Topology

437. D. G. Bourgin: *Linear topological spaces.*

Most of the results concern convex sets. Some theorems on regularly convex sets and weak closures in vector normed spaces are extended to linear topological spaces (l.t.s.) though in some cases a restriction of local convexity of the l.t.s. is required. As a consequence of these results it is shown that a necessary condition for the metrizability of the conjugate space of the locally convex l.t.s. L is that there exist a denumberable basis of bounded sets such that any bounded set in L is contained in a set of the basis. The sufficiency of this condition is already known and the local convexity requirement may be waived here. (Received July 31, 1941.)

438. S. S. Cairns: *The space of a variable geodesic complex on a sphere.*

A simplicial 2-complex on a sphere, S, will be called *geodesic* if its 2-cells are spherical triangles each of area less than a hemisphere. Consider a geodesic complex τ which covers S. Let $T(\tau)$ be the space each of whose points is a geodesic triangulation of S isomorphic with τ, orientation being preserved, where continuity is defined in terms of the positions of vertices. Theorem. *The space $T(\tau)$ is the topological product of projective 3-space and a $(2n-3)$-cell, where n is the number of vertices of τ.* The connectedness of $T(\tau)$ imples that, by continuous motions of the vertices and of the geodesic cells which they determine, it is possible to carry τ into an arbitrary isomorphic similarly oriented geodesic complex on S, no 2-cell becoming degenerate during the motion. Let $T_0(\tau)$ be the space defined precisely as above, but with the stipulation that the vertices of some particular 2-cell be self-corresponding under each isomorphism and remain fixed during each motion. *Then $T_0(\tau)$ is a $(2n-6)$-cell.* There are ready extensions of these results to geodesic complexes which do not entirely cover S and also to rectilinear complexes on a plane. (Received July 30, 1941.)

439. S. S. Cairns: *Topological mapping of a Brouwer 4-manifold on an analytic Riemannian 4-manifold.*

A *Brouwer n-manifold* is a simplicial complex on which the star of each vertex covers an n-cell and admits a piecewise linear homeomorphic mapping into euclidean n-space. The words "piecewise linear" mean linear on each simplex of the star. Theorem. *Every Brouwer 4-manifold can be homeomorphically mapped onto an analytic Riemannian manifold.* This supplements the results of an earlier paper by the writer (Annals of Mathematics, (2), vol. 41 (1940), p. 796). The deformation problem stated in that paper (p. 808) is involved in the proof of the present theorem. It has now been solved, and the solution is included in an article, not yet published, dealing with variable geodesic complexes on a sphere (abstract 47-9-438). (Received July 30, 1941.)

440. Max Dehn: *On the mapping of closed surfaces.*

A system of curves on a closed surface of genus $p>1$ is, apart from deformations, characterized by a matrix of integers with $3p-3$ rows and two columns. The mappings of the surface (conserving the indicatrix), apart from deformations, constitute a group Γ_p which can be generated by screwings parallel to $p(3p-1)/2$ curves. The elements of Γ_p may be regarded as transformations of the characterizing matrices. If these matrices are considered modulo 2, then Γ_p degenerates into a finite factor group which is for $p=2$ the symmetric group of six elements. In consequence of this, an odd number of screwings parallel to non-dissecting curves never yields a deformation. This result and the analogous results for $p>2$ still to be proved, have consequences for the determination of the group Γ_p/k_p, the factor group of the commutator group of Γ_p which is always a finite cyclic group. For instance it follows that Γ_2/k_2 is the cyclic group of order ten or of order two. (Received July 26, 1941.)

441. F. A. Ficken: *Closure-mappings of partially ordered sets.* Preliminary report.

Closure-mappings of a partially ordered set P onto subsets $f(P)$ are those single-valued mappings $x \rightarrow f(x)$ with the properties, whenever x and y are in P, that: $x \leqq f(x)$, $f(f(x))=f(x)$, and $x \leqq y$ implies $f(x) \leqq f(y)$. When a set S has a least member $y \geqq x$ for each x in P, then and only then is $f(x)=y$ a closure-mapping for which $f(P)=S$. Thus the mapping f and the set $f(P)$ determine each other uniquely, and $f(P)$ either has a least element or has no lower bound. Also associated uniquely with f by means of the equivalence relation $a \sim b$, meaning $f(a)=f(b)$, is a partition of P into sets of a very specific kind. As f varies, the solutions of $f(x)=a$ and of $f(a)=y$ are found to run over convex sets. Subsets S such that $f(S) \subset S$ are discussed. (Received July 30, 1941.)

