CORRECTION TO "TOTALLY GEODESIC EINSTEIN SPACES"[1]

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The coordinate system (p. 427) in which $H=x^n$ for some fixed value of y and $f_{n\lambda}=0$ exists if and only if $f^{ij}H_{,i}H_{,j}\neq 0$ for this value of y. Hence Theorem 3.1 is valid only if this inequality holds. The remaining case, namely,

$$f^{ij}H_{,i}H_{,j} = 0 \tag{3.15}$$

for every y can arise only if $c=0$, as may be seen by differentiating (3.15) covariantly with respect to k and using (3.7). We note that, in accordance with (3.8) and (3.9), $c=0$ implies that $a=b=0$. To obtain the analogue of Theorem 3.1 for the case in which (3.15) holds, we proceed in a manner analogous to that in H. W. Brinkmann, loc. cit., pp. 131–135 or A. Fialkow, *Conformal geodesics*, Transactions of this Society, vol. 45 (1939), p. 473. By these methods, we find a coordinate system such that $H=x^n$ for a fixed value of y and

$$f^{ns} = 0, \qquad f^{nn} = 0, \qquad f^{(n-1)n} = 1,$$
$$f_{t(n-1)} = 0, \qquad f_{(n-1)(n-1)} = 0, \qquad f_{(n-1)n} = 1,$$

where $s,\ t=1,\ 2,\ \cdots,\ n-2$. In this coordinate system, the characteristic condition (3.7) becomes $\partial g_{ij}/\partial x^{n-1}=0$. (In the Transactions paper, this last equation appears incorrectly as $\partial g_{st}/\partial x^{n-1}=0$.)

If the f_{ij} are to be the components of the metric tensor of an Einstein space E_n, then, as was shown by Brinkmann, the first fundamental form of E_n may be written as

$$\begin{aligned} &f_{st} = h_{st}(x^s,\ x^n), \qquad f_{sn} = 0, \qquad f_{nn} = 0, \\ &f_{(n-1)n} = 1, \qquad f_{s(n-1)} = 0, \qquad f_{(n-1)(n-1)} = 0, \end{aligned} \tag{3.16}$$

where $h_{st}dx^s dx^t$ with x^n constant is the first fundamental form of an Einstein space E_{n-2} of zero mean curvature, and the components of the tensor h_{st} satisfy certain partial differential equations. According to Brinkmann, the conditions (3.16) are the necessary and sufficient conditions that E_n be conformal to another Einstein space by means of a transformation $d\bar{s}=\sigma ds$ with $\Delta_1\sigma=f^{ij}\sigma_{,i}\sigma_{,j}=0$. We note that the most general solution for H of the form $H=H(x^n,\ y)$ is given by (3.13). Now this solution $H(x^n,\ y)$ must involve x^n by the hypothesis

Received by the editors April 4, 1941.

[1] This Bulletin, vol. 45 (1939), pp. 423–428.

of §3 and must also depend upon y since $g^{\alpha\beta}H_{,\alpha}H_{,\beta}\neq 0$ and, according to (3.1) and (3.15), $g^{\alpha\beta}H_{,\alpha}H_{,\beta}=g^{(n+1)(n+1)}(\partial H/\partial y)^2$. Hence the first fundamental form of the E_{n+1} which contains the ∞^1 isometric E_n's is

$$ds^2 = f_{ij}dx^i dx^j + e[\alpha(y)x^n + \beta(y)]^2 dx^{n+1^2}$$

where the f_{ij} satisfy (3.16) and $\alpha\neq 0$ and $(d\alpha/dy)^2+(d\beta/dy)^2\neq 0$. These remarks show that Theorem 3.1 must be modified in the case where (3.15) holds so that the phrase "$\Delta_1\sigma\neq 0$" is replaced by the phrase "$\Delta_1\sigma=0$." Both cases may be included in the theorem:

THEOREM. *A one-parameter family of isometric E_n's may be imbedded as ∞^1 nonparallel, totally geodesic hypersurfaces of an E_{n+1} if and only if each E_n may be mapped conformally on another Einstein space. If a and b are the mean curvatures of E_{n+1} and E_n respectively, then $nb=(n-1)a$.*

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