Geometry

319. J. J. DeCicco: *Equilong geometry of series of collineal third order differential elements.*

The direct equilong group induces at a fixed line (u, v) of the plane a six-parameter group G_6 between the differential elements of third order. Any three elements (C_1, C_2, C_3) possess the fundamental invariant $(\delta_1 r_1+\delta_2 r_2+\delta_3 r_3)^2/(\delta_1 s_1+\delta_2 s_2+\delta_3 s_3)$, where δ_k is the distance from the point of C_i to that of C_j, r_i is the radius of curvature of C_i and $s_i=dr_i/d\theta_i$ is the rate of variation of the radius of curvature per unit radian measure of the inclination θ_i at the fixed line. In general, n elements possess $3n-6$ independent invariants, of which $n-1$ are distances, and the remaining $2n-5$ are of the new type given above. Any other invariant of these n elements must be a function of these $3n-6$ independent invariants. A series $r=r(w)$, $s=s(w)$ possesses the two fundamental differential invariants $k_1=(d^2r/dw^2)/(d^3r/dw^3)$, and $k_2=(d^2r/dw^2)^2 \div (d^2s/dw^2)$. Any two series with their curvatures the same functions of the distance w are equivalent under our group G_6. (Received May 21, 1941.)

320. Edward Kasner and J. J. DeCicco: *Conformal geometry of series of third order differential elements.*

Kasner and Comenetz have shown that at a fixed point of the plane the direct conformal group induces a six-parameter group G_6 between the differential elements of third order. In this paper it is found that three elements (C_1, C_2, C_3) possess the fundamental invariant $(k_1 \sin a_1+k_2 \sin a_2+k_3 \sin a_3)^2/(l_1 \sin 2a_1+l_2 \sin 2a_2+l_3 \sin 2a_3)$, where a_λ is the angle from C_μ to C_ν, k_λ is the curvature of C_λ, and $l_\lambda=dk_\lambda/ds_\lambda$ is the rate of variation of the curvature per unit length of arc at the fixed point. In general, n elements possess $3n-6$ independent invariants, of which $n-1$ are angles and the remaining $2n-5$ are of the new type given above. Any other invariant of these n elements must be a function of these $3n-6$ independent invariants. A series $k=k(\theta)$, $l=l(\theta)$ possesses the two fundamental differential invariants $\rho_1=(d^2k/d\theta^2+k) \div (d^3k/d\theta^3+dk/d\theta)$ and $\rho_2=(d^2k/d\theta^2+k)^2/(d^2l/d\theta^2+4l)$. Any two series with the curvatures ρ_1 and ρ_2 the same functions of the inclination θ are equivalent under the Kasner-Comenetz group G_6. (Received May 21, 1941.)

321. Edward Kasner and J. J. DeCicco: *General differential geometry of second order differential elements.*

Kasner (American Journal of Mathematics, vol. 28, pp. 203–213) showed that at a fixed point of the plane the entire group of arbitrary point transformations induces an eight-parameter group G_8 between differential elements of second order. He gave a complete discussion of all the invariants of n elements with all the appropriate geometric interpretations. This new paper considers the differential geometry of a series $q=q(p)$ under this group G_8, where $p=dy/dx$ and $q=d^2y/dx^2$. The length of arc of a series is $s=\int[5q^{iv}q^{vi}-6(q^{v})^2]^{1/2}/q^{iv}dp$, and the curvature is $K=[25(q^{iv})^2q^{vii}-105q^{iv}q^{v}q^{vi}+84(q^{v})^3]/[5q^{iv}q^{vi}-6(q^{v})^2]^{3/2}$, where the superscripts denote differentiation with respect to p. Any two series with their curvatures K the same functions of the arc length s are equivalent under the group G_8. (Received May 21, 1941.)

322. D. T. McClay: *Clifford numbers.*

The system of numbers $a+b\Lambda$, with a and b real and $\Lambda^2=1$, first used by Clifford, is an algebra of Weierstrass. If these "Clifford numbers" are represented by points

(a, b) in a rectangular coordinate system, the analytic functions correspond to transformations analogous to conformal transformations, the "angle" between two directions of slopes m_1, m_2 being defined as $\tanh^{-1}(m_2-m_1)/(1-m_1m_2)$. Lorentz transformations are a particular case. By adjoining suitably two intersecting lines at infinity, the plane of Clifford numbers may be mapped on a ring-shaped second order surface in projective 3-space just as complex numbers are mapped stereographically on a sphere. The linear transformations of a Clifford variable give geometrical transformations similar to the linear transformations of a complex variable, the metric dx^2-dy^2 replacing the euclidean. The exponential function $e^{x+y\Lambda}=e^x(\cosh y+\Lambda \sinh y)$ is not periodic, whereas $\sin (x+y\Lambda)=\sin x \cos y+\Lambda \cos x \sin y$ and the other trigonometric functions are doubly periodic. Cauchy's integral theorem and other complex variable theorems hold unchanged, or with slight modifications. (Received April 2, 1941.)

323. Nelson Robinson: *A transformation between osculating curves of a rational normal curve in an odd dimensional space.*

The equations of a rational normal curve Γ_n in a linear space of n dimensions are reduced to a canonical form. Hyperquadrics Q, containing Γ_n, and osculating curves Γ_{n-j} of Γ_n at a point P are defined geometrically and their equations derived. By use of a hyperquadric Q, a transformation involving a generalized null system is set up between osculating curves. This correspondence is proved to exist if and only if the ambient space is of odd dimensions. (Received May 14, 1941.)

Statistics and Probability

324. Alfred Basch: *A contribution to the theory of multiple correlation.*

If r_{xy} and r_{xz} are the correlation coefficients between x and y, and x and z, then r_{yz} lies between the limits $r_{xy}r_{xz}\pm k_{xy}k_{xz}$, where $k_{xy}=(1-r_{xy}^2)^{1/2}$, $k_{xz}=(1-r_{xz})^{1/2}$ are the alienation coefficients. In a geometrical representation where r_{xy} and r_{xz} are the rectangular coordinates, the ellipses inscribed in the square with the sides $r_{xy}=\pm 1$, $r_{xz}=\pm 1$ are the loci of constant lower and constant upper limits of r_{yz}. Rays parallel to the three coordinate axes give the three contour ellipses of the standard ellipsoid. From these three contour ellipses can be obtained the three correlation coefficients. If two of the contour ellipses are given, then the third must satisfy restricting conditions in agreement with the limits given for the third correlation coefficient. The intersection ellipses of the standard ellipsoid with the coordinate planes are characteristic for the coefficients of the correlation between two variables, freed from the influence of the third. The "limit cases" correspond to the degeneration of the standard ellipsoid, $r_{yz,\ x}=r_{xy,\ z}=r_{xz,\ y}$. In these limit cases there exists a functional relation between the three variables. The "middle case," $r_{yz}=r_{xy}r_{xz}$ is equivalent to $r_{yz,\ x}=0$. The standard ellipsoid intersects in this case the yz-plane in an ellipse, whose chief axes are the coordinate axes. (Received April 3, 1941.)

Topology

325. H. A. Arnold: *Homology in set-intersections, with an application to r-regular convergence.*

A and B are closed subsets of compact space R. Using Vietoris cycles the following lemmas are proved: (1) If the complete r-cycle γ^r, carried by A is ~ 0 in $A+B$, then