123. P. C. Rosenbloom: *Post algebras*: I. *Postulates and general properties.*

Post algebras are the algebras which bear the same relation to the n-valued logics defined by Post (American Journal of Mathematics, vol. 43 (1921), pp. 180–185) as Boolean algebras bear to the usual 2-valued logic. They are investigated here purely from the algebraic standpoint with no consideration of their interpretation as logic. The first postulate sets for these systems are introduced and the most important general properties are deduced. A fundamental theorem analogous to the Boolean expansion of functions in normal form is proved. The definition of "prime elements" is analogous to Huntington's definition for the Boolean algebras. "Powers of primes" are also defined. A theorem analogous to the fundamental theorem of arithmetic is proved to the effect that "in a Post algebra with a finite number of elements, every element is uniquely factorable into a product of powers of primes, disregarding order and repetition of factors." An arithmetic interpretation generalizing Sheffer's "Boolean numbers" is given. Several unsolved problems are discussed. (Received January 17, 1941.)

124. Ernst Snapper: *Structure of linear sets.*

It is shown that the linear sets of a vector space of arbitrary dimension over an integral domain in which every ideal has a finite basis admit a Noether decomposition into "primary" linear sets. The "associated prime ideals" of the largest primary components are uniquely determined invariants of the linear set. The proofs are based on definitions of "quotient ideal" of a linear set by a linear set, of "quotient linear set" of a linear set by an ideal, and of "product linear set" of an ideal and a linear set. The quotient ideal of a linear set by the whole space is called its "essential scalar ideal" and is fundamental in the definition of primary linear set. The radical of the essential scalar ideal of a primary linear set is prime and is called its associated prime ideal. The "isolated component linear sets" are uniquely determined by their corresponding prime ideals and the theory becomes the ordinary ideal theory in the case of dimension one. Also, this investigation gives rise to the notions of scalar ideal, almost-primarity, radical and essential radical, closed set, and dense set. (Received January 14, 1941.)

ANALYSIS

125. R. P. Agnew: *On methods of summability and mass functions determined by hypergeometric coefficients.*

Let α, β, γ be complex constants and $\gamma \neq 0, -1, -2, \cdots$. Let $\lambda_n(\alpha, \beta, \gamma)$, $n=0, 1, 2, \cdots$, be the coefficients in the power series expansion $\sum \lambda_n z^n$ of the hypergeometric function $F(\alpha, \beta, \gamma; z)$. Let $(H, \alpha, \beta, \gamma)$ be the Hurwitz-Silverman-Hausdorff method of summability generated by the sequence $\lambda_n(\alpha, \beta\ \gamma)$. (See Garabedian and Wall, Transactions of this Society, vol. 48 (1940), pp. 195–201.) Let C_r denote the Cesàro method of order r. For certain ranges of the parameters it is shown that $(H, \alpha, \beta, \gamma) = C_{\alpha-1}^{-1} C_{\beta-1}^{-1} C_{\gamma-1}$ and that $(H, \alpha, \beta, \gamma)$ is equivalent to $C_{\gamma-\alpha-\beta+1}$. These results determine conditions under which $(H, \alpha, \beta, \gamma)$ is regular and $\lambda_n(\alpha, \beta, \gamma)$ is the moment sequence of a regular mass function. (Received December 31, 1940.)

126. E. F. Beckenbach: *On almost subharmonic functions.*

It is shown that certain integral inequalities which, in the case of continuous functions, are known to characterize subharmonic functions and functions whose logarithms are subharmonic, no longer characterize these functions when mere summability is assumed. Instead, the inequalities characterize functions which, in the terminology of Szpilrajn, are almost subharmonic or have almost subharmonic logarithms. (Received January 27, 1941.)

