(a, b) in a rectangular coordinate system, the analytic functions correspond to transformations analogous to conformal transformations, the "angle" between two directions of slopes m_1, m_2 being defined as $\tanh^{-1}(m_2-m_1)/(1-m_1m_2)$. Lorentz transformations are a particular case. By adjoining suitably two intersecting lines at infinity, the plane of Clifford numbers may be mapped on a ring-shaped second order surface in projective 3-space just as complex numbers are mapped stereographically on a sphere. The linear transformations of a Clifford variable give geometrical transformations similar to the linear transformations of a complex variable, the metric dx^2-dy^2 replacing the euclidean. The exponential function $e^{x+y\Lambda}=e^x(\cosh y+\Lambda \sinh y)$ is not periodic, whereas $\sin(x+y\Lambda)=\sin x \cos y+\Lambda \cos x \sin y$ and the other trigonometric functions are doubly periodic. Cauchy's integral theorem and other complex variable theorems hold unchanged, or with slight modifications. (Received April 2, 1941.)

323. Nelson Robinson: *A transformation between osculating curves of a rational normal curve in an odd dimensional space.*

The equations of a rational normal curve Γ_n in a linear space of n dimensions are reduced to a canonical form. Hyperquadrics Q, containing Γ_n, and osculating curves Γ_{n-j} of Γ_n at a point P are defined geometrically and their equations derived. By use of a hyperquadric Q, a transformation involving a generalized null system is set up between osculating curves. This correspondence is proved to exist if and only if the ambient space is of odd dimensions. (Received May 14, 1941.)

Statistics and Probability

324. Alfred Basch: *A contribution to the theory of multiple correlation.*

If r_{xy} and r_{xz} are the correlation coefficients between x and y, and x and z, then r_{yz} lies between the limits $r_{xy}r_{xz}\pm k_{xy}k_{xz}$, where $k_{xy}=(1-r_{xy}^2)^{1/2}$, $k_{xz}=(1-r_{xz})^{1/2}$ are the alienation coefficients. In a geometrical representation where r_{xy} and r_{xz} are the rectangular coordinates, the ellipses inscribed in the square with the sides $r_{xy}=\pm 1$, $r_{xz}=\pm 1$ are the loci of constant lower and constant upper limits of r_{yz}. Rays parallel to the three coordinate axes give the three contour ellipses of the standard ellipsoid. From these three contour ellipses can be obtained the three correlation coefficients. If two of the contour ellipses are given, then the third must satisfy restricting conditions in agreement with the limits given for the third correlation coefficient. The intersection ellipses of the standard ellipsoid with the coordinate planes are characteristic for the coefficients of the correlation between two variables, freed from the influence of the third. The "limit cases" correspond to the degeneration of the standard ellipsoid, $r_{yz,\,x}=r_{xy,\,z}=r_{xz,\,y}$. In these limit cases there exists a functional relation between the three variables. The "middle case," $r_{yz}=r_{xy}r_{xz}$ is equivalent to $r_{yz,\,x}=0$. The standard ellipsoid intersects in this case the yz-plane in an ellipse, whose chief axes are the coordinate axes. (Received April 3, 1941.)

Topology

325. H. A. Arnold: *Homology in set-intersections, with an application to r-regular convergence.*

A and B are closed subsets of compact space R. Using Vietoris cycles the following lemmas are proved: (1) If the complete r-cycle γ^r, carried by A is ~ 0 in $A+B$, then

$A \cdot B$ contains a complete r-cycle μ^r (which may be the null cycle) with $\gamma^r \sim \mu^r$ in A. (2) If the complete r-cycle γ^r in $A \cdot B$ is ~ 0 in A and ~ 0 in B, and the $(r-1)$st Betti number of $A+B$ is 0, then $\gamma^r \sim 0$ in $A \cdot B$. These and other similar lemmas are used to prove that if a sequence of continua has a γ^1-continuum as a limit, then there can be no isolated points of non 0-regular convergence. (For 0-regular convergence, see G. T. Whyburn, Fundamenta Mathematicae, vol. 25 (1935), pp. 408–426.) A point of non 0-regular convergence is defined by the present writer to be a point of the limit set, on every neighborhood of which the condition of 0-regular convergence is violated. This theorem is extended to the cases $r>1$. (Received April 3, 1941.)