442. R. H. Fox: *Homotopy type and deformation retraction.*

If $f(X) \subset Y$ and $g(Y) \subset X$ and the map gf is homotopic to the identity, then g is called a left inverse of f and f a right inverse of g. Spaces X and Y belong to the same homotopy type if and only if there is a map $\phi(X) \subset Y$ which has both a right and a left inverse. The mapping cylinder C_ϕ $(\supset X+\phi(X))$ is obtained from $X \times [0, 1]$ by identifying $(x, 1)$ and $(x', 1)$ whenever $\phi(x)=\phi(x')$. It is proved that X is a retract of $Y+C_\phi$ if and only if ϕ has a right inverse, and that $Y+C\phi$ can be deformed into X if and only if ϕ has a left inverse. These theorems constitute an analysis of a theorem proved by J. H. C. Whitehead (Proceedings of the London Mathematical Society, vol. 45 (1939), p. 278). These theorems are generalized and applied to Hopf-Pannwitz deformations and yield a new characterization of closure of a homogeneous n-dimensional polyhedron (Alexandroff and Hopf, *Topologie*, Berlin, 1935, chap. 13, § 4). (Received July 28, 1941.)

443. Orrin Frink: *Topology in lattices.*

Different non-sequential topologies which are definable in terms of the order relation in a lattice are studied and compared. These include the Moore-Smith order convergence of directed sets introduced by Garrett Birkhoff, and the interval topology which results on taking the closed intervals of the lattice as a subbasis for the closed sets of the space. It is shown that a complete lattice is bicompact in its interval topology, but not necessarily in its Moore-Smith topology. Applications are made to lattices which are direct products of chains, to function spaces, and to Boolean algebras. Modifications called the neighborhood topology and the convergence topology are defined for the lattice of all closed sets of a topological space. A topology is introduced for the space whose elements are all continuous transformations of a topological space. (Received June 27, 1941.)

444. O. G. Harrold: *A mapping characterization of Peano spaces.*

Nöbeling (*Reguläre Kurven als Bilder der Kreislinie*, Fundamenta Mathematicae, vol. 20 (1933), pp. 30–46) has established the existence of a continuous mapping of the circle onto a regular continuum X such that for each point $x \in X$ whose order $o(x)$ is finite, $m(x) \leqq o(x)$ holds, where $m(x)$ is the number of times x is covered. The existence of strongly irreducible maps of an interval or circle onto any Peano space containing no free arc has been proved by the author (Duke Mathematical Journal, vol. 6 (1940), pp. 750–752). It is shown in the present paper that if X is a Peano continuum, there is a continuous mapping of the circle onto X such that for any point y in the interior of a free arc, $m(y) \leqq 2$. Together with previous results this gives: There is a continuous mapping of the circle onto X such that for any point y of a certain dense subset Y of X, $m(y) \leqq 2$. Thus a mapping of minimal character is established. The mapping need not be strongly irreducible or even reducible. (Received July 28, 1941.)

445. E. A. Knobelauch: *Extension of homeomorphisms on the torus and projective plane.*

H. M. Gehman (Transactions of this Society, vol. 28, p. 252) showed that a homeomorphism between two Peanian continua lying in planes could be extended to the planes if the homeomorphism preserved sides of arcs. V. Adkisson and S. MacLane (Duke Mathematical Journal, vol. 6, p. 216) showed that a homeomorphism between two Peanian continua lying in spheres could be extended to the spheres if the homeomorphism preserved the relative sense of every pair of triods. In this paper a definition of arc crossings is introduced and it is shown that preservation of arc crossings is equivalent to the preservation of relative sense of triods (on a sphere). Further, let K_i $(i=1, 2)$ be a Peanian continuum on a projective plane P_i (or torus T_i). Let ϕ be a homeomorphism of K_1 on K_2. It is shown that if ϕ preserves arc crossings and bounding simple closed curves that ϕ can be extended to a homeomorphism θ of P_1 on P_2 (or of T_1 on T_2), provided that if K_1 contains no non-bounding simple closed curve and $x \in K_1 - C_1$ (C_1 a bounding simple closed curve of P_1 (or of T_1)) then $\phi(x)$ belongs to a 2-cell on P_2 (or T_2) bounded by $\phi(C_1)$ if and only if x belongs to a 2-cell on P_1 (or T_1) bounded by C_1. (Received June 7, 1941.)

