acquaint the reader with a skeleton of methods such as he is apt to encounter in the calculus of variations, rather than to formulate results as generally as possible. The first chapter is devoted to a discussion of various problems, and the derivation of the first order necessary conditions for an extremum. Chapter II treats quadratic problems; in particular, the theory of integral equations with real symmetric kernel and boundary value problems associated with a second order linear differential equation. Chapter III, which deals with sufficient conditions, is extremely brief; nowhere in the lectures is the Weierstrass $\mathcal{E}$-function mentioned. In Chapter IV the work of Lewy (Mathematische Annalen, vol. 98 (1928), pp. 107–124) on the absolute minimum for nonparametric problems in the plane is presented. Chapter V is concerned with harmonic functions and associated boundary value problems; this chapter is preliminary to the discussion of the problem of Plateau and conformal mapping given in Chapter VI. In this last chapter the problem of Plateau is solved for a single contour in three dimensions; the method of proof is that of Courant (Annals of Mathematics, (2), vol. 38 (1937), pp. 676–724).

W. T. Reid

Die Zylinderfunktionen und ihre Anwendungen. By R. Weyrich. Leipzig, Teubner, 1937. 5+137 pp.

A cylinder function may be defined as any solution of Bessel's differential equation. They include functions of the first kind, $J_p(z)$, also called Bessel functions, which are regular at $z=0$, and functions of the second kind which are not regular at $z=0$. There has been considerable confusion in the notation and canonical form of the cylinder functions of the second kind. They have been denoted by Y, G, K, and so on, by various authors and often the same notation is used with different meanings. Following the tables of Jahnke and Emde, our author uses the notation $N_p(z)$ for functions of the second kind and calls them Neumann functions. They are the same as those denoted by $Y_p(z)$ by Nielsen and also by Watson. (See Watson, *Theory of Bessel functions*, p. 57, for a further discussion.) In addition there are the Hankel cylinder functions, which are really functions of the second kind, defined by

$$H_p^{(m)}(z) = J_p(z)+i(-1)^{m+1}N_p(z), \qquad m=1,\ 2,\ i=(-1)^{1/2}.$$

The importance of the Hankel functions arises from the fact that alone among the cylinder functions $H_p^{(m)}(re^{i\theta})\rightarrow 0$ as $r\rightarrow\infty$, provided that $m=1$, $0\leqq\theta\leqq\pi$, and that $m=2$, $\pi\leqq\theta\leqq 2\pi$.

One way to introduce cylinder functions is to define them, as above, as solutions of Bessel's differential equation. Our author, however, follows a different path. His starting point is the wave equation, from which he derives first plane waves, and then, by superposition of these, spherical and cylindrical waves. This leads immediately to the Sommerfeld integral representation of the Hankel functions, which then serve as the basis for defining the J and N functions and for deriving the important properties of the cylinder functions.

The author discusses the following aspects of cylinder functions: power series, asymptotic expansions of Hankel and Debye, various integral representations, recurrence relations, zeros, definite and indefinite integrals, boundary value problems and applications. It will be noted that the author has succeeded in covering the most important topics in a remarkably small number of pages. But the value of the book must not be judged by its brevity. It contains a carefully planned exposition of the theory and will serve as a valuable study and reference book.

C. A. SHOOK

Differentialgeometrie der Kurven und Flächen und Tensorrechnung. By Václav Hlavatý. Groningen, Noordhoff, 1939. 11+569 pp.

This treatise presents a large portion of the classical differential geometry of one and two dimensional subspaces of ordinary euclidean space. Just enough vector and tensor analysis is given to enable the reader to manage profitably the abbreviated symbolism. Definitions and results are stated in such a way as to generalize readily to higher dimensions.

The first chapter is devoted to curves. After the theory is developed in terms of a general parameter, an account is given of the various specializations arising from the use of the arc length as parameter. In particular, the construction of a curve from its curvature and torsion is treated carefully.

The second chapter concerns those properties of a surface which depend only on its metric tensor. The absolute differential is used systematically. There is an unusually full discussion of the applicability of surfaces, including explicit equations for developing a surface on a plane or on a surface of revolution.

The normal to a surface leads, on differentiation, to a tensor associated with the behavior of the surface toward the ambient space. In the third chapter, those properties are discussed which depend on this (second fundamental) tensor. A feature of this chapter is the discussion of the explicit construction of surfaces having prescribed first or first and second fundamental tensors.