The book is attractively and very legibly printed on paper of good quality.

J. H. CURTISS

Le Origini del Calcolo Infinitesimale nell' Era Moderna. By G. Castelnuovo. Bologna, Zanichelli, 1938. 164 pp.

This book is number 12 of the series Per la Storia e la Filosofia della Matematiche, a collection published under the auspices of the Italian Istituto Nazionale per la Storia delle Scienze and which was founded and directed by F. Enriques, whose name no longer appears. After an introductory chapter on Archimedes it reports on the growth of the infinitesimal calculus in the sixteenth and seventeenth centuries, beginning with Maurolycus and Commandinus and ending with Newton and Leibniz. An appendix contains Newton's "Tractatus de quadratura curvarum" (1704) and Leibniz' "Nova methodus pro maximis et minimis" (1684) in an Italian translation with notes by Professor E. Carruccio. Professor Vacca has given valuable advice in the composition of the book.

The distinguished geometer of the University of Rome has given an unusually clear picture of the different steps which the mathematics of the infinitesimal took in its formative years. "I have tried to show in the clearest possible way how the new science originated when the antique conceptions of Archimedes' genius were fertilized with the new doctrines of algebra and analytical geometry on one side, and of dynamics (or better cinematics) on the other," he writes in the preface. Though he gives a brief sketch of the lives of each of the great contributors, he intentionally avoids discussion of the many personal and priority squabbles which characterized some of their work. Concentrating on the major figures, he has been able to reconstruct before our eyes the important results which gradually completed the structure of the differential and integral calculus. He uses a number of more recent papers, especially some written by Italian historians, and adds some interesting information in the concise bibliographies which end each chapter.

The reader will be wise to compare this book with the recent book of C. B. Boyer, *The Concepts of the Calculus* (Columbia University Press, New York, 1939), which discusses a larger period and also penetrates deeper into the philosophical arguments which formed an important aspect of the study of the infinitesimals from the time of Zeno to the time of Cantor, but has not Castelnuovo's wealth of mathematical details. A sociological study of the development of the calculus, which can only explain its rise and growth in the sixteenth and seventeenth centuries, still has to be written.

D. J. STRUIK

Algebren. By M. Deuring. (Ergebnisse der Mathematik, vol. 4, no. 1.) *Gruppen von linearen Transformationen*. By B. L. van der Waerden (Ergebnisse der Mathematik, vol. 4, no. 2.) Berlin, Springer, 1935. 5+143 and 3+91 pp., respectively.

The theory of algebras, now about to enter the second century of its existence, constitutes today an integrating part of algebra and arithmetics. The most fundamental step in its development seems to have been the introduction of general reference fields, essentially due to Wedderburn. In order to describe approximately the degree of generality we may say that Wedderburn's theory holds at least for those fields which obey the theory of Galois. We quote from Dickson's *Linear Algebras* (1914): "Any linear associative algebra over a field F is the sum of a semisimple algebra and a nilpotent invariant subalgebra (the radical) each over F. A semisimple algebra is either simple or the direct sum of algebras over F. Any simple algebra over F is the direct product of a division algebra and a simple matrix algebra each over F."

Other results are found in Dickson's book *Algebras and their Arithmetics* (1923), which concludes with an instructive list of unsolved problems: (I) the determination of all division algebras, (II) the classification of nilpotent algebras, the discovery of relations between an algebra and its maximal nilpotent invariant subalgebra (the radical), (III) theory of non-associative algebras, and (IV) theory of ideals in the arithmetic of a division algebra and the extension to algebras of the whole theory of algebraic numbers.

Progress in the study of problems II and III has been moderate, in the sense that we have many beautiful special results but no general theory.

As to problems I and IV, our knowledge has advanced considerably, to say the least; this advance is reported in Deuring's report *Algebren* and Albert's Colloquium lectures *Structure of Algebras*. We quote both authors in saying that R. Brauer, H. Hasse, E. Noether and A. A. Albert have given the solution of problem I for algebraic number fields. From the authors who contributed to the solution of problem IV we single out (at the expense of others) H. Brandt who provided the fundamental idea that a left ideal in one maximal domain of integrity is a right ideal in another, and his pupil Eichler