself is open to suspicion?

Jeffreys attempts in his book to place scientific method above suspicion. "There is no more ground now than thirty years ago for doubting the general validity of scientific method, and there is no adequate substitute for it. When we make a scientific generalization we do not assert the generalization or its consequences with certainty; we assert that they have a high degree of probability on the knowledge available to us at the time, but that this probability may be modified by additional knowledge. The more facts are shown to be coordinated by the law, the higher the probability of that law and of further inferences from it. But we can never be entirely sure that additional knowledge will not some day show that the law is in need of modification. The law is provisional, not final; but scientific method provides its own means of assimilating new knowledge and improving its results" (pp. 6–7).

This reasoning therefore depends on the introduction of probability considerations. "The notion of probability, which plays no part in logic, is fundamental in scientific inference. But the mere notion does not take us far. We must consider what general rules it satisfies, what probabilities are attached to propositions in particular cases, and how the theory of probability can be developed so as to derive estimates of the probabilities of propositions inferred from others and not directly known by experience" (p. 7).

The notion of probability is established as a relation between a proposition and a set of data, intelligible to everyone who ever says, "I am nearly sure of that," or similar expressions. Every probability can be associated with a real number from 0 to 1. A symbolism can be developed starting with $P(p|q)$ for the probability of the proposition $p$ on the data $q$. If $p_1, p_2, \cdots, p_n$ are a number of mutually exclusive hypotheses such that one of them must be true, and if $P(p|q\cdot h)$ is the probability that $p$ is true given $h$ and $q$, then Jeffreys' central theorem is Bayes' law:

$$P(p_r|q\cdot h) = \frac{P(q|p_r\cdot h)P(p_r|h)}{\sum_{r=1}^{n} P(q|p_r\cdot h)P(p_r|h)}.$$

"This law," writes the author, "is to the theory of probability what Pythagoras' theorem is to geometry" (p. 19).

Bayes' law enables us to pass from a priori probabilities, through superior knowledge, to new, a posteriori probabilities. We must start somewhere, with a prior probability. Scientific work tends to make the posterior probability so near zero or unity as to amount to practical certainty. This contention is exemplified by an exposition of the theory of sampling. In the process of discovery of quantitative laws, as in physics, we cannot apply the general principle without some further specification. This is found in the so-called simplicity postulate. Jeffreys establishes it in the form: "Every quantitative law can be expressed as a differential equation of finite order and degree, in which the numerical coefficients are integers" (p. 45). Such a law restricts the number of admissible laws to $\aleph_0$ and the a priori probabilities must decrease with decreasing simplicity.

Now follows an investigation of the theory of errors. There is a belief that there are true values of the quantities to be measured, and that the observed values have certain errors. The existence of true values is a postulate, and its validity has to be determined. This leads to a derivation and discussion of the normal law of errors, and of different types of errors.

The remaining part of the book is mainly an application to various fields of theoretical physics of the principles enunciated in the first chapters. They deal with the measurement of physical magnitude, mensuration (distance and angle), Newtonian dynamics, light and relativity, and contain careful studies of the different steps necessary in building the concepts and the equations, and their justification by the available evidence. In regard to the theory of relativity we meet the conclusion that there is no antagonism between the principle of relativity and the simplicity postulate. The general theory of relativity is justified as a general law up to a certain point, and the simplicity postulate entitles us to extend it further, if possible.

In the last chapter we find a discussion of miscellaneous questions. First comes the topic: "Is there a non-quantitative simplicity postulate?" The question is answered in the affirmative. The principle seems to be that if an object of a given class has $r$ properties $a, b, c, \cdots, k$, then there is a finite a priori probability that all future members of the class with any $r-1$ of these properties will also have the remaining one. This principle guides the botanist in determining a new species. Under "Ultimate concepts" the question is discussed whether the process of constructing concepts of increasing generality will ever stop. This is denied in the case of the theory of quanta, and that of life. There are also observations on the subconscious, on determinism and causality. Another question is whether "neglecting small quantities" and arguing by "orders of magnitude" can be taken as a departure from popular standards of accuracy.

In Chapter 11 the author compares his theory of scientific knowledge to other theories, notably the statistical theory of probability (Venn), Keynes' theory, Russell's theory of sense data, Whitehead's theory of events. In an appendix we find some data on probability in logic and pure mathematics, on transfinite numbers, and the analytic treatment of sine and cosine. In an "Addenda" Jeffreys gives us a summary of new results, in which his theory now can decide whether a set of observed data support or do not support the introduction of a new parameter to express them.

The book of Jeffreys is an interesting and valuable contribution to the whole problem of induction. The assertions of philosophers on this subject are seldom substantiated to such an extent by a large number of examples from many branches of natural science. The systematic way in which the author, step after step, analyzes the different experiences which justify the theoretical foundations of the sciences is quite valuable for a fundamental understanding. The student of the theory of samples and errors, of dynamics, of light, and of relativity will find much useful and stimulating material in this book, even if he has his doubts about Jeffreys' specific theory of scientific inference.

To the reviewer, at any rate, such doubts exist. He thoroughly agrees with the author's materialistic point of view, which rejects the doctrine that a physical object is nothing but a class of sensibilia. "The physical object," he argues against Russell, "and the laws of physics are anterior in knowledge to the sensibilia" (p. 226). In other words, science deals with an objective world. Jeffreys' theory now seems to state that all our scientific laws are more or less probable expressions of objective laws. It seems doubtful, however, whether it is correct to introduce the concept of probability for expressing the fact that our laws are approximations of laws in nature. Probability is a concept of natural philosophy too, expressing an objective phenomenon. It is mathematically expressed by the theory of measure, and physically realized in statistical behavior. It is a notion concerning the behavior of groups, and as such expresses facts which are not necessarily enforced for the individual. It is not clear what the meaning of probability, thus conceived, can be for the validity of, say, Newton's law. For Jeffreys, however, probability is another theory. It expresses a relation between a proposition and a set of data. Probability expresses a judgment.

This theory of scientific inference is therefore a theory of judgment, and introduces a subjective element into the expression of the laws of nature. It seems to the reviewer that the certainty of obtaining objective knowledge by induction can only be obtained by eliminating the subjective element from our observations, and not by applying a method which itself brings in a subjective element. Somehow the question: How probable are Maxwell's equations? seems to miss the point. But even if we may have our doubts concerning the success of Jeffreys' efforts in the theory of knowledge, we must agree at the same time that the careful way in which he has built up his case allows us to study in detail its different implications.

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