transformations in one, two, and three dimensions, as well as correlations in two and three dimensions are developed, including the fixed points and a considerable number of particular cases. This is followed by a more detailed study of algebraic curves and surfaces. In the plane, polarity is introduced and used to derive Plücker's numbers, with application to cubic curves. The configuration of the points of inflexion, the constant cross ratio of the tangents from a point on the curve, nodal and cuspidal forms are provided for. Space cubic curves, and both kinds of space quartics are treated briefly.

Even with the omission of many proofs, the development is so rapid and condensed that a reader must be alert and patient to get all the points of the argument. On the other hand, one who has mastered this volume will be in possession of a large part of the knowledge which goes under the generic name of geometry.

VIRGIL SNYDER

Some Integrals, Differential Equations and Series Related to the Modified Bessel Function of the First Kind. By A. H. Heatley. (University of Toronto Studies, Mathematical Series, no. 7.) Toronto, University Press, 1939. 32 pp.

Let $I_n(x)$ denote Bessel's function, and let $T(m, n)$ denote the integral over $0<t<\infty$ of the function $I_n(2at)t^{m-n}\exp(-p^2t^2)$. Differential equations are given for $T(m, n)$, for $T(m, n)\exp(-a^2/p^2)$, and for $T(m, n)\exp(-a^2/2p^2)$; and these lead to explicit formulas for $T(2n+1, n)$ and for $T(n, n)$. Then recursion formulas lead to evaluation of $T(m, n)$ for other pairs of values of m and n.

Power series expansions of $T(m, n)$ and $T(m, n)\exp(-a^2/p^2)$ are obtained. These results of Part I (pp. 7–21) are used in Part II (pp. 21–30) to evaluate an integral used by the author (Physical Reviews, vol. 52 (1937), pp. 235–238) in the Langmuir collector theory.

Part III (pp. 30–32) deals briefly with integrals of $x^{n+m}e^xI_n(x)$ and $x^{n+m}e^{-x}I_n(x)$.

R. P. AGNEW

Theory of Probability. By Harold Jeffreys. Oxford, Clarendon Press, 1939. 7+380 pp.

This book of Jeffreys is an outstanding addition to the relatively few substantial treatises of probability in English. It is in line with the author's *Statistical Inference*. At the start, it resembles Keynes' *A Treatise on Probability* in its subjective or psychical approach to probability. But it carries the implications of this approach to a great variety of problems arising in the physical sciences, in biology and in economics. This subjective axiomatic approach to probability is exemplified also by a recent paper of B. O. Koopman, *The axioms and algebra of intuitive probability*, in the Annals of Mathematics, April, 1940.

Keynes, Jeffreys, and Koopman deal first with non-numerical probability. The probability of a proposition is an extension of logic by interpolation between the false or impossible proposition and the true or certain proposition. First, these subjective probabilities are assumed to be orderable. However, Keynes does not even require the order to be linear—he gives on page 39 a two-dimensional diagram showing several curves extending from the point of impossibility to the point of certainty. Points on different curves represent propositions with probabilities not comparable. But Jeffreys writes: "Axiom 1. Given p, q is either more or less probable than r, or both are equally probable; and no two of these alternatives can be true." After Theorem 2, Jeffreys sets up his "Convention 3" which associates certainty with the number 1. But he states that cases arise where ∞ is a more suitable choice for certainty. His Theorem 8 is: "Any probability can be expressed by a real number."

Jeffreys criticizes other conceptions or definitions of probability as follows: The "classical" definition of probability leads to nothing but combinatorial analysis. Probability is not introduced until human judgment assesses the elements to be equally probable. The extension of the classical definition by Neyman and Cramer to continuous probability, using the ratios of measures of sets, faces a similar difficulty. For with $f(x)$ monotonic, $f(x_2)-f(x_1)$ may be taken as the measure of the interval from x_1 to x_2 just as well as x_2-x_1. It takes a human judgment to select. In connection with probability as the ratio of measures of sets, Jeffreys might also have mentioned Lomnicki and Steinhaus, Fundamenta Mathematicae, 1923. As to probability as a limit, Jeffreys writes (p. 304): "The existence of the limit is taken as a postulate by Mises, whereas Venn hardly considered it as needing a postulate. Thus there is no saving of hypotheses in any case, and the necessary existence of the limit denies the possibility of complete randomness." In R. A. Fisher's "infinite population," there remains implicitly in the randomness the notion of "equally probable." In this respect, this infinite population resembles the Willard Gibbs "ensemble" (p. 300).

Jeffreys also criticizes certain applications of probability notions to particular problems. He considers (p. 309) that "Student" in his z-distribution found a direct probability instead of an inverse probability. He states (p. 323) "My main disagreement with Fisher concerns the hypothetical infinite population which is a superfluous postulate since it does not avoid the need to estimate the chance in some other way, and the properties of chance have still to be assumed since there is no way of proving them. Another is that, as in the fiducial argument, an inadequate notation enables him, like 'Student,' to pass over a number of really difficult steps without stating what hypotheses are involved in them."

Jeffreys writes Bayes' theorem as: Posterior probability is proportional to the product of prior probability and likelihood. This likelihood corresponds to productive probability. He discusses in considerable detail the troublesome prior or "a priori" probability. For the range 0 to $+\infty$ he takes frequently the prior probability to be proportional to dv/v, although he calls attention (p. 100) to the fact that the integral of dv/v becomes infinite at both ends of the range.

Among applications, Jeffreys is perhaps most interested in significance tests, treated in Chapters V and VI, 107 pages. But after a preliminary discussion of direct probabilities, he takes up in Chapters II to IV: multiple sampling and the multinomial law, the Poisson law, normal law of error, Pearson laws, negative binomial law, characteristic function, t and z distributions, the method of least squares, rectangular distribution, correlation, maximum likelihood, approximation and successive approximation, expectations, effects of grouping, smoothing, rank correlation, grades, contingency, and artificial randomness.

In Appendix I are given certain tables for chi-square and t^2. And in Appendix II on the factorial function, a short table is given of the diagramma function, the derivative of log $x!$; that is, of log $\Gamma(x+1)$. There is an index jointly for topics and authors.

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