(iii) $\int_0^\omega (1-\lambda/\omega)^k dA(\lambda)$, $\int_0^\omega (1-\lambda/\omega)^k dB(\lambda)$ converge as $\omega \to \infty$, then $C_k(x, \omega)$ equiconverges with the (C, k) means of the Fourier series of $f(x)$ in $(a+\epsilon, b-\epsilon)$, $\epsilon>0$. At the same time the allied trigonometrical integral $\overline{C}_k(x, \omega)$ equiconverges with the trigonometrical series allied with the Fourier series of $f(x)$ in the same interval. (B) If lim sup $C_k(x, \omega) < \infty$ for $\omega \to \infty$ in a set of positive measure, then $C_k(x, \omega)$ and $\overline{C}_k(x, \omega)$ are bounded for almost all x of E as functions of ω and if furthermore (i) and (ii) are satisfied, then the result of the preceding theorem holds good. (C) If $|C_m(x, \omega)| \leqq \psi(x) \subset L$ in (a, b), then the result of the first theorem holds for any $k>m$. (D) If $\int_a^b |C_m(x, \omega)|^p dx < M$, then the result of the first theorem holds for any $k>m+1/p$. (Received June 23, 1941.)

409. J. W. T. Youngs: *A generalized Lebesgue integral.*

S. Banach has extended the Lebesgue integral to all bounded functions defined on a finite interval (*Théorie des Opérations Linéaires*, Warsaw, 1932). The extended integral enjoys several of the standard properties of the Lebesgue integral, but properties concerned with termwise integrability of a sequence are lacking. It is shown here that if convergence of a sequence of functions is understood to mean convergence in (a general) measure, then the Lemma of Fatou is true; that is, the extended integral is a lower semi-continuous functional on non-negative functions. This fact is used to extend further the integral to unbounded functions. All the properties mentioned above together with the usual theorems on termwise integrability of a sequence hold. Several applications are made. (Received July 24, 1941.)

Applied Mathematics

410. E. L. Buell: *On the distribution of plane stress in a semi-infinite plate with partially stiffened edge.* Preliminary report.

The concentration of stress at the straight edge of a thin semi-infinite plate near the point of application of a concentrated shear load acting in the plane of the plate is reduced if the load is applied, not directly to the plate, but to an elastic stiffening rod attached along its edge. When this rod does not extend along the entire edge the boundary value problem of bi-potential theory for the Airy stress function has non-uniform boundary conditions. To solve this problem a conformal mapping of the slit full plane into the interior of the unit circle is employed. A solution of the resulting transformed boundary value problem is obtained in the form of a Fourier series, the coefficients of which satisfy an infinite system of linear equations in an infinite number of unknowns. This system has been solved approximately for the case of a stiffening rod extending to infinity in one direction from the loading point, care being taken to improve the convergence by first separating out the discontinuous parts of the solution. The resultant expressions for those stresses which are of interest have been derived and evaluated numerically. (Received July 28, 1941.)

411. A. S. Householder: *A theory of steady-state activity in nerve-fiber networks.* IV: *n circuits with a common synapse.*

Let A_i be the product of the activity parameters for the ith circuit (for terminology see Bulletin of Mathematical Biophysics, vol. 3 (1941), pp. 63–69, 105–112). Then an arbitrary stimulus pattern (*SP*) determines uniquely an activity pattern (*AP*) if and only if every A_i, as well as the sum of any number of distinct A_i, is less than unity. In case this condition fails only for the sum of all the A_i, then the possible

AP's can be classified as "even," if it is even with respect to each circuit (see above reference), "odd," if it is odd with respect to each circuit, otherwise "mixed." Then, as for the simple circuit, given any even AP and any odd AP, there exists an SP consistent with both, but no other pairing of distinct mutually consistent AP's is possible. (Received July 25, 1941.)

412. Wilfred Kaplan: *Topology of the two-body problem.*

The differential equations of the two-body problem, when written as four first order equations, represent a flow in 4-space. If the energy constant (assumed negative) is fixed, the trajectories are restricted to a three-dimensional manifold E. The paper analyzes the topological structure of E and of the family of trajectories in E. It is found in particular that E is homeomorphic with 3-space minus a line and that the family of trajectories is built up of families on tori. (Received July 21, 1941.)

413. A. N. Lowan, Norman Davids, and Arthur Levinson: *Tables for Gauss' mechanical quadrature formula.*

The Mathematical Tables Project described in the May 1941 issue of the Bulletin has prepared a 15 place table of the zeros of the first sixteen Legendre polynomials together with the corresponding weight coefficients in Gauss' mechanical quadrature formula. The computation was carried out by a self-checking iterated process of synthetic division, and further checked by employing various properties of the zeros and the weight coefficients. The present table constitutes a considerable extension of that of Gauss. Several errors were found in the latter. (Received July 3, 1941.)

