ABSTRACTS OF PAPERS

SUBMITTED FOR PRESENTATION TO THE SOCIETY

The following papers have been submitted to the Secretary and the Associate Secretaries of the Society for presentation at meetings of the Society. They are numbered serially throughout this volume. Cross references to them in the reports of the meetings will give the number of this volume, the number of this issue, and the serial number of the abstract.

203. J. W. Calkin: *A quotient ring over the ring of bounded operators in Hilbert space.* I.

Let $\mathfrak{B}$ denote the ring of bounded everywhere defined operators in Hilbert space $\mathfrak{H}$. The subset $\mathfrak{T}$ of totally continuous operators is a two-sided ideal in $\mathfrak{B}$ in the ordinary algebraic sense, and the quotient ring $\mathfrak{B}/\mathfrak{T}$ can be constructed in the usual way; moreover, since $\mathfrak{T}$ is closed with respect to the operation *, this operation can be defined in $\mathfrak{B}/\mathfrak{T}$ too. Defining a norm in $\mathfrak{B}/\mathfrak{T}$ by the equation $|\alpha| = \text{g.l.b.}|A|$, A in α, where $|A|$ is the bound of the operator A in $\mathfrak{H}$, the author shows that $\mathfrak{B}/\mathfrak{T}$ is a complete metric space. Further, he shows that there exists a unitary space $\mathfrak{K}$ (nonseparable, however) and a set $\mathfrak{M}$ of bounded everywhere defined operators in $\mathfrak{K}$ which is a $(+, \cdot, *)$-isomorphism of $\mathfrak{B}/\mathfrak{T}$. In addition, if $T(\alpha)$ denotes the element of $\mathfrak{M}$ corresponding to α in $\mathfrak{B}/\mathfrak{T}$, the bound of $T(\alpha)$ is $|\alpha|$. Thus the correspondence $\mathfrak{B}/\mathfrak{T} \longleftrightarrow \mathfrak{M}$ is an isometry, and $\mathfrak{M}$ is an algebraic ring of operators closed in the uniform topology. Other results are: If $\mathfrak{J}$ is an arbitrary two-sided ideal in $\mathfrak{B}$, either $\mathfrak{J} \leqq \mathfrak{T}$ or $\mathfrak{J} = \mathfrak{B}$. Every self-adjoint transformation $T(\alpha)$ in $\mathfrak{M}$ has a pure point spectrum. (Received February 24, 1940.)

204. J. W. Calkin: *A generalization of a theorem of Weyl.*

The paper defines the augmented resolvent set of a bounded operator A in Hilbert space $\mathfrak{H}$ as the set of values of λ such that $\mathfrak{R}(A-\lambda I)$, the range of $A-\lambda I$, is closed, while $\mathfrak{H} \ominus \mathfrak{R}(A-\lambda I)$ and the manifold of zeros of $A-\lambda I$ each have a finite dimension number. It then proves, by recourse to very simple properties of the homomorphism $\mathfrak{B} \rightarrow \mathfrak{M}$ defined in abstract 46-5-203, that two operators A and B whose difference is totally continuous have the same augmented resolvent set. Since, for a self-adjoint A, the complement of this set is precisely the set of Häufungspunkte of the spectrum of A in the sense of Weyl (Rendiconti del Circolo Matematico di Palermo, vol. 27 (1909), pp. 373–392), Weyl's theorem to the effect that the latter set is the same for any two self-adjoint operators whose difference is totally continuous follows at once. (Received February 24, 1940.)

205. J. W. Calkin: *Functions of several variables and absolute continuity.* I.

The author studies various properties of real- and complex-valued functions of n real variables which are potential functions of their generalized derivatives in the sense of G. C. Evans. It is shown that every such function is equivalent to a function differentiable with respect to each variable almost everywhere (previously proved by

Evans for the case $n=2$) and various kinds of approximation to such functions are considered. In particular, it is shown that a sequence $\{f_n\}$ of such functions defined on an open set, which converges there together with the sequences of partial derivatives in the mean of order $p \geqq 1$, has a limit which is a function of the same type. (Received February 24, 1940.)

206. Roy Dubisch: *Non-cyclic algebras of degree four and exponent two with pure maximal subfields.*

In a paper published in this Bulletin (vol. 44 (1938), pp. 576–579), A. A. Albert proved the falsity of the converse of the well known proposition that a cyclic normal division algebra contains a quantity j whose minimum equation is $x^n=j$ in the base field of the algebra. His proof consisted of an example of a non-cyclic normal division algebra of degree and exponent four over a non-modular field containing a quantity j as described above. It is the purpose of this paper to show that the exponent does not affect the property, by constructing similarly an algebra of degree four and exponent two over the same field. (Received February 17, 1940.)

207. M. H. Heins: *A note on a theorem of Radó.*

Radó has shown that there are no $(1, m)$ conformal maps of a multiply-connected region of finite connectivity, the boundary of which consists of p disjoint continua, onto itself for $m>1$. His proof is based on the possibility of mapping such a region one-to-one and conformally onto a region bounded by p disjoint circles. It is the object of the present note to prove this theorem without recourse to the possibility of mapping the given region one-to-one and conformally onto a region of canonical type. The technique employed consists of a systematic use of the theory of iteration and of a simple extension of Nevanlinna's principle of harmonic measure. This technique may be also used to establish the theorem that the number of $(1, 1)$ conformal maps of a region of finite connectivity p (greater than 2) onto itself is necessarily finite. (Received February 20, 1940.)

208. E. V. Huntington: *Congressional reapportionment by the method of smallest divisors.*

It was shown in a former paper (Transactions of this Society, vol. 30 (1928), pp. 85–110) that there are five possible methods of apportioning representatives in Congress, and that the method of equal proportions has the property of putting each state as nearly as may be on a par with every other state, both with respect to congressional districts and with respect to individual shares. It is shown in the present note that the method of smallest divisors has the property of minimizing the sum of the congressional districts. This method may therefore be called the method of minimum (total) load. (Received February 24, 1940.)

209. E. J. McShane: *On the second variation in certain anormal problems of the calculus of variations.*

If a curve E_{12} is a minimizing curve for a Lagrange problem with variable end points, and the order of anormality of E_{12} does not exceed 1, there are multipliers with which E_{12} satisfies the Euler equations and transversality conditions and the necessary conditions of Weierstrass, Clebsch and Jacobi. Except for the Jacobi condition, this has been previously proved without restriction on the order of anormality. In problems with a single side condition the order of anormality cannot exceed 1, so for such problems there are always multipliers with which the full set of necessary conditions is satisfied. By an example it is shown that the restriction on the order of anormality cannot be weakened. (Received February 21, 1940.)

210. Karl Menger: *On Green's formula and the integral of derivatives.*

We define a multiple integral as the limit of its Weierstrass sums, for example $\int\int(\partial p/\partial y-\partial q/\partial x)dxdy$ as the limit of the sums $W(N)=\sum\{[p(x_i, y_{j+1})-p(x_i, y_j)]/(y_{j+1}-y_j)-[q(x_{i+1}, y_j)-q(x_i, y_j)]/(x_{i+1}-x_i)\}(x_{i+1}-x_i)(y_{j+1}-y_j)$ for rectangular nets $N\{x_i, y_j\}$, when the diagonals of the meshes $[(x_{i+1}-x_i)^2+(y_{j+1}-y_j)^2]^{1/2}$ converge toward O. Whenever the Riemann double integral exists, our limit exists and is equal to the integral. For a rectangular domain of integration R, whenever p and q are continuous on R', the boundary of R, except perhaps a set of linear measure O, then our limit exists and is equal to $\int pdx+qdy$ along R', thus satisfying Green's formula regardless of the behavior of p and q in the interior of R. For, by cancelling the "interior" terms of $W(N)$ we see that $W(N)$ is a Weisertrass sum of $\int pdx+qdy$ along R'. The simple integral $\int_a^b f'(x)dx$ considered as the limit of its Weierstrass sums $\sum\{[f(x_{i+1})-f(x_i)]/(x_{i+1}-x_i)\}(x_{i+1}-x_i)$ is $f(b)-f(a)$ for each finite function f, since each of the Weierstrass sums has this value. (Received February 15, 1940.)

211. D. D. Miller: *Hereditary properties of continuous transformations.* Preliminary report.

Necessary and sufficient conditions are found for non-topological continuous transformations of various types (for example, monotone, non-separating, non-alternating, and so on) to have their defining properties on all subcontinua, or on certain special subcontinua, of the space S on which they are defined. Conversely, conditions are found under which continuous transformations on a space S, having these properties on certain collections of subcontinua, have the same properties on S. (Received February 24, 1940.)

212. Rufus Oldenburger and A. E. Porges: *The minimal numbers of binary forms.*

It was previously proved by one of the authors that the range of the minimal number of binary forms of degree n for a field K of characteristic greater than n is either $1, 2, \cdots, n$ or $1, 2, \cdots, n+1$. In the present paper by a study of certain determinants it is shown that the minimal number does not exceed n for finite fields of sufficiently high order. Since the minimal number of $x^{n-1}y$ is n, there exists for each ρ in the set $1, 2, \cdots, n$ a form with minimal number ρ. These results have application to the theory of equivalence and factorability of binary forms. (Received February 24, 1940.)

213. Rufus Oldenburger: *Polynomials in several variables.*

Various aspects of the theory of minimal representation of forms are developed in the present paper. It is proved in particular that for almost every field the range of the minimal number of forms of degree q in n essential variables overlaps the range of this number for forms of lower degree in the same number of essential variables. Adding a term λL^q, where L is linear and λ is in the given field, to a form of degree q changes the minimal number by at most 1. The coefficients in a minimal representation of a form comprise an invariant minimal class. Two forms are equivalent with respect to a given field if and only if their minimal classes with respect to this field are identical. If the minimal number of a form equals the number of essential variables in the form, the form has a unique minimal representation, and the problem of its equivalence to another form can be answered in a simple fashion. (Received February 27, 1940.)

214. Rufus Oldenburger: *On a class of non-negative matrices.*

In the present paper the existence of the infinite powers of a fairly general class of non-negative matrices arising in the Hardy Cross balancing process of engineering is proved. It follows that the associated balancings converge. (Received February 27, 1940.)

215. Gordon Pall: *On the arithmetic of quaternions.*

This article includes the results of abstract 39-5-137, somewhat generalized. Conditions for quaternions to have the same right or left factors of a given norm are obtained. Among other applications the relation between the number of classes of binary quadratic forms and representations by sums of three squares is derived in a simple way. (Received February 13, 1940.)

216. Everett Pitcher: *Homology groups under continuous maps.* I.

An analogue and extension of the Mayer-Vietoris formulas in the formulation of Alexandroff and Hopf (*Topologie*, I, pp. 287–299) is obtained as follows. Let f be a simplicial map on a complex K to a complex L which covers L. Call a chain of K *vanishing* if its image under f is null. Let U^k denote the group of vanishing k-cycles modulo the subgroup which bound vanishing chains. Let N^k be the subgroup consisting of those classes of cycles which bound on K. Let $B^k(K)$, $B^k(L)$ denote the groups of homology classes of k-cycles on K and L. Let S^k denote the group of homology classes of k-cycles of L which are images under f. Let T^k be the group of homology classes of k-cycles of K whose images under f bound. The formulas are the following. I. $B^k(L)$ mod $S^k = N^{k-1}$. II. $B^k(K)$ mod $T^k = S^k$. III. U^k mod $N^k = T^k$. Of course, II is well known. Applications and connections with familiar material seem numerous. (Received February 6, 1940.)

217. L. B. Robinson: *The solution of a functional equation on the contour of the unit circle.*

The functional equation (I) $u'(x) = a(x)u(x^2)$ admits a solution converging within the unit circle and on its contour. Also the equation (II) $u'(x) = a(x)\{u(x^2)\}^2$ admits a solution converging within the unit circle. Does this solution always converge on the contour? The following example shows that the answer is negative. A solution of the equation $u'(x) = (1+x)^2\{u(x^2)\}^2$ is $u(x) = 1/(1-x) = 1+x+x^2+\cdots$, and this series diverges on the contour. It is indeed possible to demonstrate that the solution of (II) is lacunary, but, as the above example shows, only after making the assumption $a(x) \neq 0$ when $|x| = 1$. (Received February 14, 1940.)

218. Wolfgang Sternberg: *The general limit theorem for probability densities.*

Let x_i $(i=1, \cdots, n)$ be independent chance variables, v_i their probability densities, a_i, b_i, c_i their mean values, standard deviations and absolute moments of the third order, respectively. Denote the probability density of $x_1 + \cdots + x_i$ $(i=2, \cdots, n)$ by w_i, mean value and standard deviation by A_i and B_i, respectively. In order to simplify the formulas, assume that $a_i=0$ for all i, so that $A_i=0$, and $B_n=1$. Then, n being a large number, $w_n(x)$ lies near the Gaussian probability density $\phi(x)=(2\pi)^{-1/2}e^{-x^2/2}$, if very general assumptions concerning the v_i are made. Assume that the quotients c_i/b_i are "very small" and that the $v_i(x)$ have continuous derivatives up to the fourth order for all x and state the theorem as follows: For every positive ϵ there exists a positive δ, such that the inequality $|w_n(x)-\phi(x)|<\epsilon$ holds uniformly in x if the conditions $c_i/b_i<\delta$ $(i=1, \cdots, n)$ are satisfied. To prove this, introduce a function $\psi(x, y)$ defined as the solution of $\partial\psi/\partial y=(1/2)\cdot\partial^2\psi/\partial x^2$ assuming the values $w_r(x)$ on the characteristic $y=B_r$. The number r is a certain integer $(1<r<n)$ depending on ϵ. It is shown first that $|\psi(x, 1)-\phi(x)|<\epsilon/2$ because B_r is a small number (this inequality has a simple physical meaning), and second, by means of the recursion formula of the $w_i(x)$ and the properties of $\psi(x, y)$, that $|w_n(x)-\psi(x, 1)|<\epsilon/2$. (Cf. abstract 45-9-380.) (Received February 3, 1940.)

219. H. S. Wall: *A class of power series bounded in the unit circle.*

Let $f(x)=c_0-c_1x+c_2x^2-\cdots$ wherein $c_0, c_1, c_2, \cdots$ is a totally monotone sequence. The radius of convergence of the series is at least 1 so that $f(x)$ is analytic for $|x|<1$. Let $M(f)$ be the least upper bound of $|f(x)|$ for $|x|<1$. It is shown that $M(f)$ is finite if and only if there exists a positive constant h such that $hf(x)$ has a continued fraction representation of the form $g_1/1+(1-g_1)g_2x/1+(1-g_2)g_3x/1+\cdots$, $0\leqq g_n\leqq 1$, $(n\leqq 1)$, it being agreed that the continued fraction terminates with the first vanishing partial quotient; or, if and only if $f(x)$ has a Stieltjes integral representation of the form $\int_0^1(1-u)d\phi(u)/(1+xu)$ in which $\phi(u)$ is monotone non-decreasing; or, if and only if the series $\sum c_i$ converges, whereupon $M(f)=\sum c_i$. If $M(f)\leqq 1$, $w=f(x)$ maps the region $|x|<1$ into the region $|w-1/2|<1/2$, and if w_0 is any point in the latter region, there exists a function $f(x)$ and a point x_0, $|x_0|<1$, such that $w_0=f(x_0)$. It is shown that if $M(f)\leqq 1$, the functions $f_{n+1}(x)=[\gamma_n-f_n(x)]/x[1-\gamma_n f(x)]$, $\gamma_n=f_n(0)$, $f_0=f$, $(n=0, 1, 2, \cdots)$, obtained by the algorithm used by Schur are all functions of the same character as $f(x)$. (Received February 5, 1940.)

