ABSTRACTS OF PAPERS

SUBMITTED FOR PRESENTATION TO THE SOCIETY

The following papers have been submitted to the Secretary and the Associate Secretaries of the Society for presentation at meetings of the Society. They are numbered serially throughout this volume. Cross references to them in the reports of the meetings will give the number of this volume, the number of this issue, and the serial number of the abstract.

ALGEBRA AND THEORY OF NUMBERS

182. Reinhold Baer: *A unified theory of projective spaces and finite abelian groups.*

It is the object of this investigation to develop in detail a theory which contains as special cases both projective geometry and the theory of finite abelian groups. The primary abelian operator groups lend themselves in quite a natural manner for this purpose. Those partially ordered sets that are exactly the systems of all the admissible subgroups of suitable primary abelian operator groups are determined, and it is shown that the group and its operators are completely determined by the system of admissible subgroups. Those questions which are fundamental in the two theories are discussed, such as the basis-theorem, the relations between dualities and bilinear forms, and so on. (Received March 12, 1941.)

183. Richard Brauer: *On the connection between the ordinary and the modular characters of groups of finite order.*

Let G be a group of finite order $g=p^a g'$, where p is a fixed prime and $(p, g')=1$. If $\mathfrak{Z}$ is a character of G, then for every element A of an order prime to p the value $\mathfrak{Z}(A)$ is a linear combination of the modular group characters of G. The coefficients are rational integers, the decomposition numbers of G. In this paper it is shown that the value $\mathfrak{Z}(A)$ of $\mathfrak{Z}$ for elements A of an order divisible by p can be expressed by means of the modular characters of certain subgroups N_i of G. The matrix Z of all ordinary group characters of G then appears as a product DX of two square matrics. The matrix X contains in its rows the values of the modular characters of G and of the N_i while the coefficients of D are integers of the field of the p^ath roots of unity. A number of properties of the coefficients of D are given. (Received March 18, 1941.)

184. R. H. Bruck and T. L. Wade: *Bisymmetric tensor algebra.* I.

This paper lays a basis for a study of the linear associative algebra of bisymmetric tensors (H. Weyl, *The Classical Groups*, p. 98) which gives a realization of a semi-simple algebra. The ordered product of two tensors $A^{(i)}_{(j)}$ and $B^{(i)}_{(j)}$ is defined by $A^{(i)}_{(m)}B^{(m)}_{(j)}=C^{(i)}_{(j)}$, where $(i)=i_1 \cdots i_p$. The unit tensor $\delta^{(i)}_{(j)}=\delta^{i1}_{j1} \cdots \delta^{ip}_{jp}$ may be decomposed into a direct sum of immanent tensors (T. L. Wade, abstract 47-1-10). For a tensor $A^{(i)}_{(j)}$, a determinant, adjoint, and inverse are defined with the aid of a tensor $\delta^{(i_1)\cdots(i_N)}_{(j_1)\cdots(j_N)}$ which constitutes a further generalization of the familiar generalized Kronecker delta; here $N=n^p$. Also the concepts of rank and of rank tensor are introduced. Explicit algebraic construction is given of the factors of the determinant in the case of a bisymmetric tensor. The determinant of a Kronecker product (Murnaghan, *The Theory of Group Representations*, pp. 68–69) is a special case of the determinant defined in this paper. (Received March 27, 1941.)

185. R. H. Bruck and T. L. Wade: *Bisymmetric tensor algebra. II.*

If an idempotent numerical tensor $C^{(i)}_{(j)}$ has rank r (as defined in part I), its rank tensor, an absolute numerical tensor, factors into the product of two relative tensors. For a certain invariant subalgebra of the algebra of bisymmetric tensors, $C^{(i)}_{(j)}$ is the unit, and for this subalgebra the rank tensor plays the role of the generalized Kronecker delta for $p=1$ (see part I). $C^{(i)}_{(j)}$ may be thought of as the idempotent numerical tensor corresponding to an operator c, or as an immanent tensor corresponding to an ϵ, where c and ϵ are used in the sense of H. Weyl (*The Classical Groups*, pp. 120–127). In either case a determinant, adjoint, and inverse are defined, and the relation between the two cases is made explicit. In connection with this work the homomorphic correspondence between the group-ring of the symmetric group on p letters and the algebra of absolute numerical tensors is established and used considerably. (Received March 27, 1941).