326. H. A. Arnold: *On r-regular convergence of sets.*

Using the theory of r-regular convergence, created by G. T. Whyburn (see Fundamenta Mathematicae, vol. 25 (1935), pp. 408–426), the following theorems are proved for compact spaces: (1) If the sequence $\{M_n\}$ of closed γ^r-continua M_n converges s-regularly to the limit set M, for all $s \leqq r$, then M is a γ^r-continuum. (2) If the sequence $\{M_n\}$ of closed sets M_n converges r-regularly to the limit set M, then for every sequence of decompositions $M_{n_i}=A_{n_i}+B_{n_i}$ into closed sets, such that the sequence $\{X_i\}=\{A_{n_i}\cdot B_{n_i}\}$ converges to X and the sequences $\{B_{n_i}\}$, $\{A_{n_i}\}$ converge $(r-1)$-regularly to B and A respectively, then $\{X_i\}$ converges $(r-1)$-regularly to X. (3) In the above notation, if $\{M_n\}$ converges to M, and $\{A_n \cdot B_n\}$ converges to $\{A \cdot B\}$, both r-regularly, then $\{A_n\}$ converges to A, and $\{B_n\}$ converges to B, both r-regularly. (Received April 3, 1941.)

327. T. A. Botts: *On convex sets in linear normed spaces.*

This note contains a simple proof of the theorem that in a linear normed space two convex bodies without common inner points are separated by a plane. (Received May 8, 1941.)

328. F. A. Ficken: *Certain systems of subsets of quasi and partially ordered sets.*

Let S be a set which is quasi ordered, partially ordered, or a lattice, and let $U(S)$ denote the set of subsets of S. Assuming the existence of the necessary meets and joins, a subset T which is a lattice is said to be a strict sublattice if both joins and meets in T of appropriate subsets of T agree with joins and meets in S, to be a JS-sublattice (MS-sublattice) if joins but not meets (meets but not joins) agree, and otherwise to be a loose sublattice. Subsystems of $U(S)$ are investigated which consist of those subsets of S which are: lower-convex, lower-normal, convex, strict-sublattices, MS-sublattices, and so on. Among the many sublattices L of $U(S)$ thus found are many which are MU-sublattices. Many of the L are sublattices of others of the L. It has been shown by Garrett Birkhoff that each quasi ordering of S determines and is determined by a complete strict sublattice of $U(S)$. The present discussion yields the specialization of this result to partial orderings, lattices, and so on. (Received April 7, 1941.)

329. Witold Hurewicz: *On duality theorems.*

Let A be a locally compact space, B a closed subset of A, and $H^n(A)$, $H^n(B)$, $H^n(A-B)$ the n-dimensional cohomology groups of the sets A, B and $A-B$ (with integers as coefficients). Consider "natural homomorphisms" $H^n(A) \to H^n(B) \to$

$H^{n+1}(A-B) \to H^{n+1}(A) \to H^{n+1}(A-B)$. It can be shown that the kernel of each of these homomorphisms is the image of the preceding homomorphism. This statement contains Kolmogoroff's generalization of Alexander's duality theorem and has many applications. Using the preceding theorem one can prove: If A and B are compact spaces of dimensions n and m respectively, the necessary and sufficient condition that the topological product $A \times B$ be of dimension $n+m$ is the existence of an open set $U \subset A$ and an open set $V \subset B$ such that $H^n(U)$ and $H^m(V)$ contain elements α and β satisfying the following conditions: If the integer d is a factor of the order of α, then $\beta \not\equiv 0$ modulo d (that is, there is no element γ of $H^m(V)$ satisfying $\beta = d\gamma$); if the integer e is a factor of the order of β, then $\alpha \not\equiv 0$ modulo e. (Received May 3, 1941.)