220. J. L. Walsh: *Note on overconvergent power series.*

Ostrowski has proved the remarkable fact that in a power series $\sum a_nz^n$ possessing overconvergent partial sums, the sequence of coefficients must exhibit Hadamard gaps, that is to say, gaps whose relative lengths are bounded from zero. The present note emphasizes the fact that these gaps cannot be considered as sharply terminating. The coefficients have moduli decreasing gradually at the beginning of the gaps and increasing gradually at the end. In particular the radius of convergence of the series must be a *non-isolated* point of the derivative of the set $\{|a_n|^{-1/n}\}$. (Received February 7, 1940.)

221. G. T. Whyburn: *Mapping theorems.*

A slight modification of a theorem published by the author (American Journal of Mathematics, vol. 53 (1931), p. 753) is shown to yield not only the theorem that any compact locally connected continuum is the continuous image of an interval but also the proposition that any compact metric space is the continuous image of a closed 0-dimensional subset of an interval. Also it is noted that the latter of these results may be obtained readily from the former by an independent method. Further, the author's treatment of the Mazurkiewicz relative distance transformation (American

Journal of Mathematics, vol. 54 (1932), pp. 367–376) is modified so as to yield conclusions in general spaces from which it results that if R is a plane region with a connected boundary B there exists a complete space C of the form $C=S+A$ where $S\cdot A=0$ and a continuous transformation $W(C)=R+B'$ such that: (1) S maps topologically onto R; (2) A maps onto the set B' of all points of B which are accessible from R; (3) the transformation $W(A)=B'$ is non-alternating in a certain natural sense; (4) A can be mapped by a (1-1) continuous transformation onto a dense subset of a circle. (Received February 8, 1940.)

222. G. T. Whyburn: *On non-alternating transformations.*

A new treatment of this subject is given which makes possible the extension to semi-locally-connected compact continua of all major results originally established by the author (American Journal of Mathematics, vol. 56 (1934), pp. 298–300) for locally-connected continua and of some of the principal ones to arbitrary compact continua. For example, if $f(A)=B$ is non-alternating where A is a compact continuum, then for any simple link E_b in B there exists one and only one simple link E_a in A such that $f(E_a)\supset E_b$ and for any other simple link E_a' of A, $f(E_a')$ contains at most one point of E_b. Also it is shown that if A is connected and is either (a) compact and semi-locally-connected or (b) locally-connected, a continuous transformation $f(A)=B$ will be non-alternating if and only if (i) for each $y \in B$, $f^{-1}(y)$ contains every point which separates two points of $f^{-1}(y)$ in A and (ii) f is non-alternating on each cyclic element of A. (Received February 8, 1940.)

223. Y. K. Wong: *Interchange of J-processes in bilinear forms involving non-modular matrices.*

When κ^{12} is of type $\mathfrak{M}^1\overline{\mathfrak{M}}^2$ subject to the basis $(\mathfrak{A}^D, \mathfrak{P}^1, \mathfrak{P}^2, \epsilon^1, \epsilon^2)$, then κ^{12} is said to have the property I^{12} over $\mathfrak{M}_0^1$, $\mathfrak{M}_0^2$ (contained in $\mathfrak{M}^1$, $\mathfrak{M}^2$) in case the iterated integrals $J^1\overline{\mu}_0^1 J^2\kappa^{12}\mu_0^2$, $J^2(J^1\overline{\mu}_0^1\kappa^{12})\mu_0^2$ exist and are equal for every μ_0^1, μ_0^2 in the aforementioned subsets. Equivalent conditions for κ^{12} to have the property I^{12} defined above are obtained. By restricting $\mathfrak{M}_0^1$, $\mathfrak{M}_0^2$ to be the four pairs of subsets in the $(1\ \kappa^*\ 2)$-, $(2\ \kappa\ 1)$-domains, we have sixteen possible modes of I^{12} properties. It is proved that there are but four distinct modes of I^{12} properties, and their relations are discussed. The problem is then formulated in terms of idempotent matrices and positive hermitian modular matrices. Equivalent conditions in terms of certain double integrals are established. We then define the property $I(\epsilon^1\ \epsilon^2)$ for κ^{12} and show that $I(\overset{1}{\epsilon_\kappa}\ \overset{2}{\epsilon_{\kappa^*}})$ and $I(\overset{1}{\epsilon_{\kappa^*}}\ \overset{2}{\epsilon_\kappa})$ are equivalent properties. (Received February 1, 1940.)

224. Y. K. Wong: *On non-modular matrices.*

E. H. Moore's generalized Fourier theory shows that $\mathfrak{M}^{1\kappa}$, $\mathfrak{M}^{2\kappa}$ are in one-to-one orthogonal correspondence via $J^1\kappa^{*21}$, $J^2\kappa\kappa^{12}$. This paper studies the invariance of density and closure properties for corresponding subsets in $\mathfrak{M}^{1\kappa}$, $\mathfrak{M}^{2\kappa}$. By using the basis ($\mathfrak{A}^D$, $\mathfrak{P}^1$, $\mathfrak{P}^2$, ϵ^1, ϵ^2, κ^{12} by columns of $\mathfrak{M}^1$), the $(1\ \kappa^*\ 2)$-domain is defined to be the class of all μ^1 (modular as to ϵ^1) such that $J^1\kappa^{*21}\mu^1$ is modular as to ϵ^2. The intersection $\mathfrak{M}^{2\cap 2\kappa}$ of $\mathfrak{M}^2$ and $\mathfrak{M}^{2\kappa}$ is in one-to-one correspondence with the $(1_\kappa\ \kappa^*\ 2)$-domain. The $(1\ \kappa^*\ 2)$-domain is the linear extension of the logical sum of the $(1_\kappa\ \kappa^*\ 2)$-domain and the orthogonal complement of $\mathfrak{M}^1\kappa$. Density and closure properties of the $(1\ \kappa^*\ 2)$-domain and the set $\mathfrak{M}^{2\cap 2\kappa}$ are studied. When κ^{12} is of type $\mathfrak{M}^1\overline{\mathfrak{M}}^2$, the intersection $\mathfrak{M}^{1\kappa\cap 1\kappa^*}$ of $\mathfrak{M}^{1\kappa}$ and $\mathfrak{M}^{1\mu^*}$ and various domains associated with κ^{12} are discussed. (Received February 1, 1940.)

225. J. C. Abbott: *Projective theory of congruence in non-euclidean geometry.* Preliminary report.

Most assumptions made in non-euclidean geometry about congruence of segments can be derived from the projective postulates for parallelism and order of Jenks and the law of Pappus. If, in addition, Pascal's law is assumed for asymptotic hexagons, then also non-euclidean properties of perpendicularity and congruence of angles can be derived which, without this assumption, break down. The theory of congruence of segments, however, is independent of this assumption. (Received March 12, 1940.)

226. R. P. Agnew: *Some remarks on a paper entitled "General Tauberian theorems."*

The Tauberian classes T, T', T_α, T'_α, T_β, and T'_β defined by H. R. Pitt (*General Tauberian theorems*, Proceedings of the London Mathematical Society, (2), vol. 44 (1938), pp. 243–288) each fail to have the following property: if $s(x)$ is a function in the class, then, for each constant A, the function $s(x)-A$ is in the class. This fact has a bearing on the connection between the Tauberian oscillation theorems and the Tauberian convergence theorems of Pitt's paper. (Received March 11, 1940.)

227. S. P. Avann: *Lattice automorphisms.*

It is shown in this paper that every finite abelian group is the automorphism group of some lattice, in fact of a finite distributive lattice. A necessary and sufficient condition is obtained that a point lattice have the symmetric group as its automorphism group. The group of a distributive lattice is considered from the standpoint of symmetrically placed generators. Also included is a test for modularity of a lattice in terms of the number of coverings of the elements. Further study of lattice automorphisms is in progress by the author. (Received March 7, 1940.)

228. Reinhold Baer: *Nets and groups.* II.

The nets, derived from division systems with unit, satisfying $x(yz)=1$ if, and only if $(xy)z=1$, are characterized by certain local symmetry properties. (Received March 11, 1940.)

229. Reinhold Baer: *Sylow theorems for infinite groups.*

This paper investigates the results which one obtains in applying the methods used for proving the theorems of the Sylow type upon infinite groups. Particular attention is given to locally finite groups, that is, groups all of whose finite subsets are contained in finite and normal subgroups, since for these groups certain conditions turn out to be necessary which in general are only sufficient, but not necessary. (Received March 11, 1940.)

230. R. C. F. Bartels and R. V. Churchill: *An extension of Duhamel's theorem.*

The method of the Laplace transformation is applied in establishing an extension of Duhamel's theorem to a general boundary-value problem with a partial differential equation of the parabolic type having discontinuous coefficients. It is shown that the solution of a problem having boundary conditions dependent on the variable time can be expressed in terms of the solution of a simpler problem with fixed boundary conditions. Conditions on the solution of the latter problem which are sufficient to insure a solution of the general problem are discussed. The derivation makes use of a generalization of the composition integral (Faltung) in the theory of the Laplace transformation. Some properties of this generalized form are considered. (Received March 15, 1940.)

231. Harry Bateman: *Hulthen's integral equation.* Preliminary report.

Let $V(x, y)$ be a potential function regular for $y \geqq 0$. The integral equation in question is for the determination of $V(x, 0)$ and may be written in the form $f(x) = V(x, 0) + kV(x, a)$ where $a > 0$ and $f(x)$ is continuous for real values of x. If $U(x, 0)$ is the potential regular for $y \geqq 0$ which is equal to $f(x)$ for $y = 0$, the solution obtained by the method of successive approximations is $V(x, 0) = U(x, 0) - kU(x, a) + k^2U(x, 2a) - \cdots$ and if $|f(x)| \leqq f$ this series is convergent for $|k| < 1$. The case $k = 1$ may be treated by expanding $V(x, y)$ in a trigonometric series of multiples of $2\pi y/a$, but the sine terms are left undetermined by the equation. The homogeneous integral equation has in this case solutions and the nonhomogeneous equation has solutions only if $f(x)$ is a function of a certain form. Potential problems in which a correspondence between the points of two boundaries occurs in the boundary condition give rise to an interesting type of integral equation. (Received March 16, 1940.)

232. R. A. Beaumont: *Projections of non-abelian groups upon abelian groups.* Preliminary report.

A function f of the subgroups of a group G upon the subgroups of a group H is called a projectivity of G upon H if f is a single valued monotone increasing function with a single valued inverse. Groups which have projections upon abelian groups naturally fall into two classes: groups without elements of infinite order, and groups with elements of infinite order. In this paper the latter are studied. Baer has shown that if an abelian group G contains at least two independent elements of infinite order, then every projection of G is induced by an isomorphism. Hence in a survey of groups containing elements of infinite order which are projective with abelian groups, only abelian groups G where $G/F(G)$ is of rank one need be considered. The author gives a survey of such groups. The discussion of the case where $G/F(G)$ is an infinite cyclic group is important for the general problem. (Received March 12, 1940.)

233. E. T. Bell: *Note on a certain type of diophantine system.*

The system is $a_iP_i(x) = c_iy_i^{n_i}$, $(i = 1, \cdots, r)$, in which a_i, c_i, n_i are any given constant integers greater than 0; $P_i(x)$ is a polynomial of degree m_i (greater than 0) in x with integer coefficients; $P_i(x)$ has no constant term; the coefficient of the lowest power of x in $P_i(x)$ is b_i. Necessary and sufficient conditions that the system have a solution in integers $x, y_1, \cdots, y_r$ are obtained. These conditions concern $b_1, \cdots, b_r$ alone; when they are satisfied, solutions are immediately obtainable. Very special cases of such systems have been discussed by several writers, without attention to necessity or sufficiency. (Received March 11, 1940.)

234. E. T. Bell: *Postulational basis for the umbral calculus.*

As the umbral calculus for vectors has been misunderstood, a full postulational treatment is given. (Received March 11, 1940.)

235. Stefan Bergman: *On the surface integrals of functions of two complex variables.*

The author considers functions $f(z_1, z_2)$ of two complex variables meromorphic in domains $\mathfrak{M}^4$, $\mathfrak{M}^4$ being bounded by a finite number of segments of analytic hypersurfaces (see, for details, Mathematische Zeitschrift, vol. 39 (1937), p. 76, and Recueil Mathématique, vol. 1 (43) (1936), p. 851). Each domain $\mathfrak{M}^4$ possesses a distinguished boundary surface $\mathfrak{F}^2$. From an extension of the methods used in the papers mentioned, it can be shown that a surface integral over $f(z_1, z_2)\chi(z_1, z_2)$ along $\mathfrak{F}^2$ is equal to the sum $\sum_\nu R(P_\nu; f)$ of expressions (residues) $R(P_\nu; f)$ which are associated with certain points P_ν lying in $\mathfrak{M}^4$ and on the singularity surface of f. The function $\chi(z_1, z_2)$ is a weight function which depends upon $\mathfrak{M}^4$ and upon the intersection of $f(z_1, z_2)=0$ with the boundary of $\mathfrak{M}^4$. (Received March 25, 1940.)

236. B. A. Bernstein: *Groups in terms of addition and negation.*

The author defines groups and abelian groups by sets of postulates expressed in terms of addition and negation. The postulates are simple and readily yield the basic propositions of groups. The sets for abelian groups embrace, in each case, a set for groups in general. (Received March 8, 1940.)

237. R. P. Boas: *Expansions of analytic functions.*

This paper is a contribution to the problem of expanding an analytic function $f(z)$ in a series of the form $\sum c_n z^n [1+h_n(z)]$, where the $h_n(z)$ are "small." (For references to the literature, see G. S. Ketchum, Transactions of this Society, vol. 40 (1936), pp. 208–224.) The central problem is to obtain as large a circle of convergence as possible for this expansion. Most of the known criteria are obtained as consequences of a new one: if the functions $h_n(z)$ are analytic in $|z| < r$, vanish at $z=0$, and have (for large n) a common majorant $h(z)$, and if $f(z)$ is analytic in $|z| < s$, the above series represents $f(z)$ in $|z| < \min(s, t)$ if $h(t) < 1$. The method used in this paper avoids rearrangements of power series, using instead arguments depending on Cauchy's theorem. Applications are made to the study of the distribution of the values taken by an analytic function and its derivatives. A more detailed abstract of part of the paper has appeared in the Proceedings of the National Academy of Sciences, vol. 26 (1940), pp. 139–143. (Received March 18, 1940.)

238. R. P. Boas: *Univalent derivatives of entire functions.*

The following theorem is proved: If $f(z)$ is a transcendental entire function of order one and exponential type k (that is, if $|f(z)| < C \exp(k|z|)$), and $k < \log 2$, then an infinite number of the derivatives of $f(z)$ are univalent in $|z| < 1$. The proof depends on an expansion theorem of the type described in the preceding abstract. (Received March 18, 1940.)

239. Richard Brauer: *On the Cartan invariants of a group of finite order.*

E. Cartan introduced an important set of invariants $c_{\kappa\lambda}$ ($\kappa, \lambda = 1, 2, \cdots, k$) of an algebra with a principal unit. If G is a group of finite order, these $c_{\kappa\lambda}$ may be formed for the group ring of G taking the field of reference as a modular field K. It is shown that the determinant $|c_{\kappa\lambda}|$ is a power of the characteristic p of K, and the exponent of p is determined. (Received March 27, 1940.)

240. J. L. Brenner: *Well-orderable abelian groups whose elements have prime-power order.*

The author gives a new, short proof of the Kronecker decomposition theorem for finite abelian groups and a partial decomposition theorem for arbitrary torsion groups. The elements of the basis are picked out one by one. (Received January 19, 1940.)