186. D. M. Dribin: *A theorem on sets of ideals in solvable extensions of algebraic number fields.*

The following theorem is proved: Let k be an algebraic number field and $K|k$ a normal extension with solvable group. Let Π be any set of prime ideals in k which decompose into prime ideals of relative degree one in K and which are all equivalent in k in the ordinary (broad) sense. Then if Π_K is the set of all prime ideal divisors in K of the ideals in Π, all the ideals of Π_K are equivalent (in the ordinary sense) in K. (Received March 14, 1941.)

187. Fred Kiokemeister: *The parastrophic criterion for the factorization of primes.*

The parastrophic form $\pi(\xi)$ of a Frobenius algebra is defined as the determinant of the parastrophic matrix in a given basis of the algebra. In particular the parastrophic form may be computed with respect to the basis of a maximal domain of integrity K of an algebraic number field. Let p be a rational prime number. Then the prime ideal factorization $(p)=\mathfrak{p}_1^{\alpha_1}\mathfrak{p}_2^{\alpha_2}\cdots\mathfrak{p}_s^{\alpha_s}$ of p in K is paralleled by the factorization of $\pi(\xi)$ modulo p into a product of powers of irreducible factors. This result is obtained through a study of the relationship between the ideal structure of a linear algebra and the zeros of $\pi(\xi)$. (Received February 14, 1941.)

188. C. C. MacDuffee: *Products and norms of ideals.*

In an integral domain of a Frobenius algebra over the quotient field of a principal ideal ring, the number h of ideal classes is finite. To every left (or right) ideal $\mathfrak{a}$ correspond h matrices $A_1, A_2, \cdots, A_h$, one for every ideal class of the domain. The first matrix A_1, corresponding to the principal class, is a matrix which determines a minimal basis of $\mathfrak{a}$. If $B_1, B_2, \cdots, B_h$ correspond similarly to the ideal $\mathfrak{b}$ which belongs to the ith ideal class, then the matrix $C_1=A_iB_1$ corresponds to the product ideal $\mathfrak{a}\times\mathfrak{b}$. Thus ideal multiplication can always be accomplished by means of matric multiplication. Call $|A_i|=N_i(\mathfrak{a})$ the ith norm of $\mathfrak{a}$, and $N_1(\mathfrak{a})$ the principal norm. It is known that the principal norm of a product is not always equal to the product of the principal norms. It is here proved, however, that the principal norm of $\mathfrak{a}\times\mathfrak{b}$ is always equal to the product of the ith norm of $\mathfrak{a}$ by the principal norm of $\mathfrak{b}$. (Received March 31, 1941.)

189. Saunders MacLane and O. F. G. Schilling: *A formula for the direct product of cross product algebras.*

Let N be a separable normal extension of a field L with Galois group Γ. Suppose that K is a normal subfield of N belonging to the group Δ. The authors prove the formula $(N, \Gamma, F)\times(K, \Gamma/\Delta, G)=(N, \Gamma, FG^*)$ where F, G are factor sets belonging to N, K, respectively. The factor set G^* of N is obtained by "Δ-symmetric extension" of G to N. (Received March 20, 1941.)

190. Saunders MacLane and O. F. G. Schilling: *Group extensions characterizing complete fields.*

Let F be a field which is relatively complete with respect to a valuation Φ with residue class field $\mathfrak{J}$. The authors define the "dominant" F' of F as the factor group of the multiplicative group of F modulo the group of units u with $\Phi(1-u)>0$. The group F' is a group extension of the multiplicative group of $\mathfrak{J}$ by the value group. Suppose that K is a finite separable extension of F whose degree over the unramified field is prime to the characteristic of F. The dominant K' of K is then a "linked" extension of F'. The authors prove that the extensions $K\supset F$ are in 1-1 correspondence with the linked extensions $K'\supset F'$. The type number defined by Albert (Annals of Mathematics, (2), vol. 41 (1940), pp. 678–679) is essentially an integer describing a cyclic extension $K'\supset F'$. (Received March 20, 1941.)

191. Saunders MacLane and O. F. G. Schilling: *The principal divisor theorem for function fields.*

Let F be a field of algebraic functions of one variable. The authors investigate the structure of extensions $K\supset F$ in which the divisors of a given class group B of F become principal. An unramified extension $K\supset F$ in which exactly the divisors of B become principal is termed a Hilbert class field for B. In general there exist several "minimal" normal Hilbert class fields belonging to a group B. The Galois groups of such separable normal fields are abelian if an only if (i) the order of B is prime to the characteristic of F, and (ii) the characters of B can be realized in F. The methods of proof differ radically from the ones used in the theory of algebraic number fields. (Received March 20, 1941.)