446. Horace Komm: *Partial orders in euclidean spaces.*

This paper investigates certain questions raised by the definition of the dimension of a partial order (Dushnik and Miller, American Journal of Mathematics, vol. 63 (1941), p. 601). Two partial orders P_n and P_n' are defined on E_n as follows: (1) If a and b are points of E_n, then $a<b$ in P_n if and only if every coordinate of a is less than the corresponding coordinate of b. (2) $a<b$ in P_n' if and only if every coordinate of a is less than or equal to the corresponding coordinate of b, and at least one coordinate of a is less than the corresponding coordinate of b. It is shown that dimension P_n = dimension $P_n' = n$, for every finite n. The λ-dimension of a partial order P is defined just as the dimension of P except that each linear extension is required to be similar to a subset of the linear continuum. A necessary and sufficient condition that P have λ-dimension is obtained. It is shown that P_n and P_n' have λ-dimension for every finite n, and that λ-dimension $P_n = \mathfrak{c}$, while λ-dimension $P_n' = \aleph_0$, for every n. (Received July 29, 1941.)

447. J. P. LaSalle: *A characterization of topological spaces.*

A characterization of topological spaces can be given in terms of a non-negative real-valued function defined for each element of a space and each element of a partially ordered set. The partially ordered set contains subsets which are directed systems (strongly partially ordered sets) with properties in relation to the real-valued function. This "pseudo-norm" is a generalization of Hyers' pseudo-norm. (Duke Mathematical Journal, vol. 5 (1939), pp. 628–634.) In some cases the "pseudo-norm" may seem to be trivial in that it can be merely another notation for set inclusion, though the value of this characterization lies in the fact that it takes as the basic concept in topology the extension of a notion which is a familiar one, namely, that of a norm. The author has found this postulation basis convenient for some purposes, and certain topological properties imply interesting conditions on the "pseudo-norm." The converse of the latter statement is also true. The neighborhood system generated by the "pseudo-norm" is not necessarily a complete neighborhood system, but there exists always a complete neighborhood system to which it is equivalent. (Received June 18, 1941.)

448. R. G. Lubben: *Decompositions of point sets and of portions of spaces.*

A T-normal point set is a closed set of T which can be separated by mutually exclusive open sets from any closed set of T not intersecting it. (1) The Hausdorff decompositions of the perfectly compact space H of Fréchet, T, are characterized by the property of being upper semi-continuous and of having T-normal elements. (2) Let T be the space of the atomic ideal points of the completely regular Hausdorff space S. (For definitions see Transactions of this Society, vol. 49 (1941), pp. 410–466.) (a) An amalgamation point of S is regular if and only if it can be decomposed into a T-normal point set. (b) A decomposition of the maximal S-portion into ideal points is an element of $\delta(S)$ if and only if its elements are decomposable into subsets of T whose totality form a Hausdorff decomposition of T; $\omega(S)$ gives the finest of these decompositions. (c) An example answers in the negative the question of the author concerning the identity of the atomic regular and the atomic mapping points, as well as Alexandroff's special case concerning his regular and completely regular ends (Matematicheskii Sbornik, vol. 5 (47) (1939), pp. 403–423). (Received July 30, 1941.)