127. Stefan Bergman: *On a class of linear operators applicable to functions of a complex variable.*

The totality of functions obtained from the analytic functions of one complex variable regular at the origin by applying the operator $u=\mathrm{P}(f) =\int_{-1}^{+1}\mathrm{E}(z, \bar{z}, t)f[z(1-t^2)/2](1-t^2)^{-1/2}dt$ is called the "class of functions" $\mathcal{C}(\mathrm{E})$. The author studies the properties of functions $u(z, \bar{z}) \in \mathcal{C}(\mathrm{E})$, where $\mathrm{E}=1+z\bar{z}t^2\mathrm{E}_1(z, \bar{z}, t)$. Functions u possess the following property: to every point Z there exists a function $H(z, \bar{z}; Z) \in \mathcal{C}(\mathrm{E})$ such that every u satisfies the functional equation $u(z, \bar{z}) =(2\pi i)^{-1}\int_{\mathfrak{a}^1} \boldsymbol{R}(Z; u)H(z, \bar{z}; Z)dZ$; $\boldsymbol{R}(Z, u)$ being a certain operator, and $\mathfrak{a}^1$ an arbitrary closed differentiable curve lying in the regularity domain of $u(z, \bar{z})$. The theorems of the theory of analytic functions of one complex variable are classified from a certain point of view, and the duality between theorems for functions of one complex variable and those for the class $\mathcal{C}(\mathrm{E})$ is studied. In this way the author obtains results on the development of u in series $\sum_{\nu=1}^{\infty} a_\nu u_\nu$ in certain domains, $\{u_\nu\}$ being a special set of functions depending on the domain, and further results on the connection between the coefficients A_{mn} of the development $u=\sum A_{mn}z^m\bar{z}^n$ and properties of u, and so on. For example, functions satisfying a linear partial differential equation of elliptic type form a class $\mathcal{C}(\mathrm{E})$ corresponding to a particular E. (Received January 22, 1941.)

128. Lipman Bers: *Analytic functions of two complex variables in domains bounded by two analytic hypersurfaces.*

Let $\mathfrak{M}^4$ be a domain of the z_1, z_2-space $(z_i=x_i+iy_i)$ bounded by two analytic hypersurfaces whose intersection (the *distinguished boundary surface* of $\mathfrak{M}^4$) forms a closed surface $\mathfrak{F}^2$: $z_i=Z_i(\lambda_1, \lambda_2)$, $0\leqq\lambda_i\leqq 2\pi$. If certain hypotheses (chiefly about the existence of a suitable set of subdomains of the same type) are made, one can generalize theorems proved for simple special cases of such domains (S. Bergman, Compositio Mathematica, vol. 6 (1939), p. 305; S. Bergman and J. Marcinkiewicz, Fundamenta Mathematicae, vol. 33 (1939), p. 75; L. Bers, Comptes Rendus de l'Académie des Sciences, Paris, vol. 208 (1939), p. 1273 and p. 1475). If U is a biharmonic function in $\mathfrak{M}^4$ which possesses continuous boundary values u on $\mathfrak{F}^2$, then $U(z_1, z_2) =\int_0^{2\pi}\int_0^{2\pi}\Omega(z_1, z_2; \lambda_1, \lambda_2\, u(\lambda_1, \lambda_2)d\lambda_1 d\lambda_2$ where Ω depends only upon the domain. Biharmonic functions which satisfy certain conditions (for example all non-negative functions) may be represented in the form: $U(z_1, z_2)=\int_0^{2\pi}\int_0^{2\pi}\Omega(z_1, z_2; \lambda_1, \lambda_2)d\omega_{\lambda_1,\lambda_2}$; U possesses a finite non-tangential boundary value in every point of $\mathfrak{F}^2$ where the absolute additive set function ω possesses a finite strong derivative. It follows from these theorems that a function $f(z_1, z_2)$ which may be represented in $\mathfrak{M}^4$ as a quotient of two bounded analytic functions possesses almost everywhere on $\mathfrak{F}^2$ finite non-tangential boundary values. If these boundary values are constant on a set of positive two-dimensional measure, f is constant. (Received January 22, 1941.)