414. Isaac Opatowski: *On the theory of lethal irradiation of microörganisms.* I.

The theory of irradiation of a homogeneous aggregate of microörganisms is based on the hypothesis that an organism must absorb a certain number $n+1$ of lethal quanta in order to be killed. If $N_i(t)$ is the number of organisms which have absorbed exactly i quanta during a time t ($i=0, 1, \cdots, n+1$), then: $dN_i/dt=k_iN_{i-1}-k_{i+1}N_i$, where $k_0=k_{n+2}=0$ and all other k_i are positive constants dependent on the vulnerability which an organism has after an absorption of exactly i quanta. The initial conditions are: $N_0(0)=$a known constant $=N$, $N_i(0)=0$ for $i\geqq 1$. An application of the Laplace transformation gives the number of killed organisms $N_{n+1}(t)$ as the repeated Faltung (G. Doetsch, *Laplace-Transformation*, pp. 157–158): $Nk_1k_2\cdots k_{n+1}\cdot 1^*\exp(-k_1t)^*\exp(-k_2t)^*\cdots{}^*\exp(-k_{n+1}t)$, which is a symmetric function of $\{k_i\}$. This symmetry has an important consequence: the counting of killed organisms alone (which is the usual experimental method for the determination of the constants k_i) cannot yield any information as to the variation of the vulnerability of the organism due to the absorption of lethal quanta. (Received July 25, 1941.)

415. Eric Reissner: *On the equations of the stability theory of thin shells.*

Given an equilibrium state of stress in a thin shell such that the stress resultants $\boldsymbol{M}$ and $\boldsymbol{N}$ and the shape $\boldsymbol{r}$ of the deformed middle surface are known. The stability of this state against small disturbances depends on whether or not the linearized homogeneous equations of equilibrium for a shell with stress resultants $\boldsymbol{M}+\delta\boldsymbol{M}$ and $\boldsymbol{N}+\delta\boldsymbol{N}$ and a shape $\boldsymbol{r}+\delta\boldsymbol{r}$ admit solutions such that $\boldsymbol{M}+\delta\boldsymbol{M}$, $\boldsymbol{N}+\delta\boldsymbol{N}$ and $\boldsymbol{r}+\delta\boldsymbol{r}$ satisfy the same boundary conditions as those prescribed for $\boldsymbol{M}$, $\boldsymbol{N}$ and $\boldsymbol{r}$. The prob-

lem is thereby reduced to one of the determination of branch points of a system of nonlinear equations by means of a characteristic value problem for a system of linearized equations. The relations between $\delta \boldsymbol{M}$, $\delta \boldsymbol{N}$ and $\delta \boldsymbol{r}$ are, for an elastic shell, those following from Hooke's law and from Navier's hypothesis. The complete system of scalar equations of this problem is obtained by a method analogous to that employed by the author in his simplified derivation of the equations for small displacements of thin shells (American Journal of Mathematics, vol. 63 (1941), pp. 177–184). (Received July 28, 1941.)

416. Alexander Weinstein: *On the buckling of a rectangular clamped plate compressed in one direction.*

This problem can be solved by an extension of a general method of reduction of eigenvalue problems (A. Weinstein, Mémorial des Sciences Mathématiques, vol. 88 (1937)). It can be linked to the corresponding problem for a supported plate by a sequence of intermediate differential problems of the fourth order which give lower bounds for the eigenvalues. It follows from this method that the lowest eigenvalue for a square plate of area π^2 is >10.0 while an upper bound 10.4 has been computed by J. L. Maulbetsch (Journal of Applied Mechanics, vol. 49 (1937)) who used the Rayleigh-Ritz method. The present method which gives definitely lower bounds for all eigenvalues differs essentially from previous formal procedures which could not establish such results. (Received July 18, 1941.)

GEOMETRY

417. E. F. Allen: *On a triangle inscribed in a rectangular hyperbola.*

In the study of inversive geometry the following formulas: $z\bar{z}=a^2$, $a^2z+t_1t_2\bar{z}=a^2(t_1+t_2)$, $a^2z+t_1^2\bar{z}=2a^2t_1$, $\bar{p}z+p\bar{z}=2a^2$ are respectively the self-conjugate equation of a circle, the equation of a line through two points on the circle, the equation of the tangent line, and the equation of the polar line of the point p with respect to the circle, where $z=x+iy$, $\bar{z}=x-iy$, and i is defined by the equation $i^2=-1$. If a point in the xy-plane is designated by $z=x+ry$, $\bar{z}=x-ry$, and r is defined by the equation $r^2=+1$, the base $z\bar{z}=a^2$ is the rectangular hyperbola $x^2-y^2=a^2$. It is proved that the above formulas hold. They still hold if r is defined by $r^2=-k$ or $r^2=+k$, where k is a real number. For any triangle inscribed in a rectangular hyperbola there exists a nine-point hyperbola having many of the characteristics of the nine-point circle of a triangle. An anti-orthocentric group of triangles is defined and it is proved that the four triangles of the group have a common nine-point hyperbola. (Received July 17, 1941.)

418. E. F. Beckenbach: *On the analytic prolongation of a minimal surface.*

In extension of a known result concerning the interior behavior of minimal surfaces, it is shown that if a minimal surface is bounded in part by a plane curve and if the surface approaches the plane orthogonally, then the surface can be extended analytically across the plane and the plane is a plane of symmetry for the extended minimal surface. (Received August 2, 1941.)

419. Nathaniel Coburn: *Semi-analytic unitary subspaces of unitary spaces.*

Suppose a Hermitian space X_m of m-dimensions is imbedded in an n-dimensional