241. H. H. Campaigne: *A lower limit on the number of hypergroups of a given order.*

Let ν_ω be the number of hypergroups of order ω. Then it can be shown that $\nu_{\omega+1} > 7\nu_\omega$. The method of proof is to give constructions, in terms of an arbitrary hypergroup G of order ω, for seven different hypergroups of orders $\omega+1$. Six of these have G as a subhypergroup. As a corollary we have $\nu_\omega > 7^{\omega-1}$. (Received March 15, 1940.)

242. H. H. Campaigne: *Multiplication systems.* Preliminary report.

A set M of elements $m, n, \cdots$ is a multiplication system if for every pair of elements m, n there is uniquely determined a subset mn of M. This is a generalization of the notion of group. The multiplication system C is complementary to M if the elements of C are in one-to-one correspondence, $c_\eta \longleftrightarrow m_\eta$, with those of M such that $c_\lambda \in c_\mu c_\nu$ if and only if m_λ is not in $m_\mu m_\nu$. It is evident that M is isomorphic to M' if and only if C is isomorphic to C'. The set of all automorphisms of M forms a group. The group of automorphisms of C is isomorphic to that of M. Conditions on M are found that are necessary and sufficient that C be a hypergroup. It is shown that these conditions are satisfied by groups, quasi-groups, Mischgruppen, Brandt groupoids, and defective groups. Thus we have a method of uniquely representing these systems by means of hypergroups. (Received March 15, 1940.)

243. Leonard Carlitz: *A set of polynomials.*

Put $f_m(t) = \prod_c (t + c_0 u_0 + \cdots + c_{m-1} u_{m-1})$, the product extending over all sets $(c_0, \cdots, c_{m-1})$ in the finite field $GF(p^n)$; $t, u_0, \cdots, u_{m-1}$ are indeterminates. For $u_i = x^i$, $f_m(t)$ becomes $\psi_m(t)$ (Duke Mathematical Journal, vol. 1 (1935), p. 137). The purpose of the present note is to extend known formulas involving ψ_m to the more general f_m. In particular, an explicit formula representing an arbitrary "linear" polynomial $\sum \alpha_i t^{p^{ni}}$ is derived in terms of $f_m(t)$; also the inverse function to f_m is constructed. Application is made to the evaluation of $\sum (c_0 u_0 + \cdots + c_m u_m)^{p^{nk}-1}$ and $\sum (c_0 u_0 + \cdots + c_m u_m)^{-(p^{nk}-1)}$. (Received March 28, 1940.)

244. Leonard Carlitz: *On certain exponential sums.*

Certain sums, which may be briefly described as generalized Gauss sums, are defined relative to a special modular system $M = (m, A_1, \cdots, A_k)$, where A_k is a polynomial in k indeterminates. The generalized sums are reduced to ordinary sums by means of a method used previously in a special case; a generalized Jacobi symbol (quadratic character) appears incidentally. (Received March 28, 1940.)

245. Randolph Church: *Numerical analysis of certain free distributive structures.*

The elements of the free distributive structure based on n elements fall into disjoint sets of conjugates under the permutations of the symmetric group of degree n.

The 208 sets of conjugates and the structure inclusion relations among the elements constituting them have been obtained for $n=5$. The number of conjugate elements in these sets, and their rank in the structure, are here presented for $n \leqq 5$. (Received March 28, 1940.)

246. R. V. Churchill: *A problem in the conduction of heat.*

The following physical problem is treated. Find the temperatures in a semi-infinite solid $x \geqq 0$ composed of a layer $0 \leqq x \leqq a$ of one material initially at uniform temperature A in contact with a semi-infinite solid $x \geqq a$ of another material initially at temperature zero, when the face $x=0$ is kept insulated. A complete formulation of this problem as a boundary value problem is determined so that just one solution exists. The temperature formula is obtained in the form of a simple and practical series whose terms involve probability integrals. It is completely established as the solution. The Laplace transformation is used both to obtain the solution and to establish its uniqueness. (Received March 15, 1940.)

247. R. V. Churchill: *Termwise integration of Sturm-Liouville expansions.* Preliminary report.

The following theorem is established. Let $y=\phi_n(x)$, $(n=1, 2, \cdots)$, be the normalized characteristic functions of the Sturm-Liouville system $y''-[q(x)+\lambda]y=0$, $y(0, \lambda)=0$, $y(1, \lambda)=0$, where $q(x)$ is continuous in x in the interval $(0, 1)$. Let $c_n=\int_0^1 F(x)\phi_n(x)dx$, where $F(x)$ is any bounded and Riemann integrable function. Then for any function $G(x)$ of bounded variation the series $\sum_1^\infty c_n\int_0^x G(x)\phi_n(x)dx$ converges uniformly for all x in the interval $(0, 1)$ to the function $\int_0^x F(x)G(x)dx$. The Parseval equality for this system follows at once: $\sum_1^\infty c_n d_n=\int_0^1 F(x)G(x)dx$, where $d_n=\int_0^1 G(x)\phi_n(x)dx$. The method of proof is that of integration in the complex plane of the parameter λ. The result is extended to the termwise integration of the expansion based on the equation $(ry')'-[q+\lambda p]y=0$. (Received March 15, 1940.)

248. Paul Civin: *Inequalities for trigonometric integrals.* Preliminary report.

This paper concerns transforms of functions expressible as $f(x)=\int_{-R}^{R} e^{ixt}d\alpha(t)$, where $\alpha(t)$ is a function of bounded variation, and $|f(x)| \leqq M$ for all x. These transforms are of the form $T[f(x)]=g(x)=\int_{-R}\mu(t)e^{ixt}d\alpha(t)$. The problem is to find a bound for $|g(x)|$ depending only on $\mu(t)$ and M. The results obtained include generalizations of many of those in the literature for trigonometric polynomials: for example, Bernstein's theorem (where $g(x)=f'(x)$), results of G. Sokolov (Bulletin de l'Académie des Sciences de l'URSS, (7), Classe des Sciences Mathématiques et Naturelles, 1935, pp. 857–882), and results of G. Szegö (Schriften der Königsberger gelehrten Gesellschaft, vol. 5 (1928), pp. 59–70). In particular, a bound is obtained for the fractional derivative $f^{(\alpha)}(x)$, $0<\alpha<1$; $f^{(\alpha)}(x)=T[f(x)]$ where the multiplier $\mu(t)=(it)^\alpha$. This bound is $(4\alpha^{-1}+2)R^\alpha M$. A more precise but more complicated bound is also obtained. (Received March 25, 1940.)

249. A. G. Clark: *Two sample problem concerning Poisson and binomial distributions.* Preliminary report.

In an article published in Biometrika, 1939, J. Przyborowski and H. Wilenski have developed a test based upon best critical regions of the hypothesis that two variables with distributions given by the Poisson exponential limit have equal means.

The author has independently obtained some of the results given in the article quoted and has developed more extensive tables for the power of the test in both its symmetric and asymmetric forms. In addition, the effect of the relative size of the two samples upon the power of the test is determined. A corresponding test is constructed where the distribution of the variables is binomial. In this case the power of the test is expressed as a finite hypergeometric series. It is shown how the critical region may be identified when the samples are of equal size. It is hoped to find a feasible method of calculating the power of the test in the binomial case. (Received March 12, 1940.)

250. J. M. Clarkson: *An involutorial line transformation associated with a quadric.*

Given a fixed quadric H and a point O not on H, an arbitrary line t meets H in two points P_1, P_2 which are projected from O into two points P_1', P_2' on H. The line $t' \equiv P_1' P_2'$ is the transform of t by the line transformation T. The transformation T is involutorial. The invariant lines and the singular lines of T are discussed, and also the changes in the configurations when H is a cone and when H is composite. (Received March 22, 1940.)

251. Nancy Cole: *The index theorem for a calculus of variations problem in which the integrand is discontinuous.*

In this paper Morse's index theorem (Duke Mathematical Journal, vol. 4 (1938), pp. 231–246) is established for a problem in euclidean m-space in which the integrand is discontinuous. The basic curve g is a broken extremal with a finite number of corners, at each of which g is cut across by a regular $(m-1)$ manifold of class C^2, not tangent to either arc of g at the corner. At each corner g satisfies a set of "primary incidence relations." The conjugate points of a fixed point a of g are defined in terms of the zeros of the conjugate determinant $D_g(t, a)$, whose m columns represent certain linearly independent solutions of the Jacobi equations determined by g. It is also proved that if a and b are any two fixed points of g, the numbers of zeros of $D_g(t, a)$ and $D_g(t, b)$ on any interval (open or closed) of g differ by at most $m-1$. (Received March 28, 1940.)

252. H. S. M. Coxeter: *The binary polyhedral groups, and other generalizations of the quaternion group.*

The chief result of this paper is that the (largest) group defined by $R^l = S^m = T^n = RST$ is of order $4|l^{-1}+m^{-1}+n^{-1}-1|(|l|^{-1}+|m|^{-1}+|n|^{-1}-1)^{-2}$, whenever $|l|$, $|m|$, $|n|$ and $|l|^{-1}+|m|^{-1}+|n|^{-1}$ are all greater than 1. (This covers all non-trivial cases, since the group is obviously infinite when $|l|^{-1}+|m|^{-1}+|n|^{-1} \leqq 1$.) Well known cases of this group $\langle l, m, n\rangle$ are the dicyclic groups $\langle 2, 2, n\rangle$ (when $n=2$, the quaternion group), and the binary polyhedral groups $\langle 2, 3, n\rangle$ ($n=3, 4, 5$). The investigation of $\langle 2, 3, n\rangle$ for negative n was proposed by W. Threlfall as a problem (Jahresbericht der Deutschen Mathematiker-Vereinigung, vol. 46 (1936), p. 80). In most cases $\langle l, m, n\rangle$ is the direct product of $\langle |l|, |m|, |n|\rangle$ and a cyclic group; an interesting exception is the group $\langle 2, 3, -3\rangle$, of order 72. (Received March 29, 1940.)

253. H. B. Curry: *A formulation of recursive arithmetic.*

The term "recursive arithmetic," as used here, means the theory of the natural integers developed by the use of primitive recursive definitions; the emphasis is not so much on the construction of such definitions (as in work by Gödel, Péter and

Kleene) as on the technique of proving equations involving such functions and free variables. Previous formalizations of this sort of arithmetic (for example, that by Skolem and Hilbert-Bernays) have been based on a logical calculus, which is regarded as already formulated. In the present paper the theory is set up in a rigorously formal manner as an entirely independent logical system; it is then shown that the rules of the logical calculus are valid in the system, and that the system is equivalent to that of Hilbert-Bernays. (Received March 9, 1940.)

254. G. B. Dantzig: *Existence of the Neyman-Pearson unbiased test of type* A.

Consider the case where it is known that the elementary probability law $p(E|\theta)$ of a system E of n observable variables depends on only one parameter θ, the value of which is unknown, and where it is desired to test a hypothesis H which assumes that $\theta=\theta_0$. Let w denote a region which is used as a critical region to test H, such that, if $E \varepsilon w$, then H is rejected. Denote by $\beta(\theta)$ the power function of w, that is, $\beta(\theta)$ is the probability of $E \varepsilon w$ calculated under the assumption that θ is the true value of the unknown parameter. The region w is said to be unbiased of type A (Newman-Pearson, Statistical Research Memoirs, vol. 1 (1936), pp. 1–37, and ibid., vol. 2 (1938)), if it satisfies the following conditions: (1) $\beta(\theta_0)=\alpha$, (2) $d\beta/d\theta|_{\theta_0}=0$ and (3) $d^2\beta/d\theta^2|_{\theta_0}$ has for the region w a value at least equal to that corresponding to any other region w having properties (1) and (2). The existence of unbiased critical regions of type A was proved by Neyman and Pearson for a particular type of functions $p(E|\theta)$. This result is now extended by the present author to all elementary probability laws $p(E|\theta)$ which admit two differentiations under the sign of the integral over any fixed region. (Received March 9, 1940.)

255. G. B. Dantzig: *On the non-existence of tests of "Student's" hypothesis, the power function of which would be independent of* σ.

One is given a sample of n independent, normally distributed variables with a common mean ξ and a standard deviation σ. It is wished to test the hypothesis that the mean ξ is 0 when the standard deviation σ is unknown. Student's test consists in rejecting this hypothesis whenever the ratio of the mean of the sample, $\bar{x}$, to the standard deviation of the sample, s, is larger than a fixed value. The important property of this test is that the probability of rejecting the hypothesis $\xi=0$, when it is true, is independent of the unknown standard deviation σ. On the other hand, the probability of rejecting the hypothesis when some alternative hypothesis $\xi=\xi_1$ is true, depends on the value of σ. This causes difficulties in applications. It is natural, then, to consider whether there exists some other test such that the latter probability is independent of the unknown σ. This problem reduces to finding a region w in the sample space $(x_1, x_2, \cdots, x_n)$ such that the integral over w of the function $p=(\sigma(2\pi)^{1/2})^{-n} \exp\left[-\sum_{i=1}^{n}(x_i-\xi)^2/2\sigma^2\right]$ is independent of σ whatever be the value of ξ. It appears, however, that the integral of p over any such region is also independent of ξ. From the point of view of statistics such a region is unsuitable for testing the hypothesis that $\xi=0$, because it will reject this hypothesis when it is wrong and when it is correct equally frequently. (Received March 9, 1940.)

256. P. H. Daus: *Bisecting circles.*

This paper discusses from a geometric point of view the circles which bisect and the circles which are bisected by a given circle. Attention is given to the one-parameter families which bisect two circles, which are bisected by two circles, and other families which involve other conditions such as orthogonality and tangency. The paper considers the circles determined by three such conditions including, as one or more conditions, these bisection properties. It enumerates the linear, quadratic and quartic cases, and in the latter case distinguishes between those which are constructable by ruler and compass and those which are not. (Received March 6, 1940.)

257. M. M. Day: *A norm ergodic theorem.*

With fewer restrictions than in other theorems of this type, a theorem using the method of F. Riesz is proved which includes results of F. Riesz (Journal of the London Mathematical Society, vol. 13 (1938)), Cohen (this Bulletin, abstract 45-5-169), Yosida (Proceedings of the Imperial Academy of Japan, Tokyo, vol. 14 (1938)), and Dunford (Duke Mathematical Journal, vol. 5 (1939)). In fact a simple lemma on convex bodies in euclidean n-space used with the main theorem shows that the arithmetic means used by Dunford can as well be taken over all convex sets of finite nonzero measure ordered by the radius of the largest inscribed sphere rather than only over cubes ordered by edge length. The generality of the theorem lies in the greater latitude allowed in the choice of the parameter set and of the transformations used, since, as Cohen showed for sequences, the arithmetic mean turns out to be only one of a class of transformations for which the norm or "mean" ergodic theorem holds. (Received March 28, 1940.)

258. M. M. Day: *Linear methods of summability.* II.

In a previous paper the author considered the problem of extending the Silverman-Toeplitz theorem to more general limits than those of sequences. These results are now extended to functions on a directed set Y with values in a fixed Banach space B. The extension of the following theorem of Vulich (Kharkov Communications, 1938) is also considered: If B is a Banach space, if $\{a_{mn}\}$ is a matrix of real numbers, and if $U_m(f)=\sum_{n=1}^{\infty}a_{mn}f_n$, where $f=\{f_n\}$ is a convergent sequence of points of B, then $\lim_{m\to\infty} U_m(f)=\lim_{n\to\infty} f_n$ for every convergent sequence of points of B if and only if $\{a_{mn}\}$ satisfies the Toeplitz conditions; that is, regularity on real convergent sequences is equivalent to regularity on every sequence of points from every Banach space. The methods used in the investigation include some study of topologies in spaces of linear operators, and the theory of order among directed sets due to Tukey (Princeton University thesis, 1939). Also, the definition and some properties of a simple sort of a completely additive integral, analogous to the finitely additive integral of Gowurin (Fundamenta Mathematicae, vol. 27 (1936)), are given, and a regularity criterion involving it is given for use in a subsequent paper. (Received March 28, 1940.)