129. R. P. Boas: *A note on functions of exponential type.*

It has been shown by D. V. Widder (Proceedings of the National Academy of Sciences, vol. 26 (1940), pp. 657–659) that a function $f(x)$ such that $(-1)^n f^{(2n)}(x)\geqq 0$ for $n=0, 1, 2, \cdots$ and $0\leqq x\leqq 1$ must coincide over $(0, 1)$ with an entire function which is at most of the first order (and of type at most π if it is of order 1). In this note a simple proof of Widder's result is obtained by integrating $\int f(x) \sin \pi x\, dx$ repeatedly by parts; the integrated terms are all non-negative, and consequently an upper bound for $\int |f^{(2n)}(x)| \sin \pi x\, dx$ follows; from this an adequate estimate of $|f^{(2n)}(x)|$ itself is easily obtained. The same method leads to simple proofs of theorems of I. J. Schoenberg (this Bulletin, vol. 42 (1936), pp. 284–288) and J. M. Whittaker and H. Poritsky (Whittaker, Proceedings of the London Mathematical Society, (2), vol. 36 (1933), p. 455; Poritsky, Transactions of this Society, vol. 34 (1932), p. 287). (Received January 21, 1941.)

130. H. L. Garabedian: *Hausdorff methods of summation which include all of the Cesàro methods.*

In this paper are exhibited Hausdorff methods of summation which have the rare property of being more effective than all of the Cesàro methods of real orders. The regular mass functions associated with these Hausdorff methods are notably $\phi_\alpha(u) = 1-e^{\alpha u/(u-1)}$, $\alpha>0$, and $\phi_\beta(u)=A\int_0^u(1-t)^{\beta-2}e^{t/(t-1)}dt$, $\beta\geqq 0$, where A is a normalizing factor such that $\phi_\beta(1)=1$. The conjecture is made that summability $[H, \phi_\beta(u)]$ increases in efficiency with increasing β. (Received December 26, 1940.)

131. Abe Gelbart: *On the behavior of a function of two complex variables in the neighborhood of an isolated essential singularity.*

In this paper the author obtains certain upper and lower bounds for classes of functions of two complex variables in the neighborhood of an isolated essential singularity. Given a sequence of functions $\{h_\nu(z_1, z_2)\}$ regular in a finite four-dimensional domain $\overline{\mathfrak{M}}^4$ such that the analytic surfaces $\{h_\nu(z_1, z_2)=0\}$ converge to the surface $z_1=g(z_2)$ which passes through the domain $\mathfrak{M}^4$. It is assumed that the analytic surfaces $h_\nu(z_1, z_2)=0$ converge to $z_1=g(z_2)$ with sufficient rapidity. With the aid of theorems of the Runge type for functions of two complex variables for multiply-connected four-dimensional domains (see abstract 46-9-418), the author obtains an upper bound for the moduli of functions of two complex variables belonging to a class of functions analytic in $\overline{\mathfrak{A}}^4\equiv\overline{\mathfrak{M}}^4-G^4$, $G^4\equiv E[z_1=g(z_2)+\rho e^{i\lambda}, 0\leqq\lambda\leqq 2\pi, 0<\rho<\epsilon]$, and having the same zeros as $\prod_{\nu=1}^{\infty} h_\nu(z_1, z_2)$. Using a technique similar to that of Bergman (Mathematische Annalen, vol. 109 (1934), pp. 324–348), upper and lower bounds are obtained for an arbitrary function meromorphic in the neighborhood of the analytic surface $z_1=g(z_2)$ and having the same zeros and poles as $\prod_{\nu=1}^{\infty} h_\nu(z_1, z_2)/p_\nu(z_1, z_2)$ wherein $p_\nu(z_1, z_2)$ are functions regular in the domain $\overline{\mathfrak{M}}^4$. (Received January 23, 1941.)

132. H. H. Goldstine: *The prolongment and representation of multilinear functionals.*

It is first shown that every Banach space is linearly equivalent to a linear subset of the space of all bounded and real-valued functions defined on a suitably chosen range. Every linear continuous functional on this space of functions is representable by a Lebesgue-Stieltjes integral. Making use of this theorem, the Hahn-Banach theorem can be proved for multilinear, continuous functionals; that is, the range of definition of every such functional can be extended to the entire space without changing the norm of the functional. At the same time it is shown that these functionals are representable as generalized Fréchet integrals. (Cf. Goldstine, *Bilinear functionals on the space of bounded, measurable functions*, this Bulletin, vol. 43 (1937), pp. 528–531.) (Received December 16, 1940.)