192. I. M. Niven: *Equations in quaternions.*

It is proved that the equation $x^n+a_1x^{n-1}+a_2x^{n-2}+\cdots+a_n=0$, with real quaternion coefficients, has a root in the algebra of real quaternions when n is odd. Also, the number of roots of this equation may be finite or infinite, and if finite, does not exceed $(2n-1)^2$. (Received March 20, 1941.)

193. I. M. Niven: *Sums of nth powers of quadratic integers.*

The first problem is to find those quadratic fields all of whose integers are expressible as sums of nth powers. This is solved completely, criteria being given in terms of n and the basis of each field. The second problem is to determine whether any given quadratic integer is expressible as a sum of nth powers. Necessary and sufficient conditions are again given; in this case however, integers of real quadratic fields are not treated when n is even. (Received March 5, 1941.)

194. R. S. Pate: *Rings with multiple-valued operations.*

A set R of elements forming a hypergroup with respect to addition and having a distributive associative multiplication such that ab is a subset of elements of R is defined to be a ring. If R is an additive group every product contains the same number n of elements, and n is a divisor of the order of the group. A subring A of R such that $AR \subset A$ is an ideal. To a fixed element q, to every element r of R may be associated an arbitrary set of corresponds x_i and y_i such that $r+x_i \supset q$ and $r \subset y_i+q$. A Q ideal contains q, and, if it contains r, contains all corresponds of r. Usually, the necessary and sufficient condition that a set of ideals form a lattice is that they be Q ideals. For a proper correspondence Q, the classical decomposition of ideals may be derived. An analogue of an indeterminate domain may be defined and a result derived similar to the Gauss lemma. It can be shown that the "indeterminate domain" of a ring with a "well-behaved" multiplication has this same multiplication only if it is an ordinary ring. (Received March 17, 1941.)

195. Sam Perlis: *A characterization of the radical of an algebra.*

The following theorem is proved: If F is any field and A is an algebra over F with a unity element, the radical of A consists of all elements h such that $g+h$ is regular for every regular g. If A does not have a unity element, one may be adjoined without altering the radical. (Received March 14, 1941.)

196. Everett Pitcher and M. F. Smiley: *Transitivities of betweenness.*

The importance of the transitivity "*abc*, *adc*, and *bxd*→*axc*" in the theories of lattices and of metric spaces leads the authors to investigate all possible five point transitivities of betweenness on a line whose conclusions state that *just one* relation holds and whose hypotheses consist of three relations. Thirty-three distinct postulates are found. Of these, only ten fail to be equivalent to combinations of the fundamental properties (1), (2), and (3) of Huntington and Kline (Transactions of this Society, vol. 18 (1917), p. 305). A discussion of the two possible weak transitivities on four points is included. The influence of each of these transitivities, as well as the fundamental ones of Huntington and Kline, when applied to a definition of betweenness in lattices (this Bulletin, abstract 47-5-201) is investigated. Eight of the five point transitivities are shown to hold in metric ptolemaic spaces, while only one of them is valid in every metric space. (Received March 31, 1941.)

197. H. J. Riblet: *Certain theorems for symmetric differential functions.*

Symmetric differential functions, in which the n quantities $y_1, \cdots, y_n$ are not all distinct, are considered. It is shown that the symmetric function theorem still holds although in different form. For the case in which $y_1, \cdots, y_n$ are all distinct, an improved bound is given for the power of the discriminant which occurs in the expression of any symmetric differential function in terms of the elementary symmetric functions and their derivatives. (Received March 31, 1941.)

198. W. M. Scott: *On matrix algebras over an algebraically closed field.*

The matrix algebra A is considered as an (A, A) module, that is to say, as an abelian group with A as a right and left operator system. By taking A in reduced form, a set of parts C_{ij} is obtained, of which the C_{ii} are the irreducible constituents of A. These parts C_{ij} are called the simple parts of A, as each forms a simple (A, A) module. Each nonzero simple part is isomorphic to a composition factor group of the module A. A second set of simple (A, A) modules, the elementary modules of A, is defined by use of the Cartan basis system employed by C. Nesbitt in his study of the regular representations of algebras (Annals of Mathematics, (2), vol. 39 (1938), pp. 634–658). A (1-1) correspondence exists between the elementary modules and the composition factor groups of A. The simple parts of A are expressible as linear combinations of these elementary modules, and a similar statement may be made concerning simple parts of representations of A. A classification of the simple parts of A, by use of chains, gives a decomposition of A by elementary transformations into parts, each of which defines a directly indecomposable invariant subalgebra of A. (Received March 25, 1941.)