259. J. J. DeCicco: *The affinelong near-Laguerre transformations.*

This paper is concerned with the group of line transformations of the complex plane with respect to the maximum number of circles preserved. A line correspondence is (I) magnilong if it magnifies by a nonzero constant the distance between the two points of contact on the common tangent line of any two curves, (II) affinelong if it is a non-magnilong correspondence which preserves the ∞^1 parallel pencils of lines, (III) general if it is not of types (I) or (II). A general (affinelong, magnilong) transformation preserves at most $2\infty^2$ (∞^2, ∞^1) circles. (*The analogue of the Moebius group in the Kasner plane*, this Bulletin, vol. 45 (1939), pp. 936–943.) In abstract

46-1-59 are found all the magnilong correspondences which preserve exactly ∞^1 circles. The author has now succeeded in finding *all* the affinelong near-Laguerre transformations. The family preserved is a certain quadratic family F which under the group of Laguerre transformations may be classified into the three distinct types: (A) the ∞^2 circles tangent to a fixed line. (B) The ∞^2 circles of the ∞^1 linear pencils (in line geometry) which contain a fixed circle and whose vertices are on a fixed line. (C) The ∞^2 circles tangent to a fixed circle. (Received March 20, 1940.)

260. E. L. Dodd: *The substitutive mean and certain subclasses of this general mean.*

Chisini gave a very general definition of a mean, $m=f_n(x_1, x_2, \cdots, x_n)$, which required that $f_n(c, c, \cdots, c)=c$; but did not require that $\min x_i \leqq m \leqq \max x_i$. This may be called a substitution mean. Nagumo, Kolmogoroff, and B. de Finetti treated the associative mean f_n which remains invariant when each element of a set of r of its elements, with $r<n$, is replaced by the f_r for that set. As a large subclass of substitutive means, the author defines a summational mean as a solution for y of an equation of the form $F\{y, \sum f_1(c_ix_i, y), \cdots, f_k(c_ix_i, y)\}=0$; and defines, as a subclass of summational means, the quasi-arithmetic mean m determined from $\sum c_i\psi(m)=\sum c_i\psi(x_i)$, with $\sum c_i \neq 0$. The associative mean is then a special case of the quasi-arithmetic mean. By transformations, some functions become means of certain functions of the given elements. When this is applied to curve-fitting with Pearson types, various kinds of means emerge. (Received March 18, 1940.)

261. J. L. Doob and R. A. Leibler: *On the spectral analysis of a certain transformation.* Preliminary report.

The analysis of a one-to-one measure preserving transformation T on an abstract space Ω by means of the spectrum of a corresponding operator U goes back to Koopman. In the present paper a particular such transformation is studied in detail: it is supposed that there is a sequence $\cdots, f_{-1}(\omega), f_0(\omega), f_1(\omega), \cdots$ of functions, such that any measurable function can be expressed as a function of the $f_j(\omega)$, and such that $f_j(T\omega)=f_{j+1}(\omega)$. This transformation is of importance in probability. The operator U transforms ω-functions in L_2 into themselves taking $f_j(\omega)$ into $f_{j+1}(\omega)$. It is found that if $E(\lambda)$ is the resolution of the identity of U $(0\leqq\lambda\leqq 2\pi)$, then $(E(\lambda)f(\omega), g(\omega))$ is absolutely continuous except for a possible jump at $\lambda=0$. Necessary and sufficient conditions on $E(\lambda)$ are found that $E(\lambda)$ be the resolution of the identity of an operator corresponding to such a transformation T. (Received March 15, 1940.)

262. Jesse Douglas: *A new special form of the linear element of a surface.*

This paper will be published in full in the Transactions of this Society. (Received March 21, 1940.)

263. F. W. Dresch: *A mathematical model of a dynamic economic system.*

A static economic model due to G. C. Evans is made dynamic by introducing as additional variables the amounts of fixed capital tied up in the production processes of the economy. Assumed production functions and the hypothesis of strict competition permit all quantity variables (those giving rates of production, rates of employment of factors of production and the allocation of such factors to alternative production processes) to be expressed as functions of prices (including the interest rate) and their time derivatives. Introducing a demand function for consumption goods, a supply function for labor and assuming the value of money constant, one may express all variables as functions of time by equating supply and demand. The expressions thus obtained may be regarded as representing a trend situation for the system. In the general situation for which supply is not equal to demand and for which a variation in stocks is thus possible, the traditional assumption that the rate of change of prices is related to the accumulaton of corresponding stocks (or, in the case of labor, to unemployment) is sufficient to again permit all variables to be expressed as functions of time. If one may regard the general situation as a perturbation of the trend situation, the parameters of the system may be estimated from available economic data provided that the empirical functions entering the equations of the model may be assumed of suitable forms. (Received March 9, 1940.)

264. D. M. Dribin: *The norm residue symbol for solvable algebraic number fields.*

Let $K|k$ be a solvable algebraic number field and $K \supset K_{t-1} \supset \cdots \supset K_1 \supset k$ be a chain of subfields of $K|k$ such that $K_{i+1}|K_i$ is abelian. In terms of the generalized Frobenius-Artin symbol $\{K/\mathfrak{p}\}$ (which is a certain unordered vector composed of sets of automorphisms of $K|k$) and a prescribed definition for $\{K/\mathfrak{a}\}$, where $\mathfrak{a}$ is a composite ideal in k (given by means of a definite ordering $\mathfrak{o}$ of the symbols $\{K/\mathfrak{p}\}$), the generalized Hilbert norm residue symbol $\{\beta, K/\mathfrak{p}\}$ can be defined. It enjoys the usual properties, save that the symbol $\{\beta_1\beta_2, K/\mathfrak{p}\}$ is not in general the same as $\{\beta_1, K/\mathfrak{p}\}\{\beta_2, K/\mathfrak{p}\}$, since multiplication of the symbols is non-abelian. If $\mathfrak{F}_i$ is the conductor of $K_{i+1}|K_i$ and $\mathfrak{f} = N_{K|k}(\prod_{i=0}^{t-1}\mathfrak{F}_i)$, then $\{\beta, K/\mathfrak{p}) = 1$ if and only if β is a norm residue modulo $\mathfrak{f}_\mathfrak{p}$, where $\mathfrak{f}_\mathfrak{p}$ is the $\mathfrak{p}$-contribution of $\mathfrak{f}$. (Received March 8, 1940.)

265. R. J. Duffin and J. J. Eachus: *The converse of a closure theorem of Paley and Wiener.*

Paley and Wiener (*Fourier Transforms in the Complex Domain*, American Mathematical Society Colloquium Publications, vol. 19, p. 100) have formulated a criterion for a set of functions $g_n(x)$ to lie close, on the average, to a given orthonormal set $f_n(x)$. From this criterion they show that the functions $g_n(x)$ approximately satisfy Parseval's equation. The authors show that, conversely, the validity of this latter condition on a set of functions $g_n(x)$ is sufficient to insure the existence of at least one orthonormal set $f_n(x)$ which satisfies the original criterion. They prove that the closure of the set $f_n(x)$ implies the closure of the set $g_n(x)$; it is found that the closure of the set $g_n(x)$ implies the closure of $f_n(x)$. (Received March 12, 1940.)

266. Ben Dushnik and E. W. Miller: *On partially ordered sets.*

Circumstances under which a partially ordered set contains an infinite (or nondenumerable) linearly ordered set are investigated. The work is carried out using associated families of sets, and leads to certain questions about real functions of a real variable. The idea of a reversible partial order is introduced. A partial order P is called reversible if "precedes" can be redefined so that, in the resulting partial order P', any two elements a and b are comparable if and only if they are not comparable in P. Not all partial orders are reversible. Necessary and sufficient conditions that a partial order be reversible are given. (Received March 15, 1940.)

267. Samuel Eilenberg: *On spherical cycles.*

A singular n-dimensional cycle of a space X with integer coefficients is called spherical if it is homologous in X to a singular n-sphere. The following two theorems are proved for any r-dimensional simplicial manifold M^r (1) Given any $(r-n-1)$-dimensional subpolyhedron P^{r-n-1} of M^r, every n-cycle of M^r-P^{r-n-1} which bounds in M^r is spherical. (2) If $r>2n$ then every spherical n-cycle of M^r is homologous to an n-sphere topologically imbedded into M^r, simplicially with respect to some subdivision of M^r. (Received March 13, 1940.)

268. Samuel Eilenberg: *Ordered topological spaces.*

A topological space X is called ordered if an order-relation $<$ for the points of X is given such that: given $x<y$ there are neighborhoods $U(x)$ of x and $U(y)$ of y such that $x<y'$ and $x'<y$ whenever $x' \varepsilon U(x)$ and $y' \varepsilon U(y)$. It is proved that a connected nondegenerate space X can be ordered if and only if the subset $P(X)$ of the cartesian product $X \times X$ consisting of all points $(x, y) \varepsilon X \times X$ such that $x \neq y$ is not connected. The uniqueness of the order in a connected space also is established. Some other questions linking order and topology are discussed. The following application is given: a separable connected locally connected nondegenerate space X is homeomorphic with a subset of the linear continuum if and only if $P(X)$ is not connected. (Received March 13, 1940.)

269. Samuel Eilenberg and R. L. Wilder: *Uniform local contractibility.*

Domains of the n-sphere S_n, which have uniform local connectedness properties in the homology sense have been investigated by Wilder in earlier papers. The present paper is concerned with the stronger property of uniform local contractibility (ULC). Although the results obtained hold for more general spaces, the following summary will suffice to disclose their nature: Let D be a domain of S_n, $M=\bar{D}-D$, and E the interval $0 \leqq t \leqq 1$. Then in order that D be ULC it is necessary and sufficient that there exist a continuous mapping $f(D \times E) \subset \bar{D}$ such that $f(x, 0)=x$ and $f(x, t) \varepsilon D$ for $x \varepsilon \bar{D}$ and $t>0$. Consequently if D is ULC then $\bar{D}$ is LC and M is deformation-free in the sense of Wilder (Fundamenta Mathematicae, vol. 21 (1933), p. 163). If D is ULC, M is LC, and the Poincaré group $\pi_1(M)=0$, then $\pi_1(\bar{D})=\pi_1(D)=0$. In particular if M is a topological image of S_{n-1} and D is ULC then $\pi_1(\bar{D})=\pi_1(D)=0$ and $\bar{D}$ can be represented as a *membrane* in the following way: Let Q_n be an open n-element with S_{n-1} as boundary; there is a continuous mapping $f(\bar{Q}_n)=\bar{D}$ such that $f(S_{n-1})=M$ is a homeomorphism and $f(Q_n)=D$. Thus in the case of the Alexander examples (Proceedings of the National Academy of Sciences, vol. 10 (1924), pp. 8–12) the D for which $\pi_1(D) \neq 0$ is not ULC. (Received March 28, 1940.)

270. J. M. Feld: *Whirl-similitudes, euclidean kinematics and non-euclidean geometry.* Preliminary report.

Kasner's whirl group G_3 (American Journal of Mathematics, vol. 33, p. 193) is enlarged to a mixed group $\bar{G}_3$ composed of 4 distinct continuous families of line element transformations by the introduction of the concepts: non-direct whirl, direct whirl correlation and non-direct whirl correlation. Likewise Kasner's whirl-motion group G_6 is enlarged, first to a mixed $\bar{G}_6$ and then to a $\bar{G}_7$ (complete whirl-similitude group). The groups $\bar{G}_6$ and $\bar{G}_7$ are each composed of 8 distinct continuous families of line element transformations, Kasner's G_6 being one of the families in $\bar{G}_6$. The geometry of line elements, flat fields and turbines is investigated and two transfer principles are presented: (1) turbines, line elements and flat fields are mapped respectively on ordered pairs of points, euclidean planar displacements and planar symmetries. (2) Turbines, line elements and flat fields are mapped respectively on lines, points, and planes of pseudo-elliptic 3-space (Blaschke, Zeitschrift für Mathematik und Physik, vol. 60, p. 61), in consequence of which $\bar{G}_6$ is shown to be isomorphic with the group of pseudo-elliptic motions and $\bar{G}_7$ with the group of automorphisms of the pseudo-elliptic absolute. (Received March 15, 1940.)

271. R. E. Gaskell: *A problem in heat conduction and an expansion theorem.*

The problem is that of determining the distribution of heat in a right cylindrical solid, one end of which is in contact with a liquid, with lateral insulation provided so as to make the problem one-dimensional. The solution is obtained by reducing the problem, with the Laplace transformation, to an ordinary second order differential equation with end conditions, both differential equation and end conditions involving a parameter. Inversion is carried out by means of the inversion integral and the result expressed in series form. The convergence of this series at $t=0$ leads to an expansion theorem for the initial temperature distribution function $f(x)$ which is piecewise-continuous and has a bounded, integrable first derivative. The same problem for a solid in contact with a liquid at each end is also considered. (Received March 15, 1940.)

272. Abe Gelbart: *On the growth of a function of two complex variables satisfying certain partial differential equations.*

In this paper the author obtains relations between the growth of max $|U(z_1, z_2)|$ and the coefficients a_{mn} of the Taylor series $\sum_{m,n=0}^{\infty} a_{mn} z_1^m z_2^n$ of an analytic function $f(z_1, z_2)$ of two complex variables, where f belongs to the class of functions F given by the totality of functions satisfying the partial differential equation $\partial^2 U/\partial z_1 \partial z_2 + b_1 \partial U/\partial z_1 + b_2 \partial U/\partial z_2 + b_3 U = 0$ where $b_k = b_k(z_1, z_2)$ are entire functions of two complex variables. A simple form for the upper bound of the growth of max $|U(z_1, z_2)|$ is obtained, which depends on the coefficients b_k, $(k=1, 2, 3)$, of the partial differential equation and on the sequences a_{n0}, a_{0m}, $(n, m=1, 2, \cdots)$, by using the linear operators given by Bergman (Matematischeskii Sbornik, vol. 2 (44) (1937), pp 1169–1197) which transforms the class of analytic functions of one complex variable into the class F. Certain applications are made of these results. (Received March 23, 1940.)

273. H. H. Goldstine: *Linear functionals and integrals in abstract spaces.*

In this paper we consider a linear and non-negative functional I defined on a linear set $\mathfrak{X}$ of real-valued functions x, whose range p is arbitrary. Upper and lower functionals are introduced and it is shown that I may be so extended that a function is in the extension of $\mathfrak{X}$ if and only if its upper and lower functionals are equal. Outer and inner measures are then introduced and every set for which they are equal is measurable in the usual sense. If a continuity assumption is made (this hypothesis is slightly less restrictive than those of Daniell or Banach), then the outer measure is a regular one in the sense of Carathéodory and the integral defined by this measure coincides with the original functional. A generalization of the Lebesgue theorem on termwise integration is then obtained by means of semi-uniform convergence. (Received March 8, 1940.)

274. Michael Golomb: *On the boundary-value problems of the equation $\Delta u = g(x, y)$ in the infinite half-strip.*

In this paper some boundary-value problems of the equation $\Delta u = g(x, y)$ in the infinite half-strip $0 < x < 1$, $y > 0$ are explicitly solved. The studied boundary conditions are: either u or $\partial u / \partial n$ on each of the three sides of the half-strip are given. The method of solution applied makes use of Laplace's transformation, as outlined by G. Doetsch (*Theorie und Anwendung der Laplace-Transformation*, Berlin, 1937, pp. 378–383) for the homogeneous equation, with an incorrect result. The condition that the solution has a Laplace transform, convergent in a certain half-plane, restricts the number of possible solutions, and yields theorems of uniqueness. (Received March 27, 1940.)

