SCHOUTEN AND STRUIK ON DIFFERENTIAL GEOMETRY

Einführung in die neuren Methoden der Differentialgeometrie. By J. A. Schouten and D. J. Struik. 2d completely revised edition. Vol. 1. *Algebra und Übertragungslehre.* By J. A. Schouten. 12+202 pp. Vol. 2. *Geometrie.* By D. J. Struik. 12+338 pp. Groningen, Noordhoff, 1935 and 1938.

As a revision and extension of the authors' previous treatises, this second edition of their joint work should be compared, not merely with the first edition, a short monograph published in 1924, but also with Struik's *Grundzüge der mehrdimensionalen Differentialgeometrie in direkter Darstellung,* published in 1922 and now out-of-print. The monograph dealt primarily with the algebraic and analytic aspects of the subject, whereas Struik's book was concerned essentially with Riemannian geometry.

Perhaps the most striking difference between the present work and those cited is in the methods employed. Schouten's system of direct analysis was used concurrently with the method of tensors in the original monograph and was employed almost exclusively in Struik's *Grundzüge.* In the book under review, this system has been completely discarded and tensor analysis given full sway.

In the notation for the components of geometrical objects Schouten's *Kern-Index-Methode* is consequentially applied throughout the treatise. The essence of this method consists in representing the effect on the components of a geometrical object due to a change of the system of reference, not by a different or primed central letter with indices of the original type, as is frequently the custom, but by the original central letter with indices of a different type. The change in the central letter with the preservation of the type of indices is reserved to represent the transformation of the object resulting from an actual transformation of the space. The method seems logically sound in principle and makes for definiteness and conciseness in practice. Readers to whom it is new will find that it is readily followed once they accustom themselves to scanning each formula for the full import of all the letters, central and appended, that are involved.

The outstanding advance in general methods since the appearance of the authors' first books has undoubtedly been the exploitation of nonholonomic systems of reference. These are introduced at the earliest opportunity in the first volume of the new work and are used throughout wherever fitting. In fact, the ordinary sign of equality is employed to indicate the validity of a relationship for all types of systems of reference. To indicate that an equation is true only for holonomic systems, an h is placed above the sign of equality, and a * serves a similar purpose in the case of an equation which holds only for the specific system of reference in use at the moment. Here, also, the ends of clarity and conciseness are well served.

Whereas the volumes of the early twenties treated only ordinary Riemannian geometry, the present work covers also the general linear connection and Hermitian connections, as well as the extension of Riemannian geometry to the general case in which the metric is not necessarily definite. In particular, in the latter connection, isotropic subspaces are given more than the customary passing mention.

The first volume begins with the algebraic foundation, including the fundamentals of affine geometry and of unitary geometry. The second part of this volume, in which the tools from analysis are developed, brings in at once nonholonomic systems of reference, develops the general linear connection with respect to them, discusses the D-symbolism of van der Waerden and Bortolotti in relation to the machinery for treating subspaces, treats geodesics and normal coordinates, curvature tensors and related topics, and ends with a discussion of the differential operators of importance in problems of variation and deformation.

The second volume falls into four parts. The first, on curves, deals with individual curves with respect to both a Riemannian connection and the general linear connection, and considers congruences of curves and natural systems of curves in a Riemannian space V_n. The second treats hypersurfaces in a V_n, including the study of individual curves and congruences of curves which lie in them, and the Gauss-Codazzi equations. In the third part is the corresponding material in the case of a V_m in a V_n, together with a discussion of the conditions under which a V_m may be imbedded in a Riemannian space of constant curvature S_n, and applications of the Gauss-Codazzi-Ricci equations to special problems pertaining to the class of a V_m and the possibility of deforming it in an S_n. The last part treats a diversity of selected topics: the curvature properties of nonholonomic spaces with respect to the general linear connection; the fundamentals of the theory of infinitesimal deformation; isotropic subspaces, in general nonholonomic, of a V_n; path-preserving transformations of affine connections with an introduction to projective connections; conformal transformations and connections; Hermitian connections.

Scattered through each volume there are numerous problems, ranging from simple exercises designed for beginners to substantial problems incorporating the results of research. The solutions of the problems which are given at the ends of the volumes will be welcome both to novice and expert. Both volumes are provided with excellent bibliographies and indices.

The exposition is generally maintained on so high a plane that criticism, even of a minor nature, seems out of order. It should, nevertheless, be pointed out that the reader, and particularly the beginner, would have been better served if in the early pages of the first volume the material had not been so condensed, from the original edition, as to force important definitions into the footnotes and to eliminate entirely the discussion of the geometrical significance of contragredient transformations.

The term *second edition* must here be interpreted as reincarnation, both in flesh and in spirit. The new book should prove an invaluable addition to the literature of differential geometry and an incentive to further progress.

W. C. GRAUSTEIN