199. H. A. Simmons: *Classes of maximum numbers associated with symmetric equations in n reciprocals.* IV.

The equation considered here is $\sum_{i=1}^{n}(1/x_i)+\sum_{i=1}^{m}a_i(\prod(x))^{-i}=b/a,\ a\equiv(c+1)b-1$, in which n is an integer >1, b, c, m are arbitrary positive integers, $\prod(x)=x_1x_2\cdots x_n$, and the a_i are non-negative integers. Using the terminology of the author's article with W. E. Block (Duke Mathematical Journal, vol. 2, p. 317), the results of the present paper are as follows: The largest product of the numbers in any E-solution of the equation is the product of the w_i of the Kellogg solution w of the equation; this product is not attained by the numbers in any E-solution except w. With certain restrictions on the a_i $(i=2, 3, \cdots, m)$, the Kellogg solution has the two remarkable properties that were established for the Kellogg solutions employed in the article referred to above. (Received March 12, 1941.)

200. H. A. Simmons: *Use of matrices in solving linear Diophantine equations.*

Given the equation $\sum_{i=1}^{n}a_ix_i=k$, $n>1$, in which the a_i are integers different from zero and k is an integer. It is known that this equation has a solution (in integers) if and only if $(a_1, a_2, \cdots, a_n)$ divides k. The purpose of this paper is to express in matricial form the substitutions that are used in solving the equation by the method of substitutions (as given, say, in E. Cahen, *Théorie des Nombres*, vol. 1, chaps. 9, 10). Each matrix here used is nonsingular and every matrix employed except the last one has determinant ± 1. A special order of carrying out the desired matric multiplications is advantageous. The method used in this paper is usually briefer than even the more refined recent methods that employ continued fractions. (Received March 12, 1941.)

201. M. F. Smiley: *Betweenness in general lattices.* Preliminary report.

V. Glivenko (*Contributions à l'étude des systèmes de choses normées*, American Journal of Mathematics, vol. 59 (1937), pp. 941–956, and *Géométrie des systèmes de choses normées*, American Journal of Mathematics, vol. 58 (1936), pp. 799–828) studied the relations between the properties of the lattice operations and metric betweenness in a metric lattice. He showed that c was metrically between a and b if and only if $(a\cap c)(b\cap c)=c=(a\cup c)(b\cup c)$. In particular, he characterized distributive lattices as those in which betweenness is "transitive," that is, whenever c is between x and y and both x and y are between a and b, then c is between a and b. Starting with this criterion for betweenness as a definition to be applied to general lattices, this result is obtained without the use of a metric. It is also shown that the transitive property of Pasch which holds in any metric space, namely, b between a and c with c between a and d implies that b is between a and d (see K. Menger, *Untersuchungen über allgemeine Metrik*, Mathematische Annalen, vol. 100 (1928), pp. 78–80), characterizes modular lattices. (Received February 7, 1941.)

202. B. M. Stewart: *Left-associated matrices with elements in an algebraic domain.*

The n by n matrices A and B with elements in an algebraic domain of order k are said to be left associates if there exists a unimodular matrix P with elements in the domain such that $PA=B$. If the domain is a principal ideal ring, a necessary and sufficient condition that A and B be left associates is that A and B have the same Hermite normal form. For a domain whose class number is greater than one the presence of non-principal ideals prevents such a direct solution. But if each element of A is replaced by its k by k second matric representation there is produced an enlarged matrix A' of order kn by kn with elements in the rational domain, so that for A' the Hermite form is well-defined and easily found. This paper investigates the possibility that a necessary and sufficient condition that A and B be left associates is that the corresponding enlarged matrices A' and B' be left associates. The necessity is shown, and the sufficiency is proved for nonsingular matrices, for matrices whose column class is the principal class, and for matrices of rank 1. Use is made of the results of Steinitz concerning the equivalence problem $PAQ=B$. (Received March 8, 1941.)