275. O. G. Harrold (National Research Fellow): *Characterizations of a class of continua by means of continuous functions.*

Many of the known types of continuous transformations when acting on the unit interval I, $0 \leqq t \leqq 1$, produce only image sets which are topological images of the interval, although the mapping function itself need not be topological. For instance, monotone, non-alternating, and interior mappings (which do not reduce to a single point) have this property. In this paper the nondegenerate continua without continua of condensation are characterized as the class of separable, metric spaces (M) for which there exists a continuous map f defined on the unit interval, $f(I) = M$, satisfying either (a) for X closed in I, $\dim X = \dim f(X)$; or, (b) for a nondegenerate continuum X in I, $f(X)$ contains an open set, and, for a nondegenerate continuum Y in M, $f^{-1}(Y)$ contains an open set. (Received March 28, 1940.)

276. O. G. Harrold (National Research Fellow): *Minimal coverings of Peano spaces by maps of a circle.*

Let M denote a regular continuum in the sense of Menger. There exists a continuous mapping f of the circle (interval) A onto M such that there is a $G_{\delta\sigma}$-set dense in M each point of which has at most two inverses in A. This is related to a theorem due to Nöbeling on mappings of a circle onto a regular curve (Fundamenta Mathematicae, vol. 20, pp. 30–46). An analogous theorem to the above holds for an arbitrary Peano space provided the phrase *at most two* is replaced by *at most a finite number*. For a Peano space M in which there are no free arcs, there exists a mapping f of the unit interval I onto M such that there is a G_δ-set dense in M each point of which has a single inverse on I. Thus there exists a continuous map of the interval onto the fundamental cube, $I^{\aleph_0}$, with the property which has just been mentioned. (Received March 28, 1940.)

277. A. E. Heins: *On the solution of partial difference equations: symmetric boundary conditions.*

It is shown that Nörlund's solution of a linear difference equation with constant coefficients may be reduced to a finite sum if boundary conditions are considered. The solution has two groups of terms. The first group depends on the assigned boundary conditions; the second group depends on the inhomogeneous solution (which now appears as a finite sum). This result is now applied to the solution of a boundary value problem in difference equations. The difference equation is a partial difference equation of parabolc type. (See abstract 45-5-184.) This paper considers (a) vanishing finite boundary conditions, nonvanishing initial condition; (b) infinite boundary conditions, nonvanishing initial condition. Case (a) reduces to a double finite sum of sines and cosines; case (b) reduces to a single finite sum of factorials. Limiting cases are also considered. The Laplace transform is used to reduce the partial difference equation to an ordinary inhomogeneous equation and this equation is solved by classical methods (Nörlund, *Differenzrechnung*). (Received March 11, 1940.)

278. Olaf Helmer: *A new type of transcendence proof.*

This paper is based on the following two theorems from the arithmetic of integral functions. (i) Let S be the set of integral functions of finite order with rational coefficients; the greatest common divisor of two elements f and g of S is expressible in the form $Af+Bg$ where A and B are also in S (this follows from a result obtained by R. Miche in his dissertation, Zurich, 1920). (ii) An irreducible polynomial with rational coefficients is also irreducible in S (cf. the author's abstract in this Bulletin, vol. 44, p. 338). With the help of these theorems it can be shown very easily that certain numbers N (such as π, and $\log_e r$ for rational $r\neq 0$) are either rational or transcendental, and that others (such as arc sin r and arc cos r for rational r) are algebraic at most of degree 2 or transcendental. The method used here is very natural from the standpoint of the algebraist inasmuch as it makes essential use of the fact that the numbers in question are roots of certain integral functions with rational coefficients, thus establishing the algebraic character of the roots in relation to the algebraic character of the coefficient field. (Received March 25, 1940.)

279. L. K. Hua: *On the number of partitions of a number into unequal parts.*

Let $q(n)$ be the number of partitions of an integer n into unequal parts, or into odd parts. Let $\epsilon_{h,k}=\exp\{-\pi i[(h'^2-1)((1-hh')/k-1)/8+h'(1-hh')/8h+h(k+(1-hh')/k)(hh'-2)/24]\}$ for $2|k$, and let $\epsilon_{h,k}=\exp\{-\pi i[(h^2-1)/8-hk/8+(h+h')(hh'k-(hh'-1)/k)/24]\}$ for $2\nmid k$; also let $\omega_{h,k}=\epsilon_{h,k}\exp[(-\pi i/12k)(h+h')]$ for $2|k$ and let $\omega_{h,k}=\epsilon_{h,k}\exp[(-\pi i/24k)(2h-h')]$ for $2\nmid k$ where $hh'\equiv 1 \pmod k$ and $2\nmid h'$ for $2\nmid k$. Then $q(n)=2^{-1/2}\sum_{k=1,2\nmid k}^{\infty}\sum_{(h,k)=1,\ 0<h<k}\{\omega_{h,k}\exp(-2\pi ihn/k)\cdot(d/dn)J_0(i\pi[2(n+1/24)/3]^{1/2}/k)\}$ where $J_0(x)$ is the Bessel function of order 0. (Received March 30, 1940.)

280. L. K. Hua: *On Waring's problem for cubic polynomial summands.*

Let $f(x)=a(x^3-x)/6+b(x^2-x)/2+cx+d$, $a>0$, where a, b, c, d are integers and $(a, b, c)=1$. Then every sufficiently large integer is the sum of eight values of $f(x)$, $x\geqq 0$. Further, almost all positive integers are the sum of four values of $f(x)$, $x\geqq 0$, except the following two cases: (1) $f(x)\equiv 2(2a'+1)x^3+(2b'+1)x^2+2(2c'+1)x+d' \pmod{16}$; (2) $f(x)\equiv a''x^3+3b''x^2+3c''x+d'' \pmod 9$, where $b''^2\equiv a''(c''+1) \pmod 3$. For the first exceptional case almost all positive integers are the sum of seven values of $f(x)$, $x>0$, and in the second case almost all positive integers are the sum of five values of $f(x)$, $x>0$. (Received March 30, 1940.)

281. Henry Hurwitz: *On certain wave equations analogous to the Dirac equations.*

G. D. Birkhoff (Congrès International des Mathématiciens, Oslo, 1936, p. 213) has shown that wave equations for multiple-component wave functions whose associated multiplier equations have the form $P\equiv(W^2/c^2-p^2-1)(W^2/c^2-p^2-m)=0$ are of special interest because of the wave packet phenomenon they exhibit. In this paper particular examples of such four-component wave equations are obtained for arbitrary m. They may be written $(W/c-\alpha_x p_x-\alpha_y p_y-\alpha_z p_z-\beta_L)\psi_L=0$, where W and p_i have their usual quantum mechanics definitions and α_i are ordinary Dirac matrices (Dirac, *Quantum Mechanics*, 2d edition, Oxford, 1935, pp. 254–255). With each member of the Lorentz group a β_L may be associated which reduces to Dirac's α_m when $m=1$. Since the β_L found are not Hermitian, it is convenient to consider also the equation $(W/c-\alpha_x p_x-\alpha_y p_y-\alpha_z p_z-\beta_L\dagger)\chi_L=0$. Then the charge and current four-vector has the form $\chi_L^*\alpha_i\psi_L+\psi_L^*\alpha_i\chi_L$, $i=0, 1, 2, 3$, where $\alpha_0=1$ and the ψ_L and χ_L are suitably related. A given β_L remains unchanged only in certain types of Lorentz transformations (for example, rotations about, or translations along, a particular axis). Therefore, to obtain a relativistic theory, it is necessary to solve the equations corresponding to all the elements of the Lorentz group subject to identical boundary conditions and then average the results over the group by means of the Hurwitz invariant integral. In a Lorentz transformation, S, $\psi_L'=\gamma_S\psi_{S^{-1}L}$ and $\chi_L'=\gamma_S\chi_{S^{-1}L}$. (Dirac, loc. cit., pp. 257–258.) (Received March 2, 1940.)

282. W. H. Ingram: *A generalization of Erhard Schmidt's solution of the nonhomogeneous integral equation.* Preliminary report.

Using results obtained by Frazer, Duncan and Collar and Fredholm's tentative limiting process, a particular solution of the linear integral equation of the second kind with non-symmetrical kernel is found to be $u(x)=\sum\lambda_r(\lambda_r-\lambda)^{-1}\phi_r(x)\int_a^b\psi_r(\alpha)f(\alpha)d\alpha \div\int_a^b\psi_r\phi_r d\alpha$. (Received March 11, 1940.)

283. F. P. Jenks: *Order and parallelism in the non-euclidean geometry of joining and intersecting.*

If, in the author's foundation for Bolyai-Lobachevsky geometry in terms of joining and intersecting (Reports of a Mathematical Colloquium, (2), issue 1, pp. 45–48), postulate VII is sharpened and another assumption added, then the whole theory of order (including Pasch's axiom) can be derived. Two nonintersecting lines a and b and called parallel if there exists a point P such that through P there is at most one line which intersects neither a nor b. These postulates then yield that parallelism is transitive, that there are not more than two parallels to a line through a point, and other theorems on non-euclidean parallelism. (Received March 12, 1940.)

284. Fritz John: *The Dirichlet problem for a hyperbolic equation.*

Recently D. G. Bourgin and R. Duffin (this Bulletin, vol. 45, pp. 851–858) considered the Dirichlet problem for the equation $u_{xx}-u_{yy}=0$ in the case of a rectangle with sides parallel to the coordinate axes. This paper discusses the Dirichlet problem for the same equation for general contours. The main result is the following: Let C be an arbitrary convex curve, for which the Dirichlet problem can be solved for every sufficiently regular set of boundary values. Then the region bounded by C can be mapped on a rectangle with sides parallel to the coordinate axes in such a way that the differential equation (and hence the Dirichlet problem) is preserved. (Received March 7, 1940.)

285. Wilfred Kaplan: *Singularities of a curve-family on a surface, with applications to differential equations.*

Let F be a family of curves filling an open region on a (2-dimensional) surface S. Let F be regular, that is, locally homeomorphic with parallel lines. Let P be an isolated singularity of F. Then all the possible configurations of F in the neighborhood of P are listed. The method of analysis consists in regarding a deleted neighborhood of P as a doubly-connected region, hence homeomorphic with the surface of a sphere from which two points have been removed. Methods previously developed by the author (see abstract 45-9-328) are then applied to the curve-family on the sphere. As an application, F can be taken as the set of trajectories of a differential equation on S (Received March 14, 1940.)

286. Edward Kasner and J. J. DeCicco: *Families of curves conformally equivalent to circles.*

Any three-parameter family of curves which is obtained by applying a conformal transformation to the ∞^3 circles of the plane is called an Ω family. Several geometric characterizations (in the general case) of such a family are obtained. Thus a family of curves is an Ω family if and only if: (I) the foci of the osculating parabolas of the ∞^1 curves which contain a lineal element E generate a lemniscate, (II) the centers of the orthogonal pairs of equal circles which define the lemniscates of (I) as E is rotated about its point P generate an equilateral hyperbola; and (III) the foci of the equilateral hyperbolas of (II) are connected to the point P by a direct conformal transformation. Also the only curves of an Ω family which are hyperosculated by their osculating circles consists of two orthogonal isothermal families of curves. As a corollary of abstract 46-1-92, those families of curves are determined whose hyperosculated isothermal nets are circles. Geometric characterizations are obtained of two-parameter families of curves which are conformally equivalent to the ∞^2 circles orthogonal to a given circle. Finally, by a study of Schwarzian reflection with respect to these families of curves, many interesting generalizations of ordinary inversion are obtained. (Received March 20, 1940.)

287. J. L. Kelley: *On the hyperspaces of a continuum.*

Let A be a compact continuum and let 2^A and $C(A)$ be respectively the spaces of all closed subsets and of all subcontinua of A, metricized by the Hausdorff metric. It is proved: (1) $C(A)$ is arc-wise connected, (2) 2^A and $C(A)$ are locally p-connected in the sense of Lefschetz for $p>0$, (3) any mapping of a polyhedron into $C(A)$ or 2^A is homeotopic to a constant, (4) the homology groups of 2^A of dimension greater than 0 vanish, (5) (theorem of Wodjslawski) A is locally connected if and only if 2^A (or $C(A)$) is an absolute retract. Two new proofs of this latter result are given, depending on the results previously stated and on characterizations of absolute retracts by Lefschetz and by Borsuk respectively. (Received March 25, 1940.)

288. B. O. Koopman: *Intuitive probability and sequences.*

This paper forms a second part of the study of the foundations of probability regarded as a branch of intuitive logic, the first part having appeared in the author's paper, *The axioms and algebra of intuitive probability* (abstract 45-1-92; Annals of

Mathematics, (2), vol. 41 (1940), pp. 269–292). The object is to set forth its connection with the objective notion of statistical weight or frequency in a sequence of trials. The chief theorems assert that when the values of such frequencies are given (for example, by physical laws) and when the conditions of the trials in each sequence are "essentially similar" (in the precise language of the intuitive theory), then the intuitive probabilities of success are to one another as the frequencies. No general principles beyond those assumed in the earlier paper are required, nor does the theory seek an objective definition of random as do the theories of collectives. The present theory thus reveals itself as perfectly capable of dealing with the objective conception of probability involved in statistics and quantum mechanics. It is shown moreover that every experimental application of collectives presupposes in last analysis the intuitive concept of probability. (Received March 21, 1940.)

289. H. L. Krall: *Orthogonal polynomial solutions of a certain fourth order differential equation.*

It is found that there are four classes of nonclassical orthogonal polynomials which satisfy a differential equation of the type $\sum_{i=0}^{4}\sum_{j=0}^{i} l_{ij}x^j y_n^{(i)}(x) = \lambda_n y_n(x)$. One of these classes is a new set of orthogonal polynomials whose derivatives are also orthogonal polynomials. (Received March 19, 1940.)

290. J. P. LaSalle: *Applications of the pseudo-norm to the study of linear topological spaces.*

The pseudo-norm of Hyers, which may be defined for any linear topological space (l.t.s.), is here used as a "pseudo-metric" to define closure for a l.t.s., and it is shown that this definition of closure is equivalent to that of von Neumann. This of course enables one to state necessary and sufficient conditions for the continuity of functions, convergence, and so on, in terms of the pseudo-norm. In particular a necessary and sufficient condition is given that the weak convergence of a sequence of linear functions on T to T', T and T' l.t.s.'s, implies that the limit of the sequence be linear. Also a characterization of a l.t.s. on which there is defined a non-null linear functional is given in terms of a pseudo-normed linear space in which there is defined an operation of "multiplication." (Received March 9, 1940.)

291. P. E. Lewis: *Characters of abelian groups.*

Any homomorphism of an abelian group A in an abelian group V is termed a character of A in V. The problem under consideration is to characterize those pairs of groups A and V satisfying one of the following conditions: (a) the character group of A in V is isomorphic to A; (b) the character group of B in V is isomorphic to B for every subgroup B of A; (c) the character group of A/B in V is isomorphic to A/B for every subgroup B of A. Among other results it is found that (c) implies the finiteness of A and that (a) likewise implies the finiteness of those groups A which contain only elements of finite order. If condition (b) holds, then A is either finite or a direct sum of a finite number of infinite cyclic groups. In each case an almost obvious condition has to be imposed on V. The theory is developed so as to include not only ordinary abelian groups but also certain classes of groups admitting a ring R (not necessarily commutative) as operator system. (Received March 9, 1940.)