133. P. R. Halmos: *On the decomposition of measures.*

If S is a strictly separable measure space and A is a strictly separable Borel field of measurable sets then, except possibly for a set (in A) of measure zero, S is a direct sum of measure spaces Y_x formed with respect to a measure space X in such a way that the Borel field of all measurable sets depending on x alone coincides with the given Borel field A. This general decomposition theorem yields an easy proof of a result of von Neumann on the decomposition of an arbitrary measure preserving transformation into ergodic parts. Another application of the theorem is to the spectral theory of measure preserving transformations: it is possible by means of it to construct examples of transformations with mixed spectrum—that is, transformations having neither pure point spectrum nor pure continuous spectrum—and to decompose, at least partially, an arbitrary transformation into transformations with pure spectrum. (Received January 22, 1941.)

134. D. H. Hyers: *On the stability of the linear functional equation.*

Let E and E' be linear normed spaces and let δ be a positive number. A transformation $f(x)$ of E into E' will be called δ-linear if $\|f(x+y)-f(x)-f(y)\|<\delta$ for all x and y in E, and linear if $f(x+y)=f(x)+f(y)$. S. Ulam has proposed the following problem of the "stability" of the last equation: Does there exist for each $\epsilon>0$ a $\delta>0$ such that to each δ-linear transformation $f(x)$ there corresponds a linear transformation $g(x)$ satisfying the inequality $\|f(x)-g(x)\|\leqq\epsilon$ for all x in E? In this paper this question is answered in the affirmative for the cases where E and E' are finite dimensional spaces or real Hilbert space, and where $f(x)$ is continuous. It is shown that δ may be taken equal to ϵ in these cases. As Ulam has pointed out, the problem has many generalizations to be investigated. (Received January 6, 1941.)

135. H. N. Laden: *On Kryloff-Stayermann interpolation.*

This paper deals with an interpolation polynomial $F_n(x)$ of degree less than or equal to $4n-1$ which, at the preassigned abscissas $x_1, x_2, \cdots, x_n$ takes preassigned values $y_1, y_2, \cdots, y_n$ respectively and is such that $F_n'(x_i)=F_n''(x_i)=F_n'''(x_i)=0$, $(i=1, 2, \cdots, n)$. If the abscissas are the zeros of the nth Jacobi polynomial and $y_i=f(x_i)$, $(i=1, 2, \cdots, n)$, where $f(x)$ is an arbitrary function continuous on $[-1, +1]$, it is shown that $F_n(x)$ converges to $f(x)$ as $n\rightarrow\infty$ uniformly on any closed interval wholly inside $[-1, +1]$. The convergence is uniform on $[-1, +1]$ if proper restrictions are placed on the parameters α, β of the general Jacobi polynomial. Similar results are obtained for interpolation on an infinite interval by means of Laguerre and Hermite polynomials. The first part of the results provides a correction and extension of a theorem of Kryloff and Stayermann. The methods employed are due to Fejér, Shohat, and Szegö. (Received January 11, 1941.)

136. Dorothy Maharam: *On measure in abstract sets.*

Let m be a Boolean σ-ring of sets; n, a σ-ideal in m; Φ, a class of σ-isomorphisms between principal ideals in m/n. It is assumed that Φ contains all products and inverses of its elements. Define the elements of m (n) to be measurable (null) sets, the isomorphisms of Φ to be measure-preserving transformations, and the measure of an element a of m/n to be the totality of division images of a under Φ. The element a is bounded if a is not division-equivalent to any proper sub-element of itself. The set C of bounded measure values is then described; in particular, conditions are given under which C is isomorphic to a set of positive numbers. (Received January 22, 1941.)

137. Szolem Mandelbrojt: *On a Dirichlet series.*

This paper gives necessary and sufficient conditions on $M(\sigma)$ such that there exist a function $f(s) = \sum a_n n^{-s}$, $f(s) \not\equiv 0$, $f(-2q) = 0$ $(q \geqq 1)$, max $|f(\sigma + it)| < M(\sigma)$. (Received December 16, 1940.)

