ence of various branches of science and human activity on one another.

While the reviewer can recommend this book as interesting, clear and stimulating for the moderately well informed reader, he feels that it is unnecessarily long due to redundance of examples, and that its lack of references is an unfortunate weakness.

J. K. L. MacDonald

Punktreihengeometrie. By E. A. Weiss. Leipzig and Berlin, Teubner, 1939. 8+232 pp.

The purpose of this book is to serve as an introduction to the projective geometry of higher dimensional spaces. The author adopts the point of view of Reye in regarding such space as the map space of various three-dimensional configurations and he thereby succeeds in bringing together a large variety of topics.

After a brief recapitulation of the elementary geometry of the line, the author introduces the concept of a (linear) point range (punktreihe) on a line obtained by equating to zero a bilinear form in variables (ξ_1, ξ_2) and (τ_1, τ_2), which he writes using the Clebsch-Arnold symbolic notation as $(\gamma\xi)(\mu\tau)=0$. He interprets (τ_1, τ_2) as a parametrization of the points (ξ_1, ξ_2) of the line, and he subsequently defines a point range more generally as a "one-dimensional rational manifold provided with a definite parametrization." By means of the coefficients $\gamma_i\mu_j$ linear point ranges on the line can be put in 1-1 correspondence with the points of projective 3-space; singular ranges map into a ruled quadric, pencils and bundles of ranges into lines and planes. From the properties of singular ranges the author deduces the elementary properties of the quadric.

The two main chapters of the book deal with point ranges (and their duals) in two and three dimensions. By using symbolic notation, the Clebsch correspondence principle, and similar devices, the author derives easily such fundamental results as the harmonic properties of a quadrilateral, the projective generation of a conic, and the polar theory in the plane. Further properties of a conic follow by considering it as a point range of the second order defined analytically by $(um)(\gamma\tau)^2=0$. Pascal's configurations are obtained by mapping binary quadratic forms on the points of a plane.

Linear point ranges in the plane can be mapped on R_5, projective space of five dimensions, and singular ranges correspond to a Segre manifold of three dimensions and of order 3. The singular ranges of a pencil or bundle will map into rational cubic curves and surfaces.

The third chapter deals with geometry in R_3 and the development parallels that of the second. Now the map space of linear point ranges is an R_7, the Segre manifold is quartic and four-dimensional, and properties of quartic curves and surfaces follow by considering the singular ranges in pencils and bundles. Here, as in the preceding chapters, the author studies the collineations of the map space which leave fixed the corresponding Segre manifold. By noting that quadric hypersurfaces in R_7 are the maps of linear complexes, the author leads naturally to the principle of triality of Study and Cartan, null systems, the line sphere transformation, and properties of oriented spheres.

The final chapter deals with trilinear forms and some applications to the preceding situations as well as to non-euclidean geometry.

The author has succeeded in bringing together a large variety of geometrical configurations and studying them in a way which will stimulate the student's interest and arouse his admiration for geometrical methods.

HARRY LEVY

Mathematical Recreations and Essays. 11th edition. By W. W. Rouse Ball. Revised by H. S. M. Coxeter. New York, Macmillan, 1939. 16+418 pp.

For almost half a century the earlier editions of this book[1] have provided a rich supply of mathematical topics commonly known as recreations, which although they often involve fundamental mathematical methods and notions, yet make their appeal in the spirit of a game or a puzzle, rather than with an eye to the usefulness of their conclusions. Since no knowledge of the calculus or of analytic geometry is presupposed, many of these recreations make excellent subjects for student talks in undergraduate mathematics clubs.

The eleventh edition, revised by H. M. S. Coxeter after Ball's death, not only "aims to preserve the spirit of Ball's delightful book," but does. It is, nonetheless, a thorough-going revision of the tenth edition. The chapters on Mechanical Recreations, Bees and their Cells, and String Figures have been omitted. In their stead we find the following material.

(1) A large new section of arithmetical recreations (chap. II), in-

[1] Editions of this book have also appeared in French and Italian. A three-volume edition in French with considerable additional material, especially in the history of numbers, was edited by J. Fitz-Patrick in Paris (1907–1909), and a new edition appeared in 1926–1927 with new material by A. Margossian, Reinhart, J. Fitz-Patrick and A. Aubry. Two Italian editions were printed in Bologna, the first in 1911 by D. Gambioli, 398 pp., and the second in 1927 by D. Gabioli and G. Loria.