parallels that of the second. Now the map space of linear point ranges is an R_7, the Segre manifold is quartic and four-dimensional, and properties of quartic curves and surfaces follow by considering the singular ranges in pencils and bundles. Here, as in the preceding chapters, the author studies the collineations of the map space which leave fixed the corresponding Segre manifold. By noting that quadric hypersurfaces in R_7 are the maps of linear complexes, the author leads naturally to the principle of triality of Study and Cartan, null systems, the line sphere transformation, and properties of oriented spheres.

The final chapter deals with trilinear forms and some applications to the preceding situations as well as to non-euclidean geometry.

The author has succeeded in bringing together a large variety of geometrical configurations and studying them in a way which will stimulate the student's interest and arouse his admiration for geometrical methods.

HARRY LEVY

Mathematical Recreations and Essays. 11th edition. By W. W. Rouse Ball. Revised by H. S. M. Coxeter. New York, Macmillan, 1939. 16+418 pp.

For almost half a century the earlier editions of this book[1] have provided a rich supply of mathematical topics commonly known as recreations, which although they often involve fundamental mathematical methods and notions, yet make their appeal in the spirit of a game or a puzzle, rather than with an eye to the usefulness of their conclusions. Since no knowledge of the calculus or of analytic geometry is presupposed, many of these recreations make excellent subjects for student talks in undergraduate mathematics clubs.

The eleventh edition, revised by H. M. S. Coxeter after Ball's death, not only "aims to preserve the spirit of Ball's delightful book," but does. It is, nonetheless, a thorough-going revision of the tenth edition. The chapters on Mechanical Recreations, Bees and their Cells, and String Figures have been omitted. In their stead we find the following material.

(1) A large new section of arithmetical recreations (chap. II), in-

[1] Editions of this book have also appeared in French and Italian. A three-volume edition in French with considerable additional material, especially in the history of numbers, was edited by J. Fitz-Patrick in Paris (1907–1909), and a new edition appeared in 1926–1927 with new material by A. Margossian, Reinhart, J. Fitz-Patrick and A. Aubry. Two Italian editions were printed in Bologna, the first in 1911 by D. Gambioli, 398 pp., and the second in 1927 by D. Gabioli and G. Loria.

cluding discussions of derangements of n quantities, repeating decimals, rational right-angled triangles, finite arithmetics, and the distribution of primes; (2) a new chapter (chap. V) on polyhedra—a subject in which the author is expert—written in a manner which is intelligible to a beginner equipped with a modicum of ability in space perception, and supplemented by two excellent plates showing models of the regular and semiregular solids; (3) a more extended treatment of magic squares and their generalizations; (4) a discussion, in the chapter on map colouring problems, not only of the famous four-colour problem in the plane, but of the seven-colour problem on the torus, and of other topics connected with the stereographic projections of the regular polyhedra.

The first edition of 1892 was entitled "Mathematical Recreations and Problems" and consisted of two parts, the one on Recreations and the other on Problems and Speculations. Of the five chapters in the latter part only the one on Three Classical Problems has been retained in all editions. The chapters on Astrology, Hyper-space, Time and its Measurement, and the Constitution of Matter, became the basis for a much enlarged second half of the fifth edition (1911) which bore the title of Mathematical Essays. The space devoted to essays was considerably reduced in the tenth edition (1922). It now includes the Three Classical Geometrical Problems (chap. XII), an essay on calculating prodigies (chap. XIII), and an essay on cryptography (chap. XIV), the latter being completely revised for the present edition by Abraham Sinkov, a cryptanalyst in the U. S. War Department.

A partial summary of the contents of the book is as follows. *Chapter I* includes arithmetical recreations whose interest is mainly historical rather than arithmetical. Some of these are of the "think of a number" type, others involve digit notations, and still others are tricks with cards or games with counters. *Chapter II* opens with a series of arithmetical fallacies, continues with problems of probability derangements and arrangements, decimal expansions, rational triangles, finite arithmetics, D. H. Lehmer's number sieve for prime factors, and concludes with a discussion of perfect numbers, Mersenne's numbers, and Fermat's theorem. *Chapter III* is composed mainly of geometrical fallacies and paradoxes, problems in dissection, cyclotomy and area-covering. The deltoid solution to Kakeya's minimal problem, erroneously attributed to Kakeya (p. 100) was really suggested by Professors Osgood and Kabota according to Question 39, American Mathematical Monthly (1921), p. 125. *Chapter IV* is concerned with statical and dynamical games of position.

Among topics discussed are some extensions of the game of three in a row, tessellations of the plane, problems with moving counters, and the effect of cutting a Möbius strip in various ways. *Chapter V* gives a comprehensive elementary discussion of the relations between the faces, edges, and vertices and the associated angles of the regular solids and the Archimedean solids, which is well illustrated by good figures. Stellated polyhedra, solid tessellations, and the kaleidoscope each receive some attention. The use of the term Platonic for the regular solids might be questioned since they were known before Plato. *Chapters VI* and *VII* contain familiar recreations associated with the chessboard and with magic squares. Similar problems with dominoes and with magic cubes are also discussed. *Chapter VIII* treats the general theory of the four-colour problem more elaborately than the earlier editions of this book, mentions briefly such matters as orientable surfaces and dual maps, and more fully the seven-colour mapping problem on the torus, and finally considers various colouring problems on the regular polyhedra. *Chapter IX* discusses mazes and other similar problems whose solutions depend on the unicursal tracing of a route through prescribed points (nodes) over various given paths. *Chapter X* features certain combinatorial problems known under the title of Kirkman's school-girl problems, and ends with a similar problem about arranging members of a bridge club at tables so that different members shall play together in successive rubbers. *Chapter XI*, on Miscellaneous Problems, contains an account of the Fifteen Puzzle, the Tower of Hanoï, Chinese Rings, and various mathematical card tricks. *Chapter XII* contains the famous classical problems concerning the duplication of the cube, trisection of an angle, and quadrature of the circle. *Chapter XIII* is an essay on calculating prodigies which introduces over a dozen famous mental calculators beginning with Jedediah Buxton and Thomas Fuller in the eighteenth century, and including two American calculators Zerah Colburn and Trueman Henry Safford, and gives something of their histories and the type of problems they could solve. *Chapter XIV* is a chapter on cryptography and cryptanalysis written by Dr. Abraham Sinkov. It presents in easily understandable form the chief elements in a cryptographic system, and gives various possible ways for attempting to solve such a cipher.

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Einführung in die Zahlentheorie. By Arnold Scholz. (Sammlung Göschen, no. 1131.) Berlin, de Gruyter, 1939. 136 pp.

The topics which should be taken up in an introduction to the