203. H. P. Thielman: *Groups of linear fractional transformations.*

G. A. Miller (American Journal of Mathematics, vol. 22, p. 185) has shown that two operations s_1 and s_2, each of period two, can be selected in such a way as to give a product s_1s_2 of any desired period. In the present paper this theorem is applied to linear fractional transformations of the form $T\equiv(ax+b)/(cx+d)$. If T is of period two it must be of the form $S\equiv(ax+b)/(cx-a)$, $|a|+|c|\neq 0$. Two transformations $S_1\equiv(a_1x+b_1)/(c_1x-a_1)$ and $S_2\equiv(a_2x+b_2)/(c_2x-a_2)$ are selected so that the product will be of any given period n. The dihedral group is then formed by extending the cyclic group generated by S_1S_2 by one of the elements S_1 or S_2. If in S_1 $c_1=0$, and in S_2 $a_2=0$, the groups of subtraction and division are obtained (G. A. Miller, Quarterly Journal of Pure and Applied Mathematics, 1906, pp. 80–87; E. J. Finan, American Mathematical Monthly, 1941, pp. 3–7). Simple representations of all dihedral groups are obtained. The results are consequences of the functional equations satisfied by the coefficients of S_1 and S_2. (Received March 13, 1941.)

204. R. M. Thrall: *On Young's semi-normal representation of the symmetric group.*

The main body of the paper is devoted to a new (and shorter) derivation of Young's semi-normal orthogonal form for the irreducible representations of the symmetric group S_m on m letters. The procedure is such as to admit an immediate extension which yields a specific unitary form for the irreducible representations of the alternating group A_m. This new derivation is based on a definition of the semi-normal idempotents of the group ring R_m of S_m, directly in terms of the regular Young diagrams ("standard tableaux") associated with the partitions of m. (Received March 11, 1941.)

205. G. L. Walker: *The elementary divisors of a direct product of matrices.*

If $A=(a_{ij})$ and $B=(b_{\alpha\beta})$ are square matrices, the direct product (or Kronecker product) of A and B is $A\otimes B=(a_{ij}B)$, that is, having elements $a_{ij}b_{\alpha\beta}$ arranged in rows by lexicographic order on i and α and in columns by lexicographic order on j and β. Known results on the characteristic roots of $A\otimes B$ are extended to the determination of the elementary divisors of $A\otimes B$ in terms of the elementary divisors of A and of B. The problem is first reduced to the case of A and B having the single elementary divisors, say $(\lambda-\rho)^m$ and $(\lambda-\sigma)^n$ respectively. Then $A\otimes B$ has elementary divisors: (i) $(\lambda-\rho\sigma)^{m-n-1+2t}$ where $t=1, 2, \cdots, n$ if $\rho\sigma\neq 0$ and $m\geqq n$; (ii) λ^m repeated n times if $\rho=0$, $\sigma\neq 0$; (iii) λ^k, λ^k where $k=1, 2, \cdots, n-1$ and λ^n repeated $m-n+1$ times if $\rho=\sigma=0$ and $m\geqq n$. (Received March 31, 1941.)

206. Morgan Ward: *Finite operators of formal power series.*

Let R be a ring of formal power series $A=\sum a_n x^n$ with coefficients in a commutative field. A finite operator ω transforms A into another series $\omega A=\sum a_n' x^n$ where a_n' is a function of $a_0, a_1, \cdots, a_{n-1}, a_n$ alone. The ring of all such operators is studied and in particular the subring of linear operators. The results include as special cases a previously developed "calculus of sequences" (Ward, American Journal of Mathematics, vol. 58 (1936), pp. 255–266), a large part of the symbolic calculus of Blissard, Lucas and Bell and Hurwitz' theory of integral power series. (Received March 8, 1941.)

207. Louis Weisner: *Polynomials whose roots lie in a sector.*

From two polynomials $A(z)=\sum a_k z^k$ and $B(z)=\sum b_k z^k$, form the composition-polynomials $f(z)=\sum a_k b_k z^k$ and $g(z)=\sum k! a_k b_k z^k$. The author proves that if the roots of $A(z)$ are real while those of $B(z)$ lie in a sector S with vertex at the origin and aperture less than or equal to π, then the roots of $f(z)$ and $g(z)$ lie in S or the sector vertical to S. When S is the positive or negative real axis, these results reduce to well known theorems of Malo and Schur. The author shows further that when the roots of $A(z)$ are negative, the roots of $f(z)$ and $g(z)$ lie in S. (Received March 13, 1941.)