292. D. T. McClay: *On certain manifolds of somas.*

This paper contains the complete classification of chains of somas with respect to a 27-parameter continuous subgroup of the pseudo-conformal group; the results are applied in the consideration of one-parameter families of somas. Analogues are found for the configuration of six linear complexes in involution and other configurations in projective line-geometry. (Received March 28, 1940.)

293. E. J. McShane: *On the theory of relative extrema.*

The problem of minimizing a function while giving assigned values to p other functions is discussed in a sufficiently general setting to be useful in the calculus of variations. The necessary conditions on the first derivatives are obtained. For $p=1$, the necessary conditions on the second derivatives are obtained without assumptions of normality. (Received March 15, 1940.)

294. Saunders MacLane and O. F. G. Schilling: *Normal algebraic number fields.*

Let K be a normal extension with Galois group $\{\sigma, \cdots\}$ over the algebraic number field k. Let p denote the prime divisors of k. A set of local algebra classes $H=\{H_p, \text{all } p\}$ is termed an "ideal algebra" if $H_p \sim k_p$ for all but a finite number of prime divisors. The group of all H contains the subgroup $H(K)$ of all H with $H_p \times K^p \sim K^p$. Thus, $H(K)$ contains the group of all actual algebras S_K which are split by K. Restricting H_K, S_K to subgroups H_K', S_K' which are "relatively prime" to a suitable module M (determined by K) one finds that the index $J=[H_K' : S_K']$ equals the l.c.m. of the orders of the elements σ. The index J can be interpreted in terms of factor sets of ideals. Finally, another expression for J is obtained by direct computation, using the arithmetic theory of index reduction. (Received March 12, 1940.)

295. Margaret P. Martin: *A sequence of tests for the convergence and divergence of infinite series.*

The well known de Morgan and Bertrand sequence of tests for the convergence and divergence of infinite series involves an expression for the ratio r_n of one term of the series being tested to the preceding term. This paper contains a similar sequence of tests involving an expression for the ratio $R_n = r_{n+1}/r_n$ of one ratio to the preceding ratio. The proof is based on one of a series of integral tests developed by R. W. Brink (Annals of Mathematics, (2), vol. 21 (1919), pp. 39–60) and on a method of generalization of Brink's tests given by C. T. Rajagopal (this Bulletin, vol. 43 (1937), pp. 405–412). The applicability of the tests to series which would be difficult to test by other known tests is illustrated. (Received March 26, 1940.)

296. Karl Menger: *On shortest polygonal approximations to a curve.*

In an euclidean space, let A be an arc of length $l(A)$, and let $F_1, F_2, \cdots$ be finite subsets of A getting indefinitely dense in A. If for each n, P_n is a shortest polygon through the set F_n, and λ_n the length of P_n, then the polygons $P_1, P_2, \cdots$ converge toward A, and the numbers $\lambda_1, \lambda_2, \cdots$ toward $l(A)$. The theorem holds for any continuous curve A whose length does not surpass that of any continuous curve passing through all points of A. As was previously proved (Mathematische Annalen, vol. 103, p. 467), each arc is in this sense a shortest path through all its points. (Received March 12, 1940.)

297. A. B. Mewborn: *Abstract local geometry of paths.* II.

In the local geometry of paths defined in terms of distinguished (normal) coordinates (this Bulletin, vol. 46 (1940), p. 31) a new linear connection form, defined by the use of a modified and somewhat more general Fréchet differential in the coordinate Banach space, has been found. The curvature form based on this connection does not vanish identically, and hence this space of paths is not necessarily even locally flat. Also some theorems on the local domains of paths as open sets have been found. (Received March 11, 1940.)

298. A. D. Michal: *Higher order differentials of functions with arguments and values in topological abelian groups.*

The author (see abstract 46-1-25) has already studied first order differentials of functions $f(x)$ of a topological abelian group variable x with values in a topological abelian group. The present paper gives an inductive definition of nth order differentials of $f(x)$ at $x=x_0$ by assuming the existence of the $n-1$ previous higher order differentials at $x=x_0$. Several fundamental theorems are proved—including a unicity theorem for all values of the increments and theorems on higher order differentials of functions of functions. As in the case of first order differentials, the real number system does not enter into the theory and so a new flavor is given to an ancient subject and its generalizations. (Received March 27, 1940.)

299. A. D. Michal and Max Wyman: *Characterization of complex couple spaces.*

In considering generalizations of classical complex analysis for complex Banach spaces it seems desirable to consider complex couple spaces. This paper attempts to characterize such spaces. In this connection the following two theorems can be proved. I. A necessary and sufficient condition that an arbitrary complex Banach space B be a complex couple space is that there exist a function $\bar{Z}$ on B to B with the properties: (a) $\overline{Z_1+Z_2}=\bar{Z}_1+\bar{Z}_2$, (b) $\overline{iZ}=-i\bar{Z}$, (c) $\bar{\bar{Z}}=Z$, (d) $\|\bar{Z}\|=\|Z\|$. II. A necessary and sufficient condition that an arbitrary complex Banach space B be a Hermitian couple space is that: (α) B must possess a Hermitian inner product $[Z, U]$, (β) there exist a function $\bar{Z}$ on B to B such that: (β_1) $\overline{[Z_1, Z_2]}=[\bar{Z}_1, \bar{Z}_2]$, ($\beta_2$) $\bar{\bar{Z}}=Z$. By a Hermitian couple space is meant one that is generated from a real Banach space with a real inner product. (The paper will appear in the Annals of Mathematics.) (Received March 9, 1940.)

300. A. N. Milgram: *On the length of continuous curves.*

A Jordan continuum C in a given metric space may be in many ways considered as a continuous curve, that is, continuous image of a finite interval $[a, b]$. Call $\mu(C)$ the greatest lower bound of the lengths of these curves. If $C \subset C'$ are two Jordan continua, it frequently happens that $\mu(C)>\mu(C')$. It is shown however, that $\mu(C)<2\mu(C')$ and that 2 is the best estimate; that is, for each $\epsilon>0$, there are pairs of continuous curves $C\subset C'$ such that $\mu(C')=1$ and $\mu(C)>2-\epsilon$. If M is a subset of a compact convex metric space S, the Jordan continuum C is called shortest join of M if $C\supset M$ and for each C' containing M gives $\mu(C)\leqq\mu(C')$. Then for each subset M of S there exists a shortest join C of M, and C is obtained by adding an at most denumerable number of straight line segments to the closure of M. (Received March 12, 1940.)

301. A. N. Milgram: *Partially ordered sets and the covering theorems of topology.*

Let P be a partially ordered set with a unit 1, that is, an element such that for each x of P, $x\neq 1$ implies $x<1$. If P has a denumerable separating system and P'

is any upper inductive subset of P such that, for each $x \varepsilon P'$, if $x \neq 1$ there exists $y \varepsilon P'$ such that $x < y$, then P' contains the unit. Let P be the class of open subsets of the space S, let $\{O\}$ be a covering of S and P' the set of sums of denumerably many elements of $\{O\}$. Then, the hypotheses of the theorem hold. The conclusion, that is, that $1 \varepsilon P'$, means that S is the sum of denumerably many sets in $\{O\}$. That is the Borel covering theorem for completely separable spaces. Another theorem about partially ordered sets yields the Heine-Borel covering theorem for compact spaces. The definitions of separating system and of inductiveness are given in an earlier paper of the author (Reports of a Mathematical Colloquium, (2), issue 1 (1939), p. 18). (Received March 9, 1940.)

302. W. L. Mitchell: *Topological rings and infinite matrices.*

It is noted that the set of all continuous linear transformations of a topological group into itself forms a ring. In particular, if the topological group admits a topological ring as a ring of operators, and relative to this ring possesses a basis, finite or infinite, the ring of continuous linear transformations forms a ring of matrices, finite or infinite. Some simple properties of these matrices are developed. Some properties of special topological rings are also discussed. (Received March 14, 1940.)

303. R. L. Moore: *Concerning separability.*

It is shown that if a non-separable space satisfies Axioms 0 and 1 of the author's *Foundations of Point Set Theory* (American Mathematical Society Colloquium Publications, vol. 13, 1932), then it contains uncountably many mutually exclusive domains. (Received March 21, 1940.)

304. D. C. Murdoch: *A characterization of abelian quasi-groups.*

An abelian quasi-group is one which satisfies the generalized associative law $(ab)(cd) = (ac)(bd)$. This paper contains a complete characterization of all quasi-groups of this type. It is shown that every abelian quasi-group is the direct product of a self-unit quasi-group (one in which every element is a right unit) with one which contains an idempotent element. All quasi-groups of the latter type can be constructed by defining certain new operations in abelian groups while all self-unit ones can be formed by defining similar new operations in those quasi-groups, already constructed, in which all elements are idempotent. (Received March 11, 1940.)

305. Lewis Nelson: *An Abel integral equation with constant limits of integration.*

Applying the method of T. Carleman (Mathematische Zeitschrift, vol. 15 (1922), pp. 111–120) this paper shows the uniqueness of and gives the solution $\phi(\xi)$ of the integral equation $\int_0^x (x-\xi)^{-\alpha}\phi(\xi)d\xi - \int_x^1 (\xi-x)^{-\alpha}\phi(\xi)d\xi = f(x)$ where $0<\alpha<1$, $0<x<1$, $|x-\xi|^{-\alpha}\phi(\xi)$ is integrable over $0<\xi<1$, and $f(z)$ is analytic in a domain of the complex plane containing the real axis from 0 to 1 of $z=x+iy$. (Received March 9, 1940.)

306. Philip Newman: *The geometry of the* (2, 2) *planar connex.* Preliminary report.

A convex $f(x_1x_2x_3, u_1u_2u_3)$ (Clebsch, *Vorlesungen über Geometrie*) of second order and second class defines two quadratic systems of ∞^2 conics each, point and line respectively. The equations $\rho v_i = f_{x_i}$, $\sigma y_i = f_{u_i}$, $i = 1, 2, 3$, define an element (y, v) conjugate to the element (x, u). These same equations define a multiple valued contac transformation of the element (u, y) into four elements (x, v). A pencil of lines (u) on the point (y) will be transformed into a series of conics quadritangent to a general quartic. A geometry thus appears in the (x) plane with conic and quartic as dual elements. Properties of the systems of conics and quartics with respect to the discriminant curves are studied. A related (3, 3) connex, the Jacobian of the first polar nets, is introduced. The representation of the system of (2, 2) connexes (35-parameter) on Veronese surfaces in 5-space obtained by projective transformations of two fundamental dual surfaces is also discussed. (Received March 25, 1940.)

307. K. L. Nielsen: *Concerning general boundary value problems for linear differential equations.* Preliminary report.

Trjitzinsky (Acta Mathematica, vol. 67 (1936), pp. 1–50) has obtained asymptotic representation of the solutions of the system, which in matrix notation is written $Y^{(1)}(x, \lambda) = Y(x, \lambda)D(x, \lambda)$, for x in the interval (c, d) and certain extending to infinity regions R for the parameter λ. This system is associated with a linear differential equation of order n, $L(x, \lambda; y) \equiv \sum_{k=0}^{n} \lambda^{H(n-k)} a_{n-k}(x, \lambda) y^{(k)} = 0$. The two-point boundary value problem is formulated by considering the above system with the boundary condition $Y(c, \lambda)W_c + Y(d, \lambda)W_d = 0$, where W_c and W_d are matrices of constant terms. The author obtains theorems determining the values of λ for which the non-homogeneous boundary problem is possible when (c, d) is the interval for which Trjitzinsky's existence theorem holds. He first considers the two-point problem; then the problem formulated by taking the conditions at a finite number of points in the x interval (c, d); that is, $\sum_{i=1}^{n} Y(a_i, \lambda) W_{a_i} = 0$; and finally obtains the restrictions necessary when considering the conditions at a denumerably infinite number of points in the interval (c, d). (Received March 4, 1940.)

308. Rufus Oldenburger: *Binary forms.*

In the present paper the theory of minimal numbers and representations introduced elsewhere is developed for binary forms, and is used to give a solution of the problem of equivalence of these forms. The range of the minimal number for binary forms of degree n and field K, subject to minor restrictions, is $1, 2, \cdots, n$. The maximum value n is attained for the complex field if and only if the form has a repeated linear factor of degree $n-1$. The minimal number of a binary form F for a field K exceeds a class of 2-way ranks of the form which have maximum values. For the whole set of these ranks, formally studied elsewhere, no simple applications were found until the present paper. The linear forms in a minimal representation of F with respect to K are factors of a form transforming covariantly with F. If the minimal number of a form is not too large, the associated representation is unique, in which event there is a simple answer to the problem of the equivalence of this form to another form. For the complex field the minimal number can be expressed in terms of resultants and 2-way ranks only. (Received March 28, 1940.)

309. C. D. Olds: *On the number of representations of the square of an integer as the sum of an odd number of squares.*

Let $N_r(n^2)$ denote the number of representations of the square of a positive integer n as the sum of r squares. By analytical means it is possible to derive formulas for $N_r(n^2)$ when $r=3, 5, 7$ which involve only the divisors of n (see G. Pall, Journal of the London Mathematical Society, vol. 5 (1930), pp. 102–105). In this paper it is shown that these results can be obtained in an elementary manner by using only arithmetical reasoning. Use is made of certain fundamental identities for arbitrary parity functions due originally to Liouville and which have since been proved arithmetically. When $r=3$ and 7, the method is an extension of that used by A. Hurwitz (*Mathematische Werke*, vol. 2, pp. 5–7) in his solution of the case when $r=5$. When $r=3$, a second derivation is given, the method being a generalization of T. J. Stieltjes' solution of the case when $r=3$ and $n=p^k$ where p is a prime number identical to 1 (mod 8). (See *Correspondance d'Hermite et de Stieltjes*, vol. 1, pp. 89–94.) (Received March 8, 1940.)

310. Oystein Ore: *Remarks on structures and group relations.*

This paper gives the conditions for three elements in an arbitrary structure to form a Dedekind structure. The results are applied to show that two normal subgroups and an arbitrary third always form a Dedekind structure. (Received March 25, 1940.)

311. Oystein Ore: *The extension problem for groups.*

This paper contains a discussion of the properties of extensions and factor sets and it is shown that by a special reduction method any extension may be reduced to abelian extensions. Among the applications are criteria for splitting extensions. (Received March 25, 1940.)

312. E. W. Paxson: *Strictly convex metric spaces.*

Extending an idea used by J. A. Clarkson in linear spaces (Transactions of this Society, vol. 40 (1936), pp. 396–414), a metric space is called strictly convex if triangles degenerate uniquely, that is, if the equations $\rho(x, u)+\rho(u, y)=\rho(x, y)$, $\rho(x, u)=\alpha\leqq\rho(x, y)$, for example, have a unique solution $u(x, y; \alpha)$. It is then shown that multiplication by real numbers and addition may be defined, the latter via a parallelogram construction. These defined entities verify the properties of those in a linear space. The only exceptional point is that associativity of addition is equivalent to the trisection theorem on the medians of a triangle, which may be postulated in a purely metric way. (Received March 15, 1940.)

313. P. M. Pepper: *Concerning pseudo-planar-quintuples.*

A pseudo-planar-quintuple is a metric space of five points not congruently imbeddable into the euclidean plane, although each four point subset is so imbeddable. A convex space into which each member of the five-parameter family of pseudo-planar-quintuples in congruently imbeddable consists of three euclidean semi-planes joined along a common line. The triangular dihedra are a three-parameter family of convex spaces such that each pseudo-planar-quintuple is congruently imbeddable into some member and a two-parameter family of the quintuples into each member. (Received March 29, 1940.)