449. A. W. Tucker: *A boundary-value theorem for harmonic tensors.*

In a 3-dimensional euclidean region, bounded by regular surfaces and R_1+1-ply connected, a harmonic vector v (i.e., a vector field v such that curl $v=$div $v=0$) is uniquely determined when one assigns (1) the normal component of v on the boundary and (2) the periods of the line-integral of v along R_1 independent 1-cycles (i.e. circuits). This is a vector formulation of a proposition of Lord Kelvin on cyclic irrotational motion of an incompressible fluid. It has the following tensor generalization. In an n-dimensional orientable (positive-definite) Riemannian manifold, with regular boundary and p-dimensional Betti number R_p, a harmonic alternating p-tensor A is uniquely determined when one assigns (1) the boundary value of the dual $n-p$-tensor of A and (2) the periods of the p-fold integral of A over R_p independent p-cycles. If the periods are all zero, A is the alternating derivative of a "$p-1$-tensor potential" which furnishes a solution to an analogue of the classical problem of Neumann. The proof of the general theorem rests on an extension of the multiple-integral theory of de Rham and Hodge through the use of relative methods of the sort introduced by Lefschetz to obtain duality and fixed-point theorems for non-closed manifolds. (Received July 12, 1941.)

450. A. D. Wallace: *Regularly ordered systems of sets.*

If X, Y are subsets of a topological space the symbol $X\leqq Y$ means $X=Y$ or

$\overline{X}\subset \text{int } Y$. A system $[X]$ is regularly ordered if for each pair X_1, X_2 either $X_1 \leqq X_2$ or $X_2 \leqq X_1$. Regular systems are self-dual under complementation. These systems are useful in the structural analysis of spaces. The same concept may be applied to certain families of subsets. With the aid of this notion it is possible to extend to compact (=bicompact) connected T_1-spaces the well known proposition on the existence of non-cutpoints. The paper also proves the existence of minimal continua satisfying certain conditions (under the assumption of normality) and gives a generalization of certain theorems of Janiszewski, Mullikin, Moore and others. (Received July 2, 1941.)

451. G. S. Young: *On the outer boundaries of certain connected domains.*

A study is made of the properties of the outer boundaries of connected domains D such that $\overline{D}$ is a continuous curve (not necessarily compact) in a space S satisfying Axioms 0–5 of R. L. Moore's *Foundations of Point Set Theory*, particular attention being paid to the resemblances between them and simple closed curves. Typical results are: (1) If B is an outer boundary of such a domain D, then every point of B is arcwise accessible from the component of $S-B$ that contains D; (2) $S-B$ is the sum of two mutually separated connected domains; (3) No subcontinuum K of B contains more than two limit points of $B-K$; (4) If the set H irreducibly separates S and D is a component of $S-H$, then $D+H$ is a continuous curve if and only if every closed and compact proper subset K of H lies in an arc which is a subset of $D+K$. Necessary and sufficient conditions for local compactness, complete separability, topological flatness, and so on, of B or subsets of B are also given. (Received July 28, 1941.)

452. G. S. Young: *On topologically flat subsets of certain spaces of R. L. Moore.*

In a space S satisfying Axioms 0–5 of R. L. Moore's *Foundations of Point Set Theory*, a point set M is said to be topologically flat provided that if P is a point and R is a region containing P there is a simple domain D containing P such that the boundary of D plus the common part of D and M is a subset of R. If S is topologically flat in this sense, it is topologically flat in the sense of F. B. Jones (Transactions of this Society, vol. 42 (1937), pp. 53–93). The principal results are: (1) if S is completely separable, it may be imbedded in a completely separable space S' also satisfying Axioms 0–5 in which every flat point set is compact; (2) there is a continuous and reversible transformation which throws S' into a sphere and which is reversibly continuous on every flat subset of S'. Use of these two theorems permits immediate generalization of many plane theorems by omission or weakening of conditions of compactness or connectivity. (Received July 28, 1941.)

453. J. W. T. Youngs: *Concerning arc-curves and basic sets.*

W. L. Ayres has introduced the terms "arc-curve" and "basic set" to the theory of continuous curves (Transactions of this Society, vol. 30, pp. 567–578). In this paper an abstraction is made from the concept of arc-curve upon noticing the fact that if $\overline{X}$ is the arc-curve of X, then: (1) $X\cdot Y\neq 0$ implies that $\overline{X+Y}=\overline{X}+\overline{Y}$, (2) $\overline{X}=X$ if X is empty or a single point, (3) $\overline{\overline{X}}=\overline{X}$. The similarity to the Kuratowski closure axioms is obvious and is extensively used. For example, the arc-curve of a set is comparable to the closure of a set, while a basic set is analogous to an everywhere dense set of points. (Received July 24, 1941.)