208. L. R. Wilcox: *Extensions of semi-modular lattices.*

Let L be a semi-modular lattice in which $a, b\in L$ implies the existence of $c\in L$ with $a+c=a+b$, $(a, c)\perp$, and in which there are no points (L. R. Wilcox, Annals of Mathematics, (2), vol. 40 (1939), pp. 490–505). Restricted dual-ideals are sets $S\subset L$ such that $a\in S$, $b\geqq a$, implies $b\in S$, and $a, b\in S$, a not parallel to b, implies $ab\in S$. All restricted dual-ideals form a lattice L' in which L is order-isomorphically embedded. It is proved in this paper that the restricted dual-ideals determined by two-element sets $[b, c]$ form a complemented modular sublattice $\bar{L}$ of L', which contains the image of L. Properties of $\bar{L}$ are studied. (Received March 14, 1941.)

209. M. A. Woodbury: *On ordered groups.*

A group G is said to be ordered if it is simply ordered and the group operation preserves order. If the group G has the property that it contains an element u such that for every $g \in G$ there exists a natural number n for which $u^n > g$, then G is homomorphic (preserves the group operation and $\leqq$) to a subgroup of the additive group of real numbers. If G has the further property that for any $g \in G$ preceded by the identity there is a natural number n for which $g^n > u$, then the homomorphism becomes an isomorphism. These two properties are equivalent to the Archimedean postulate. The Archimedean property is necessary for an isomorphism as is shown by an example of Reidemeister (*Grundlagen der Geometrie*, pp. 40–41) of a non-Archimedean ordered quasi-field. (Received March 14, 1941.)

210. Leonard Carlitz: *Some interpolation formulas connected with polynomials in $GF(p^n)$.*

The interpolation formulas discussed are of two types, namely, formulas arising from interpolation at $x^{p^{ni}}$, and secondly, interpolation at arbitrary $M(x)$ in $GF(p^n, x)$. This paper is closely connected with a previous paper (this Bulletin, abstract 47-1-80). (Received April 1, 1941.)

211. Leonard Carlitz: *The coefficients of the reciprocal of certain series.*

Given the "linear" function $f(t) = \sum_{i=0}^{\infty}(A_i/F_i)t^{p^{ni}}$, where the A_i are polynomials in $GF(p^n, x)$, define β_m by means of $t/f(t) = \sum_{m=0}^{\infty}(\beta_m/g_m)t^m$. In this paper various properties of β_m are discussed. Use is made of an identity of the form $f(xt) - xf(t) = \sum_{i=1}^{\infty}\alpha_i f^{p^{ni}}(t)$ and also of the inverse function of $f(t)$. (Received April 1, 1941.)

212. L. W. Griffiths: *The minimum number of variables in universal functions of polygonal numbers.*

The universal functions of polygonal numbers of order $m+2$, in which the sum of the coefficients is not greater than $m+2$, were determined in an earlier paper (Annals of Mathematics, (2), vol. 31 (1930), pp. 1–12). In each such universal function the number n of variables was at most $m+2$. In this paper a universal function is exhibited for each m which involves the smallest number of variables appearing in any of these universal functions for that m, and the value of this minimum n is determined in terms of m. (Received February 28, 1941.)

213. D. H. Lehmer: *On the values of certain Hurwitzian continued fractions.*

Euler was the discoverer of a number of remarkable regular continued fractions representing certain simple combinations of e and its rational powers as, for example, $(e+1)/(e-1) = [2, 6, 10, 14, \cdots]$ or $(e^2+1)/(e^2-1) = [1, 3, 5, 7, \cdots]$. In this paper we find the value of continued fraction $[b, a+b, 2a+b, \cdots]$, whose partial quotients form a general arithmetic progression, in terms of quotients of Bessel functions of imaginary argument. Only in case $2b/a$ is an odd integer is the continued fraction simply expressible in terms of rational powers of e. All continued fractions like Euler's $(e-1)^{-1} = [0, 1, 1, 2, 1, 1, 4, 1, 1, 6, \cdots]$, whose quotients are periodic except for a subsequence in arithmetic progression, are also evaluated as Bessel quotients, as well as certain continued fractions whose partial quotients are made up of two or more arithmetic progressions. Semiregular continued fractions whose numerators are all equal to -1 likewise may be evaluated in terms of Bessel quotients of real arguments. (Received March 8, 1941.)