314. I. E. Perlin: *Sufficient conditions that polynomials in several variables be positive.*

In this paper the author considers polynomials in n variables with real coefficients. By inserting parameters he obtains polynomials in the parameters which satisfy certain recursion relations. Sufficient conditions that the original polynomial in the n variables be positive for all real values of the arguments are obtained. These conditions are expressed in terms of the coefficients of the given polynomial and the roots of the polynomials in the parameters. (Received March 14, 1940.)

315. Sam Perlis: *Scalar extensions of algebras with exponent equal to index.*

If A is a normal simple algebra with equal index and exponent, one may inquire whether all scalar extensions A_K also have this property. It is shown, in essence, that this property is preserved for all separable extension fields K of finite degree if and only if this is true for all cyclic fields K of prime degree. For purely inseparable extensions K an example is constructed in which A is a cyclic division algebra of index and exponent four over an appropriate field of characteristic two, but A_K has index four and exponent two. The example also shows that for every integer $r>1$ there exists a modular field of degree of imperfection r such that not all p-algebras over this field have equal index and exponent. This is in contrast to the case $r=1$. (Received March 5, 1940.)

316. W. T. Puckett: *On arc-preserving transformations.*

Let $T(M)=M'$ be an arc-preserving transformation (G. T. Whyburn, American Journal of Mathematics, vol. 58 (1936)). In case M is cyclicly connected and contains a simple closed curve J such that $T(J)$ is not an arc, the inverse transformation $T^{-1}(M')$ is single-valued. If, in addition, M is strongly arcwise connected, then T is continuous and consequently topological. The set M is said to be strongly arcwise connected, provided every infinite collection of its points contains an infinite subcollection which lies on an arc in M. (Received March 11, 1940.)

317. H. A. Rademacher and A. L. Whiteman: *On Dedekind sums.*

The sums under consideration appear in the transformation formula of log $\eta(\tau)$, and were first investigated by Dedekind in his *Erläuterungen zu den Riemannschen Fragmenten über die Grenzfälle der elliptischen Modulfunktionen* (Riemann's *Werke,* 1876, pp 438–447). The present paper consists of three parts. The first part contains proofs of arithmetical formulas which Dedekind partly derived by analytic methods and partly stated without proofs. The second part verifies the equivalence of Riemann's and Dedekind's results. The last part contains new arithmetical formulas involving Dedekind sums. These formulas are used to obtain simple proofs of Lehmer's results about the sums $A_k(n)$ which appear in the theory of partitions (Transactions of this Society, vol. 43 (1938), pp. 271–295). (Received March 25, 1940.)

318. H. J. Riblet: *Factorization of differential ideals in an algebraic differential field.*

This paper gives conditions sufficient to insure the unique factorization of differential ideals. The problem of extending these properties to algebraic differential fields is discussed and sufficient conditions are given for a factorization theorem. (Received March 25, 1940.)

319. J. H. Roberts: *A theorem on dimension.*

Hurewicz has shown that if X is a compact metric space, and $Y=f(X)$, where f is an at most $(k+1)$-to-1 continuous mapping, then dim $Y\leqq$ dim $X+k$. He has raised the following question: Given a compact metric space Y of dimension n $(n>0)$, does there exist for every k ($0 \leqq k \leqq n$) a compact metric space X of dimension $n-k$ and an at most $(k+1)$-to-1 continuous mapping f such that $f(X)=Y$? The present paper gives an affirmative answer to this question. This result, combined with the above theorem of Hurewicz, yields the following characterization of dimension: The compact metric space Y is of dimension n ($n>0$) if and only if Y is the image of an $(n-1)$-dimensional compact metric space X under a continuous, at most 2-to-1, mapping. The hypothesis that X and Y are *compact* can be replaced throughout by the hypothesis that they are *separable*, provided that f is restricted to be reciprocally continuous (*beiderseits stetig*). (Received March 29, 1940.)

320. Barkley Rosser: *An additional criterion for the first case of Fermat's last theorem.*

It has been proved in a succession of papers due to numerous authors that if p is an odd prime and $a^p+b^p+c^p=0$ has a solution in integers prime to p, then $m^{p-1}\equiv 1$ (mod p^2) for each prime $m \leqq 41$. The methods which work for $m \leqq 41$ lead to excessive computation when applied to $m=43$. In this paper a new method especially adapted to m's of the form $6n+1$ is developed, and applied in the case of $m=43$ to prove the result quoted above for this case also. (Received March 28, 1940.)

321. A. C. Schaeffer and Gabor Szegö: *Inequalities for harmonic polynomials in two and three dimensions.*

The first part deals with two dimensional, the second part with three dimensional harmonic polynomials. I. The basic result is a formula of interpolatory character representing a given linear combination of the coefficients of a harmonic polynomial $U(n, \phi)$ of degree n in terms of $U(1, \phi)$ at $2n$ equidistant points. Here only the following application should be mentioned: let $U(1, \phi) \geqq 0$ and let $\sigma_n(\phi)$ be the nth Cesàro means of $U(1, \phi)$. Then $n\sigma_n(\phi) \geqq |\text{grad } U(1, \phi)|$. This theorem includes Szegö's "gradient theorem" which states that $|\text{grad } U(1, \phi)| \leqq n$ provided $|U(1, \phi)| \leqq 1$. II. The second part deals with the following problem. Consider all three dimensional harmonic polynomials $U(P)=U(x, y, z)$ of degree n satisfying the condition $|U(P)| \leqq 1$ for $x^2+y^2+z^2 \leqq 1$. If Q is a fixed point in space at a distance $R>1$ from the origin, an explicit expression for the maximum of $|U(Q)|$ is obtained in terms of R and n. The asymptotic value of this maximum as $n \to \infty$ is $(\pi^{1/2}/2)(1-R^{-2})^{1/2}n^{1/2}R^n$, whereas the corresponding maximum in the two dimensional case (as already known) is exactly R^n. (Received March 9, 1940.)

322. G. E. Schweigert: *Equivalence of pointwise periodic and interior transformations on dendrites.*

Given that $T(A)=B$ is continuous, A and B are dendrites, and $T^{-1}(b)$ is finite for each point b in B, what are the conditions under which it is possible to define a pointwise periodic homeomorphism $h(A)=A$ such that for each b in B and each x in $T^{-1}(b)$ the point-orbit of x is exactly the set $T^{-1}(b)$? Since an interior transformation S which carries A into the orbit space is induced whenever h exists, one looks to the preservation of open sets and other known necessary conditions to answer this question. A sufficient set of such conditions is found and h is effectively defined to within the choice of permutation for certain inverse sets. The conditions are simple and represent a kind of symmetry necessary in A; they may be varied slightly to suit a given case. Given T the h defined induces T; given h the induced S allows us to rediscover h to within the occasional choice of permutation in an orbit. Special care was taken throughout to avoid methods that would not be useful in a similar problem for Peano spaces. (Received March 20, 1940.)

323. R. W. Shephard: *The length of production and related dynamic aspects of a simplified economic system.* Preliminary report.

By consideration of mean values and totals of integrals over the length of production, the equations of equilibrium of an economic system can be replaced approximately by those of a simplified system involving a time lag. The plans of the producers, taken with reference to certain technical functionals, are given in terms of current and expected future prices. Under somewhat general assumptions concerning the relation of stocks to the demand for capital and consumption goods, it is found that the quantities of the system may be completely determined. The character of this determination is investigated by a study of small changes. (Received March 11, 1940.)

324. Max Shiffman: *The Morse relations in the Plateau problem for several boundaries.*

Let $\Gamma_1, \Gamma_2, \cdots, \Gamma_k$ be k prescribed non-intersecting Jordan curves in space. This paper establishes the Morse relations for minimal surfaces bounded by these k curves if degenerate as well as nondegenerate surfaces are included. The space $\mathfrak{R}_k$ of circular domains of representation with k boundaries are introduced and those points on each circle which are mapped into three specified points of Γ_ν are marked and considered parts of the domain. Degenerate domains are also introduced and limit is defined. It is shown that $\mathfrak{R}_k$ is a metric space and its connectivity numbers are determined. The paper then deals with the behavior of the Dirichlet functional over the space $\mathfrak{P}$ of potential surfaces defined over the domains of $\mathfrak{R}_k$. This behavior permits carrying over the results obtained by the author in previous work on the case of one boundary (Annals of Mathematics, (2), vol. 40 (1939), pp. 834–854). A similar theory was developed independently by Morse and Tompkins (Annals of Mathematics, (2), vol. 40 (1939), pp. 443–472). (Received March 29, 1940.)

325. W. S. Snyder: *Functions of simple figures.* Preliminary report.

Let F be a function defined on a class S of subsets of euclidean n-space. Assuming certain simple properties of the elements of S it is possible to develop a comprehensive theory of the derivatives and the Burkill integrals of F. These results include and extend the principal results of Banach, Burkill, Saks and Kempisty. The purpose of the paper is to establish a theory which is sufficiently general to cover the most important applications known at the present time. (Received March 15, 1940.)

326. D. C. Spencer: *On finitely mean valent functions.*

Suppose that $f(z)$ is regular in the unit circle $|z|<1$, and that $W(R)$ is the area (multiply covered regions being counted multiply) of the portion of the transform of $|z|<1$ by f which lies in the circle $|w|\leqq R$. Then if $W(R)\leqq p\pi R^2$ for all $R>0$, where p is a positive number (not necessarily integral), $f(z)$ is said to be mean p-valent. Let $A(r)$ be the area of the transform of $|z|<r$ by $f(z)$, $M(r,f)$ the maximum modulus of $f(z)$ on the circle $|z|=r$. Functions f satisfying the condition (less restrictive than the one given above) that $A(r)\leqq p\pi M^2(r, f)$ are described as having weak mean valency p. These definitions were first suggested to the author by Professor J. E. Littlewood. In this paper it is shown that many of the properties of p-valent functions

are possessed by mean p-valent functions, but *not* by the wider class of weak mean p-valent functions. Inequalities are obtained for the mean values of a mean p-valent function f and its derivative analogous to those already known for p-valent functions, and bounds are deduced from these inequalities for the coefficients in the power series of f. (Received March 18, 1940.)

327. D. C. Spencer: *On finitely mean valent functions.* II.

This paper is a sequel to one of the same title of which the abstract appears above. Here the rate of growth of mean p-valent functions is discussed. It is shown, for example, that if f is mean p-valent and $0 \leqq r < 1$, then $|f|$ on the circle $|z| = r$ is of order $(1-r)^{-2p}$ at most. Another (and related) result is that *schlicht* functions which fill only an infinitesimal part of the plane are of infinitesimal order. Theorems concerning the behavior of mean p-valent functions on arbitrary paths tending to the circumference $|z| = 1$ are also included. The results of the paper remain true under hypotheses less restrictive than that of mean p-valency. If the theorems of this paper are combined with those of the paper above, it is found, for example, that if f is mean p-valent and of the form $f = \sum_{\nu=0}^{\infty} a_{\nu k+1} z^{\nu k+1}$, then $|a_n| = O(n^{2p/k-1})$ when $p > k/4$ (the restriction that $p > k/4$ is actually necessary). (Received March 18, 1940.)

328. Alvin Sugar: *On a result of Hua for cubic polynomials.*

L. K. Hua showed that every integer can be additively represented by seven values of certain cubic polynomials (Tôhoku Mathematical Journal, vol. 41 (1935–1936), pp. 361–366). The writer by a shorter method, with more general cubic polynomials, proves that five, and in some cases four, values suffice. (Received March 30, 1940.)

329. J. L. Synge: *On the electromagnetic two-body problem.*

The system under discussion consists of two charged particles (hydrogenic atom). The argument is relativistically invariant (in the sense of the special theory), but classical in the sense that quantum mechanics is not involved. The field due to a particle is that of the usual retarded potential, and the force on a particle the usual ponderomotive force, without any "radiation" term. The problem has been treated previously in two limiting cases: (1) small velocities (Darwin), (2) small mass-ratio (Sommerfeld). Neither of these approximations gives any degeneracy of the motion (radiation of energy). In the present paper a general method of successive approximations is sketched, and applied to the Sommerfeld approximation, but with inclusion of higher-order terms arising from the motion of the nucleus. A full calculation is carried out for the case where the orbit of the lighter particle is approximately circular. It is found that the orbit decreases steadily in radius, remaining circular. Thus there is degeneracy, but the radiation of energy is much less than that usually quoted, which is derived from a computation of flux of energy at infinity. (Received March 16, 1940.)

330. Olga Taussky and John Todd: *On determinants of quaternions.*

The usual definitions of determinants whose elements are (real) quaternions do not ensure the reality of a "hermitian" determinant. A definition which does this can be given. With this definition the so-called Gram's determinants are investigated and the results are applied to extend recent work of H. and F. J. Weyl on unitary metrics in projective space (Annals of Mathematics, (2), vol. 39 (1938), pp. 516–538 and vol. 40 (1939), pp. 141–148 and 634–635). The metric thus obtained has the interesting property that a point need not have a zero distance from itself. (Received March 25, 1940.)

331. H. P. Thielman: *On the convex solution of a certain functional equation.*

In this paper the following theorem is proved: The only convex solution of the functional equation $1/f(x+a)=x^p f(x)$ $(x>0, a>0, p>0)$ is $F(x)=[\mathrm{B}(x/2a, 1/2)/(2a\pi)^{1/2}]^p$. This is an analogue of a theorem by E. Artin (*Einführung in die Theorie der Gammafunktion*, Hamburger mathematische Einzelschriften, no. 11, 1931, p. 12), which states that if $f(x)$ is a continuous solution of $f(x+1)=xf(x)$, and if $\log f(x)$ is convex, then $f(x)=a\Gamma(x)$, where a is an arbitrary constant. A. E. Mayer (Acta Mathematica, vol. 70 (1938), pp. 61, 62) has shown that Artin's theorem cannot be improved by replacing the convexity of $\log f(x)$ by the mere convexity of $f(x)$. He has given examples of functions which are convex, satisfy the equation $f(x+1)=xf(x)$, but are essentially different from $a\Gamma(x)$, where a is any constant. For the result of this paper it was not necessary to assume the convexity of the logarithm of the solution. The special case $a=1$, $p=1$ of the present theorem was given by Mayer in the paper referred to above. (Received March 14, 1940.)

332. R. M. Thrall: *A note on a theorem by Witt.*

Let G^c denote the cth member of the lower central series of any group G. If F is the free group with n generators, E. Witt (Journal für die reine und angewandte Mathematik, vol. 177 (1937), pp. 152–160) has shown that F^c/F^{c+1} is a free abelian group with N generators and $N=1/c\sum\mu(c/d)n$ where μ is the Möbius μ-function and the summation is over all divisors of c. Let $Q=F/H_q$ where H_q is the smallest group containing all qth powers in F. In this note it is proved that if p is a prime less than c and $q=p$, then Q^c/Q^{c+1} is abelian and of order q^N. This result is applied to the commutator calculus for groups of small class. (Received March 14, 1940.)

333. Gerhard Tintner: *Fourier integrals as Bessel differential operators.* Preliminary report.

Let $g(v)=\int[\exp(-ivx)]f(x)dx$, where the limits are $-\infty$ and $+\infty$ and $f(x)$ can be developed into a Taylor series in terms of $1/x$, $f(x)=h(1/x)=\exp(z/x)\rightarrow h(t)$ for $t=0$, where $z=d/dx$. Then by Cauchy's theorem: $g(v)=z\sum_{k=0}^{\infty}(-izv)^k/k!(k+1)!\rightarrow g(t)|_{t=0}$ $=(z/(-izv)^{1/2})I_1(2(-izv)^{1/2})\rightarrow g(t)|_{t=0}$ where I_1 is a Bessel function for purely imaginary argument. Let also $G(v_1, v_2, \cdots, v_n)=\int\cdots\int(\exp-\sum_{j=1}^{u}iv_jx_j)F(x_1, x_2, \cdots, x_n)dx_1dx_2\cdots dx_n$, where each integral runs from $-\infty$ to $+\infty$. Define $z_j=\partial/\partial x_j$. Then by a similar argument $G(v_1, v_2, \cdots, v_n)=\prod_{j=1}^{n}z_j\sum_{k=0}^{\infty}(-iz_jv_j)^k/k!(k+1)!$ $\rightarrow H(t_1, t_2, \cdots, t_n)|_{t_i=0}$, which can also be expressed as the product of n Bessel differential operators. It is believed that this method may yield useful approximations in the evaluation of statistical distributions by the method of characteristic functions. (Received March 14, 1940.)