138. N. M. Oboukhoff: *On the validity of total differential as the principal part of the increment of a function of two or more variables.*

K. Weierstrass defined total differential as principal part of the increment of a function. Let $du = Adx + Bdy$ where A and B are finite functions of x and y only, $|dx|$ and $|dy|$ being infinitely small or just small enough relative to $|A|$ and $|B|$. If the terms are all positive or negative, du retains its rank of principal part. However there are cases in which this principal part degenerates. Suppose $du = Adx - Bdy$ and $Adx > Bdy > 0$ or $Bdy > Adx > 0$. Let $|Adx - Bdy| = K(Adx + Bdy)$, where $0 \leqq K \leqq 1$. If K is finite or great enough relative to $|dx|$ and $|dy|$ then du does not degenerate. However it does if K vanishes or is of the same order of magnitude as $|dx|$ and $|dy|$. Thus $K = \big|\,|dx/dy|\,|A| - |B|\,\big| : (|dx/dy|\,|A| + |B|)$; du degenerates if $|dx/dy| = |B/A| \pm \eta$, where positive η either vanishes or is of the same order of magnitude as $|dx|$ and $|dy|$. For all other values of $|dx/dy|$ from zero to infinity (where dy vanishes) principal part holds. The favorable cases are greatly predominant. Closely similar results obtain for functions of several variables. (Received December 5, 1940.)

139. George Polya: *On converse gap theorems.*

Let $\lambda_1, \lambda_2, \lambda_3, \cdots$ *be integers,* $0 < \lambda_1 < \lambda_2 < \cdots$. *The* necessary *and sufficient condition that* each *analytic function defined by a series of the form* $a_1 z^{\lambda_1} + a_2 z^{\lambda_2} + \cdots + a_n z^{\lambda_n} + \cdots$ *having a finite radius of convergence be uniform and its domain of definition be simply connected is that* (1) $\lim \inf_{n \to \infty} n \lambda_n^{-1} = 0$. The much deeper half of this theorem concerning the sufficiency of the condition (1) has been proved previously (Annals of Mathematics, (2), vol. 34 (1933), pp. 731–777). The necessity of the condition (1) may be shown by the series (2) $\sum_{m=1}^{\infty} \mathcal{J}(m) z^m / m = \sum_{n=1}^{\infty} (-1)^{\lambda_n} z^{\lambda_n} / \lambda_n^2 \mathcal{G}'(\lambda_n)$ where $\mathcal{J}(z)$ and $\mathcal{G}(z)$ are defined by $\pi z \mathcal{J}(z) \mathcal{G}(z) = \sin \pi z$, $\mathcal{G}(z) = \prod_{n=1}^{\infty} (1 - z^2/\lambda_n^2)$. If (1) is not fulfilled, the analytic continuation of the series (2) is infinitely many valued (for proving this use Mathematische Zeitschrift, vol. 29 (1929), pp. 604–608). An analogous theorem has been proved by the author in the Comptes Rendus de l'Académie des Sciences, Paris, vol. 208 (1939), pp. 709–711. (Received January 20, 1941.)

140. Tibor Radó: *On the semi-continuity of double integrals in the calculus of variations.*

The purpose of this paper is to extend the results of McShane (*On integrals over surfaces in parametric form,* Annals of Mathematics, (2), vol. 34 (1933), pp. 815–838) to a wide class K of surfaces which may be characterized as follows: A surface S, of the type considered by McShane, belongs to the class K if it admits of a representation where its Lebesgue area is given by the usual integral formula. All the results of McShane are shown to remain valid for this class K. (Received December 16, 1940.)

