
The purpose of this book, according to the author, is “to present the most elementary course possible on this subject” and at the same time “to emphasize those notions which seem to be proper to linear spaces.” Despite the assertion that these aims are not antagonistic, the exposition would be pretty tough going for the average graduate student. Although the reader is not assumed, except in an isolated section, to know about Lebesgue integration, and although the proof of such a comparatively elementary fact as that a continuous image of a compact set is compact is given in detail (p. 48), many parts of the book assume a great deal more sophistication.

The discussion is almost entirely unmotivated: the beginner might like to know why one studies spectral families, or the adjoints of operators. Even to one familiar with the theory it requires proof that von Neumann's definition of $T^*$ is equivalent to the easier one usually given for bounded transformations; $T^*$ is defined as the negative of the transformation whose graph is the orthogonal complement of the graph of $T$.

Concerning the author's choice of the order of the material, it is questionable whether or not it is pedagogically advisable to aim the
discussion at unbounded operators from the very beginning, particularly since the main theorem (the spectral representation of self adjoint operators) is first proved for the bounded case. It would seem better to the reviewer to expound all the easy theory of bounded operators first, and thus prepared to call attention to the delicate considerations necessary to study the unbounded case. Also, in Chapter II we find practically the same proof used twice, once to establish the Riesz theorem on the representation of a bounded linear functional by an inner product (Theorem IV), and once to prove the possibility of projection on any closed linear manifold in Hilbert space (Theorem VI). The extremely elementary derivation, due to Riesz (Acta Szeged, vol. 7 (1934–1935), p. 37), of the former from the latter, could have been used here to good advantage.

The book contains many minor slips and typographical errors which may cause serious confusion. Thus in the statement of the axioms for a linear space (p. 4) the assumption \(1f = f\) is omitted, and on p. 34, Theorem I, which is stated for an arbitrary transformation, is proved by reference to a lemma valid only for the additive case. Regrettable also is the author's reluctance to give to well known theorems their usual names: Schwarz's inequality, Bessel's inequality, Parseval's identity, and the Riesz-Fischer theorem are all indiscriminately referred to as Theorem \(n\) of Chapter \(m\).

In Chapter VII (footnotes, pp. 67–68) there are some pretty examples of spectral families, and the Hellinger integral is treated courageously from the modern point of view and not reduced by means of weak convergence to the classical numerical case. The last two chapters are an excellent idea, carried out unfortunately too rarely: they contain quick sketches of further developments and applications, and references to the literature. On the whole the book is a compact and unified presentation of a well defined part of Hilbert space theory, and as such may appeal to the reader interested in learning only that part of the theory which even a non-specialist often needs.

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