165. Peter Scherk: *On real closed curves of order $n+1$ in projective n-space.* II. Preliminary report.

In the first part of this paper (abstract 46-11-502) the author discussed differentiable closed curves K^{n+1} of real order $n+1$ in R_n by means of a certain single-valued correspondence of the K^{n+1}. He proved that $S \leqq n+1$, $S \equiv n+1$ (mod 2) if S is the sum of the multiplicities of the singular points, and he characterized the case $S=n+1$. Extending a simple remark on rotation numbers to multi-valued correspondences, the author discusses a two-valued and a three-valued correspondence defined on certain arcs of the K^{n+1} and on the whole K^{n+1} respectively, and connected with the projections of the K^{n+1} from its osculating $(n-2)$-spaces and $(n-3)$-spaces respectively. The study of these two correspondences yields: (1) the first estimates of the number of osculating $(n-2)$-spaces which meet the K^{n+1} again; (2) the classification of the K^{n+1} with $S=n-1$; (3) the classification of the K^5; (4) a more systematic access to the classification of the K^4 (previously obtained by the author). (Received January 24, 1941.)

166. Alexander Wundheiler: *Abstract algebraic definition of an affine vector space.* Preliminary report.

A linear set over the field of real numbers will be called a simple vector space, and its elements, simple vectors. Two simple vector spaces A and B are cogrediently coupled if for any a in A and b in B a real number $f(a, b)$ is defined, such that $f(ka, b)=f(a, kb)=kf(a, b)$; $f(a, b'+b'')=f(a, b')+f(a, b'')$; $f(a'+a'', b)=f(a', b)+f(a'', b)$. The a's and b's are then contragredient vectors. If A and B are of the same dimension, the set $A+B$ is called an affine vector space, a is a contravariant affine vector, b a covariant one, or vice versa. Various illustrations are given, as electrical networks, the space of fruit juice cocktail cans, and so on. (Received January 24, 1941.)

167. Oscar Zariski: *Pencils on an algebraic variety and a new proof of a theorem of Bertini.*

The theorem of Bertini-Enriques states that if a linear system of W_{r-1}'s on a V_r is reducible (that is, every W_{r-1} of the system is reducible) and is free from fixed components, then the system is composite with a pencil. In this paper a new proof of this theorem is given, together with an extension to irrational pencils. With every pencil $\{W\}$ there is associated a field P of algebraic functions of one variable, a subfield of the field Σ of rational functions on V_r. The essential point of the proof is the remark that $\{W\}$ is composite if and only if P is not maximally algebraic in Σ. The rest of the proof, in the case of pencils, follows from the fact that an irreducible algebraic variety V_r over a ground field K is absolutely irreducible if K is maximally algebraic in Σ. In the case of linear systems of dimension >1, the proof is based on the following lemma: if K is maximally algebraic in Σ and if x_1, x_2 are algebraically independent elements of Σ, then for all but a finite number of elements c in K the field $K(x_1+cx_2)$ is maximally algebraic in Σ. (Received December 12, 1940.)

Logic and Foundations

168. Alvin Sugar: *Postulates for the calculus of binary relations in terms of a single operation.*

In a recent paper (*Postulates for the calculus of binary relations*, Journal of Symbolic Logic, vol. 5 (1940), pp. 85–97) J. C. C. McKinsey gave a set of postulates for the calculus of binary relations in terms of the two operations $|$ and $<$. In his paper McKinsey shows that $<$ is definable in terms of $|$ but not conversely. In this paper the author develops a set of independent postulates for the calculus of binary relations in terms of the single operation $|$. (Received January 25, 1941.)

Statistics and Probability

169. Archie Blake: *The exploratory determination of statistical distributions.*

The most powerful statistics are not always the most "efficient" or those whose distributions have already been tabled, but their distributions can be computed in small samples without inordinate labor. To find the distribution of $g(X=x_1, \cdots, x_n)$ subject to the condition $P(X)$, compute requisite values of g^{-1} (multiple-valued) and require $\int^{P, g^{-1}} f(X)dX$, f being the given distribution function. The chief task is the computation of many values of the functions involved; this is alleviated by modern machine methods (especially punched cards). Tables of f, g, and so on, with their derivatives or the required fractions of the latter are prepared once for all on cards; thereafter the work consists only of interpolation. (Taylor's series recommends itself in this problem, as it converges more rapidly than ordinary interpolation formulae, and in the case of multivariate interpolation is much less complicated.) For a statistic whose asymptotic distribution is known, we can interpolate approximately between this and the results of the computation for small n to obtain an estimated distribution for any n. (Received January 23, 1941.)

170. W. G. Madow: *The distribution of the general quadratic form in normally distributed random variables.*

The distribution of the general quadratic form in normally distributed random variables is obtained. This distribution is used to obtain the distribution of Neyman's estimate in the theory of the representative method of sampling, and it is also used to obtain a generalization of P. L. Hsu's distribution of Student's ratio when the true means and variances are unequal. The distribution is also used in tests occurring in the analysis of variance with non-orthogonal data, and the study of differences of various orders. In the latter use, a test for periodicity is obtained. (Received January 25, 1941.)

171. Henry Scheffé: *An inverse problem in correlation theory.*

An $m\times n$ matrix Y may be used to represent m sets of measurements on n variables. The $n\times n$ matrix R of correlation coefficients r_{ij} is a function of the matrix Y, $R=F(Y)$. Necessary conditions (C) that $R=F(Y)$ are that R be real symmetric with diagonal elements unity, and positive (rank=index). Given any matrix R satisfying the conditions (C), does a "statistics problem Y" exist such that $R=F(Y)$? It is proved by matrix methods that there are no solutions Y with $m\leqq$ rank R, but ∞ solutions for each $m>$ rank R. Particular solutions are constructed and the most general solution is characterized. Some corollaries are drawn. (Received January 8, 1941.)

172. Jacob Wolfowitz: *Tests of statistical hypotheses where the distribution forms are unknown.*

The likelihood ratio criterion for testing composite statistical hypotheses, discovered by Neyman and Pearson and recently proved by Wald to be asymptotically