141. Raphaël Salem: *On absolute convergence of trigonometrical series.* I.

The following theorems are proved: If the series (S) $\sum \rho_n \cos(nx - \alpha_n)$, $(\rho_n \geqq 0)$ converges absolutely at two points x_0, x_1, the series $\sum \rho_n |\sin n(x_1 - x_0)|$ converges. If

$\sum \rho_n = \infty$ the series (S) cannot converge absolutely at more than one point if $\rho_{n+1} \leqq \rho_n$ or even if ρ_{n+p}/ρ_n is bounded ($p>0$). If $\sum \rho_n = \infty$, the series $\sum \rho_n \cos (k_n x - \alpha_n)$ cannot converge absolutely at two points whose abscissae differ from δ if the numbers $k_n \delta$ are uniformly distributed (mod 2π). If $\sum \rho_n = \infty$ and the series (S) converges absolutely in a perfect set N, then for every bounded function F non-decreasing, constant in every interval contiguous to N but increasing from one interval to another, the cosine Fourier-Stieltjes coefficients of even rank of dF have their upper limit equal to $F(2\pi) - F(0)$. This condition is sufficient to ensure the absolute convergence of (S) "almost everywhere" in N, with $\sum \rho_n = \infty$. Further properties of sets of the type N are discussed. (Received January 30, 1941.)

142. Raphaël Salem: *On absolute convergence of trigonometrical series. II.*

The following extension of the Denjoy-Lusin theorem is proved: The set for which the ratio (I) $\sum_1^n \rho_n | \cos (nx - \alpha_n) | / \sum_1^n \rho_n$ has its upper limit less than or equal to α is of measure zero if $\alpha < 2/\pi$. Moreover any perfect set P in which the ratio (I) has its upper limit less than or equal to α less than $2/\pi$ has the property that every bounded function F, non-decreasing, constant in every interval contiguous to P, but not everywhere, is such that the Fourier-Stieltjes coefficients of dF do not tend to zero. The constant $2/\pi$ is the best possible one; in fact, if $\rho_n = O(1)$, the ratio (I) tends to $2/\pi$ almost everywhere. (Received January 30, 1941.)

143. I. J. Schoenberg: *On integral representations of completely monotone and related functions.*

In a recent paper W. Feller proved very elegantly the chief results of S. Bernstein and Widder concerning the nature of functions $f(x)$ allowing a Laplace-Stieltjes representation (1) $f(x) = \int_0^\infty e^{-xt} d\alpha(t)$ with various restrictions as to $\alpha(t)$. In the first part of the paper, Widder's original tools are used for the purpose of an equally short presentation of this theory. In particular, Widder's inversion formula for (1) appears in a very natural way. In the second part a similar theory is developed for the representation (2) $f(x) = \int_0^\infty K_n(xt) d\alpha(t)$ ($x>0$; n a positive integer) where $K_n(x) = (1-x)^n$ if $0 \leqq x \leqq 1$, $K_n(x) = 0$ if $x > 1$. Since $K_n(x/n) \to e^{-x}$ as $n \to \infty$, (1) is seen to be a limiting case of (2). The results may be described very roughly as follows: The role played by the *completely monotone functions* $f(x)$ (that is, satisfying the conditions (3) $(-1)^n f^{(n)}(x) \geqq 0$ ($x>0$; $n = 0, 1, 2, \cdots$)) in the theory of the representation (1), is taken over in the case of (2) by the $(n+1)$-*times monotone functions* $f(x)$. Here a function is called $(n+1)$-times monotone if $f(x) \geqq 0$, $-f'(x) \geqq 0$, $f''(x) \geqq 0, \cdots,$ $(-1)^{n-1} f^{(n-1)}(x) \geqq 0$, ($x>0$), the last function $(-1)^{n-1} f^{(n-1)}(x)$ being also required to be non-increasing and convex for $x>0$. (Received January 27, 1941).