334. W. J. Trjitzinsky: *Developments in the analytic theory of algebraic differential equations.*

This work to appear in the Acta Mathematica presents an extensive analytic theory of algebraic differential equations $F=0$, where F is a polynomial in $y, y^{(1)}, \cdots, y^{(n)}$. In F the coefficients are polynomials in x or, more generally, they are functions, analytic in certain regions extending to infinity and therein asymptotic to series of the form $x^m[a_0+a_1x^{-1}+\cdots]$ (integer m). The case is also considered when the coefficients are continuous in x on an interval and contain a parameter. The main results amount to the following. When the equation $F=0$ has a formal solution s of the same type as occurs in the corresponding linear case, one can always construct regions R and "actual solutions" y of $F=0$ for which $y^{(i)}\sim s^{(i)}$ $(i=0, \cdots, n$; in R; to $k(t)$ terms; $k(t)\rightarrow\infty$ with t). Essentially, the regions are determined by the character of a certain linear problem associated with $F=0$. (Received March 19, 1940.)

335. V. J. Varino: *A note on matrices over a principal ideal ring.*

Let A and B be square matrices with elements in a principal ideal ring $\mathfrak{P}$. An algorithm for finding the greatest common right divisor and least common left multiple of A and B is given. It is also shown that any two greatest common left divisors are left associates, and similarly for any two least common left multiples. (Received March 14, 1940.)

336. C. W. Vickery: *On cyclically invariant graduation.*

Let $f(t)$ be a function of a real variable t. Let Z be a random variable symmetrically distributed about the mean $\mathcal{E}(Z)=0$ and having a (cumulative) distribution function $F(z)$. The graduation $f^*(t)$ of $f(t)$ is defined as follows: $f^*(t)=A\cdot\mathcal{E}\{f(t+z)\}$ $=A\int_{-\infty}^{\infty}f(t+z)dF(z)$. If $A=\{\mathcal{E}(\cos Z)\}^{-1}=\{\int_{-\infty}^{\infty}(\cos z)dF(z)\}^{-1}$, then $\sin t$ and $\cos t$ are invariant with respect to this operation. In applications, Z may be assumed to have a normal distribution, a symmetrical Bernoulli distribution, and so on. The coefficient A corrects Spencer's and similar graduation formulae so that $\sin t$ and $\cos t$ are invariant with respect to their application. (Received March 9, 1940.)

337. C. W. Vickery: *On spaces ($\mathcal{E}$) and Moore spaces.*

In order that a space S be a Moore space (F. B. Jones, this Bulletin, vol. 43 (1937), p. 675) it is necessary and sufficient that it satisfy the following conditions: (1) S is a space ($\mathcal{E}$) of Fréchet (*Les Espaces Abstraits*, p. 214); (2) the derived set of every point set is closed; (3) S is regular. These conditions are independent. (Received March 9, 1940.)

338. T. L. Wade: *Subgeometries of projective geometries as theories of tensors.*

This paper is concerned with the consideration of a subgeometry of classical (flat) projective geometry as the theory of a tensor in lieu of the customary consideration of such a geometry as the theory of a subgroup of the general linear projective transformation group. Affine, euclidean (parabolic), elliptic, and hyperbolic geometries are each treated as the theory of a tensor. Fundamental theorems are given whereby all algebraic concomitants in these geometries in 2-space can be constructed as tensors of order zero; also a large number of examples connecting the tensor-invariant method with the existent literature are given. Further, it is pointed out how a presentation of the metric subgeometries of projective geometry as the theories of tensors gives additional insight into their differences. "Distance tensors" for the three metric geometries are introduced. (Received March 28, 1940.)

339. A. D. Wallace: *An analysis of non-alternating transformations.*

If $T(A)=B$ is a non-alternating transformation on the locally connected continuum A, then there exists a unique A-set A_0 such that A_0 is irreducible with respect to the property of mapping onto B under T. If $r(A)=A_0$ is the hereditarily monotone transformation which retracts A onto A_0 and t is the transformation T restricted to A_0, then $T=tr$ and $t(A_0)=B$ is a non-alternating transformation which maps cut-points into cut-points. Further if $T=T_2T_1$ and $t=t_2t_1$ are factored in accordance with the Whyburn-Eilenberg theorem, then T_2 and t_2 are equivalent as are T_1 and t_1r. Moreover the transformation T_2 takes cut-points into cut-points. (Received March 25, 1940.)

340. A. D. Wallace: *On certain classes of subcontinua.*

In what follows S will denote a metric continuum and Roman numerals will be used to designate classes of closed sets in S defined as I: all subcontinua of S; II: all closed sets X in S such that if $K \in$ I then $X \cdot K \in$ I; III: all retracts $X=r(S)$ such that if $K \in$ I and $X \cdot K \neq 0$, then $X \cdot K = r(K)$; IV: all hereditary monotone retracts of S; V: all sets X such that if Y is a closed set in X which separates X between two points, then Y separates S between these same points. In this paper alternative characterizations are given for these classes of sets and the transformations involved in their definition. The invariance of these classes is proved for certain continuous transformations. These classes may be ordered in accordance with the theorem: *For any continuum S we have* V $\subset$ IV $\subset$ III $\subset$ II $\subset$ I. *If S is locally connected, the inclusions (except the first) become equalities, but this does not hold in general.* (Received March 11, 1940.)

341. J. L. Walsh and W. E. Sewell: *On the degree of convergence of harmonic polynomials to harmonic functions.*

Let R be the interior of an analytic Jordan curve C, and let the function $u(x, y)$ be harmonic in R, continuous in $R+C$, and satisfy a Lipschitz condition of order $\alpha<1$ on C. Then there exist harmonic polynomials $p_n(x, y)$ of respective degrees n such that (1) $|f(x, y)-p_n(x, y)| \leqq M/n^\alpha$ for (x, y) on $R+C$, where M is independent of (x, y) and n. Conversely, inequality (1) implies the continuity of $u(x, y)$ on C and a Lipschitz condition of order α there. (Received March 27, 1940.)

342. M. S. Webster: *Maximum of certain fundamental Lagrange interpolation polynomials.*

This paper extends some of the results given in a previous paper (this Bulletin, vol. 45 (1939), pp. 870–873). Additional properties are also given for the maximum of certain fundamental Lagrange interpolation polynomials based on the zeros of Jacobi polynomials. Important use is made of the asymptotic expressions given by Szegö. (Received March 9, 1940.)

343. Louis Weisner: *Moduli of the roots of polynomials and power series.*

Let $f_m(z)=a_0z^{n_0}+\cdots+a_mz^{n_m}$ $(0\leqq n_0<\cdots<n_m)$ be a polynomial with zero coefficients suppressed, and let $\nu_s=n_s-n_{s-1}$. For a fixed k, let $p_1, \cdots, p_m$ be positive numbers, $p_0=1$, satisfying the inequality $p_0+p_0p_1+\cdots+p_0p_1\cdots p_m\leqq 2p_0p_1\cdots p_k$, and let $u_s=|p_sa_{s-1}/a_s|^{1/\nu_s}$. The author proves that if $R\leqq u_s$ $(s=1, \cdots, k)$, $R\geqq u_s$ $(s=k+1, \cdots, m)$, then the circle $|z|<R$ includes just n_k roots of $f_m(z)$. Various applications are made to the moduli of the roots of polynomials and power series. (Received March 27, 1940.)

344. Louis Weisner: *Power series, the roots of whose partial sums lie in a sector.*

With the aid of a generalization of the familiar inequality concerning the harmonic, geometric and arithmetic means of a number of positive numbers, the author proves the following theorem: If, for each $n \geqq n_0$, the roots of the partial sum of degree n of the formal power series $f(z) = \sum a_n z^n$ lie in some sector with vertex at the origin and aperture $\alpha < \pi$, then $f(z)$ is an entire function of order zero. This theorem was proved for the case $\alpha = 0$ by G. Pólya (Rendiconti di Palermo, vol. 36 (1913)). (Received March 26, 1940.)

345. D. W. Western: *A method for analytic continuation.*

The results obtained in this paper provide an expansion for the analytic continuation of a function of a complex variable with finite singularities beyond the limits of the ring of convergence of the Laurent series. The region of convergence is determined by definition of major and minor circles, a notion introduced by Flora Streetman and L. R. Ford (American Mathematical Monthly, vol. 38 (1931), pp. 198–201). In the limiting position about the origin, the region of convergence has an inner boundary composed of the smallest circles through the origin and the inner singular points. The outer boundary has as its limiting form the straight lines through the outer singular points perpendicular to the radius vectors of these points. The expansion involves the parameter used in construction of the major and minor circles. It is in the form of a double series of integral terms, one series being a polynomial in z and the other a rational fractional function of z. (Received March 28, 1940.)

346. F. J. Weyl: *On the defect relation for meromorphic curves.*

The theory of meromorphic curves (see Herman and F. J. Weyl, Annals of Mathematics, (2), vol. 39 (1938), pp. 516–538) is an extension of R. Nevanlinna's theory of meromorphic functions. Quantities $m^*(r; a)$, closely resembling in their behaviour the Nevanlinna defects of meromorphic functions, can be attached to the points a of the complex k-dimensional space $\mathfrak{R}$ in which a meromorphic curve is defined. The theory of meromorphic curves culminates in a stringent estimate (analogous to R. Nevanlinna's second main theorem) of the sum of such defects over any finite number of points a in $\mathfrak{R}$. In its previous formulation the validity of this so-called defect relation depends upon the assumption that the sum be extended over points which satisfy no accidental linear relations, that is, any $(k+1)$ of which are linearly independent. In the present paper the defect relation has been reformulated so as to make this restriction unnecessary without sacrificing essential parts of the original stringency. (Received March 29, 1940.)

347. P. A. White: *On certain relatively non-alternating transformations.*

This paper considers certain properties of "G-non-alternating transformations," as defined by A. D. Wallace (this Bulletin, vol. 46 (1940), p. 56). It is shown that ordinary non-alternating transformations and G-non-alternating transformations are the same when defined on a locally connected continuum A: (1) when G is the collection of all finite sets, if and only if A is unicoherent; (2) when G is the collection of all continua, if and only if A is a boundary curve. The question of the equivalence of G-n.a. and monotone transformations is also answered for the collections defined above. Finally, it is shown that the property of being a linear graph is invariant under a G-n.a. transformation, where G is the collection of all sets containing at most two points. (Received March 27, 1940.)

348. Hassler Whitney: *On regular families of curves.*

A new condition that a family of curves be "regular" is given. This is the same as that in Annals of Mathematics, (2), vol. 34 (1933), p. 244, Theorem 7A, except that the relation (2) is omitted. (Received March 19, 1940.)

349. R. L. Wilder: *Characterization of the lower dimensional generalized manifolds by positional properties in* S_n.

In the earlier papers the generalized closed manifold of dimension $n-1$ (g. c. $(n-1)$-m.) has been characterized by its positional topological properties in n-space. The g. c. 1-m. (simple closed curve) in n-space has been similarly characterized by P. Alexandroff (Annals of Mathematics, (2), vol. 36 (1937), p. 19). In the present paper the following characterization is obtained for all dimensions k: In order that a closed point set M in the n-sphere S_n should be a g. c. k-m., it is necessary and sufficient that (1) $p^{n-k-1}(S_n-M)=1$ and $p^{n-k-1}(F)=0$ for all closed proper subsets F of M; (2) S_n-M be uniformly locally i-connected for $i>n-k-1$; and (3) S_n-M be uniformly locally connected in terms of its bounding $(n-k-1)$-cycles. (Received March 15, 1940.)

350. A. R. Williams: *On a certain Cremona transformation between two* $(n-1)$*-spaces in* S_n.

The author takes a linear system of ∞^{n-1} quadrics in n-space. Then if a point P describes a $(n-1)$-space, its conjugate P' with respect to all the quadrics of the system describes a variety V of order n and dimension $n-1$. But this variety is also the locus of the poles of the $(n-1)$-space with respect to the individual quadrics of the system. Hence P' is the pole of the $(n-1)$-space with respect to some quadric of the system, and since the latter may be represented by a point P'' in a $(n-1)$-space one has a Cremona transformation between two $(n-1)$-spaces, which if one wishes may be taken coincident. Thus V appears in two aspects and the loci on it occur in pairs. To a locus of order r on V will correspond two loci of orders r_1 and r_2 on the two $(n-1)$-spaces. If r_1 is not equal to r_2, there will be another locus also of order r on V corresponding to loci of orders r_2 and r_1, respectively, in the two $(n-1)$-spaces. The study of the loci on V is facilitated by the Cremona transformation between the two $(n-1)$-spaces. (Received March 9, 1940.)

351. J. R. Woolson: *The mean of the iteration of linear operators in reflexive Banach spaces.* Preliminary report.

There exists a complex valued bilinear interspace inner product $[F, f]$ on (B), B to the complex numbers. B is a complex Banach space and (B) is the space of linear functionals on B of bound $\|F\|$. (B) is a space of the same type as B. If $[F, f]=0$ for all F, then $f=0$, and if $[F, f]=0$ for all f, then $F=0$. Let B and (B) be reflexive and define the adjoint of a linear operator A on B to B as the operation A^* on (B) to (B) such that $[A^*F, f]=[F, Af]$. Define a projection as a linear operation such that $P^2=P$. Using these concepts it is possible to demonstrate that $n^{-1}\sum_{i=1}^{n}A^i f$ approaches weakly a linear operation which is a projection. Weak convergence is functional convergence. (Received March 9, 1940.)

352. J. W. T. Youngs: *On parametric representations of surfaces.* Preliminary report.

The paper is concerned with the study of certain concepts in topology and area. A continuous vector function $\mathfrak{x}=\mathfrak{x}(a)$, defined on the surface of a sphere A (say) with values in 3-space, is a representation. If B is the image of A, then the totality of components of the inverse sets $\mathfrak{x}^{-1}(b)$, $b \varepsilon B$, constitutes an upper semi-continuous collection Σ. The set Σ can be topologized, and, by the Whyburn factor theorem, $\mathfrak{x}(A) = L(M(A))$, where $M(A)=\Sigma$ is monotone, and $L(\Sigma)=B$ is light. Two representations $\mathfrak{x}_1$ and $\mathfrak{x}_2$ are K-equivalent (Kerékjártó, Acta Szeged, vol. 3 (1927), pp. 49–67) if there exists a homeomorphism $H(\Sigma_1)=\Sigma_2$ such that $L_1(\Sigma_1)=L_2(H(\Sigma_2))$. They are F-equivalent if the Fréchet distance between them is zero. This paper discusses the interdependence of these ideas and affiliated concepts in the theory of area. Kerékjártó has shown that F-equivalence implies K-equivalence. Here it is shown that the theorem is still true if A and B are Peano spaces. Kerékjártó and Morrey (American Journal of Mathematics, vol. 57 (1935), pp. 17–50) both prove the converse in special cases. Unfortunately an example shows this to be false even in the simple cases they consider. (Received March 11, 1940.)