144. W. T. Scott and H. S. Wall: *The transformation of series and sequences.*

The transformation $S_n = \sum_{p=0}^n (1/2)^{2n+1} C_{2n+2,n-p} \cdot s_p$, ($n = 0, 1, 2, \cdots$), which arises from the power series transformation $\frac{1}{2}(1-x) f(x) = F(z)$, $z = 4x/(1-x)^2$, is shown to define a regular method of summation, that is, $s_p \to s$ implies $S_n \to s$. Let this be called Schur summability and denoted by the letter S. It is proved that $S \supset VP$, but $VP \not\supset S$, where VP denotes de la Vallée Poussin summability; S sums the geometrical series in precisely the region in which VP sums this series; $S \approx M$, where M is the summability introduced by Mersman (this Bulletin, vol. 44 (1938), pp. 667–673). It is shown that if $\phi(u) \in BV[0, 1]$, and $g(x) = \int_0^1 d\phi(u)/(1+xu)$, then the sequence $\{g(n)\}$, $(n=0, 1, 2, \cdots)$, is a *C-Folge* in the sense of Hausdorff, and the Hausdorff mean $[H, g(n)]$ is regular if and only if $\phi(1)-\phi(0)=1$. If $\phi(u)$ is real and monotone and $\phi(1)-\phi(0)=1$, then $H, g(n) \subset (C, 1)$, and is equivalent to convergence if and only if the function $g(x)$ is bounded away from zero in the half-plane $R(x) > -\frac{1}{2}$. Let $\theta(u) = \sum_{p=1}^{\infty} q_p u^p$, $q_p \geqq 0$, $\sum q_p < \infty$. Then $a_n = 1 + n\int_0^1 u^n d\theta(u)$, $(n=0, 1, 2, \cdots)$, defines a Hausdorff mean $[H, a_n]$ equivalent to convergence if and only if $\sum nq_n < \infty$. (Received January 24, 1941.)

145. J. A. Shohat: *On Bernoulli numbers and polynomials.*

Certain relations are established between Bernoulli polynomials $B_n(x)$ and those of Legendre and symmetric polynomials of Jacobi; and a new proof is given of Jacobi's theorem concerning the expression of $B_{2n}(x)$ in powers of $x(1-x)$. A reasoning of Mandl is utilized and made rigorous in order to obtain in a new way the well known remarkable relation between the Bernoulli number B_{2n} and the zeta-function $\zeta(2n)$. Applications are made to $S_n(p) = 1^p + 2^p + \cdots + (n-1)^p$. (Received January 8, 1941.)

146. S. E. Warschawski: *On conformal mapping of infinite strips.*

Let S denote the strip $\phi_-(u) < v < \phi_+(u)$, $-\infty < u < +\infty$ ($\phi_+(u)$, $\phi_-(u)$ continuous), in the w-plane, $w = u + iv$. Let $z = Z(w)$ ($\lim_{u \to +\infty} \Re Z(w) = +\infty$) map S conformally onto the strip $|y| < \pi/2$ of the z-plane, $z = x + iy$. The principal object of this paper is to obtain asymptotic expressions for $Z(w)$ and its derivative $Z'(w)$ as $u \to +\infty$. For this purpose two inequalities are established, which are similar to those of L. Ahlfors (Acta Societatis Scientiarum Fennicae, new series A, vol. 1 (1930) p. 10 and p. 16), but which, due to some assumptions regarding the smoothness of the boundary of S, yield sharper estimates for large values of $\Re w_1$ and $\Re w_2$. The asymptotic expressions for $Z(w)$ and $Z'(w)$ are then applied, after suitable transformations, to the study of the order of magnitude of the mapping function of a region R onto a circle in a neighborhood of a finite boundary point P of R. These applications include the cases where P is the vertex of a corner, or of a cusp, or is the asymptotic point of two "concurrent" spirals, and contain as special cases the results presented by the writer in two previous abstracts (42-5-219 and 42-11-409). (Received December 9, 1940.)

Applied Mathematics

147. E. S. Allen and Harvey Diehl: *The enumeration of glycols.*

Recently the authors extended the method of Henze and Blair for enumerating certain organic compounds; in particular, the alcohols. This work is used as a basis for a recursive method of enumerating isomeric glycols, that is, compounds whose molecules have two oxygen atoms. The basic numbers which result are the number of structurally asymmetric glycols possessing n carbon atoms and having α enantiomorphically distinct forms, and the number of structurally symmetric glycols possessing n carbon atoms and having α distinct forms, of which β are completely symmetric. In all cases symmetry indicates identity of aspect of the molecule when viewed from the two hydroxyl radicals. (Received December 31, 1940.)