which recourse may be had generally in attempts to reduce inductive definitions to explicit definitions in terms of recursive predicates and quantifiers. (Received March 3, 1942.)

Statistics and Probability

216. J. H. Bushey: *The distribution function of the mean under the type α hypothesis.*

An orthogonal expansion (type α series) with the Pearson type III function as weight function is obtained in a form suitable to utilize the tables of Salvosa in representing population frequencies. The expansion differs in certain respects from that of Romanovsky. The distribution function of the mean is obtained for samples of n drawn at random from a population represented by the type α series. In special cases this distribution function reduces to that obtained for the Charlier type A series by Baker and to that obtained by Church for the type III function. (Received March 6, 1942.)

217. J. H. Bushey: *The distribution function of the sample total under the type β hypothesis.*

The orthogonal polynomials $\phi_n(x)$ are defined by the weight function $p(x)=C_{s,x}p^x(1-p)^{s-x}$, $(x=0, 1, 2, \cdots, s)$ and the orthogonal relation $\sum_{x=0}^{s}p(x)\cdot\phi_m(x)\phi_n(x)=0$, $m\neq n$, or $=1$, $m=n$. The orthogonal expansion (type β series) $f(x)=\sum_{i=0}^{k}A_i p(x)\phi_i{}'x)$ may be used as a statistical hypothesis. The Charlier type A series is a special case of both the type β and the type α series (the type α series is reported in another abstract). Another special case of the type β series is the Charlier type B series with the Poisson weight function $p(x)=(e^{-sp}(sp)^x)/x!$. The distribution function for the total $z=n\bar{x}$ for samples of n drawn at random from a population represented by a type β series is obtained. This result includes, as special cases, the distribution function of z for the Charlier type B series and that obtained by Baker for the Charlier type A series. (Received March 6, 1942.)

218. J. H. Bushey: *The products of certain discrete and continuous orthogonal polynomials.*

The discrete orthogonal polynomials $\phi_n(x)$, $(x=0, 1, 2, \cdots, s)$, are defined by the weight function $p(x)=C_{s,x}\cdot p^x(1-p)^{s-x}$ and the orthogonal relation $\sum_{x=0}^{s}p(x)\cdot\phi_m(x)\phi_n(x)=0$, $m\neq n$, or $=1$, $m=n$. The polynomials $\phi_n(x)$ are closely related to the continuous polynomials of Jacobi, Hermite, and Laguerre and have applications in statistics. The product $\phi_m(x)\cdot\phi_n(x)$ is developed in terms of the polynomials $\phi_i(x)$ for $p=q=1/2$ (symmetric polynomials). This development permits the evaluation of the sum $\sum_{x=0}^{s}p(x)\phi_m(x)\phi_n(x)\phi_r(x)$. The corresponding results of Feldheim for Hermite polynomials follow as special cases. (Received March 6, 1942.)

Topology

219. E. G. Begle (National Research Fellow): *Intersections of absolute retracts.*

Aronszajn and Borsuk have shown (Fundamenta Mathematicae, vol. 18 (1932) pp. 193–197) that if A and B are compact metric spaces such that their intersection, $A\cap B$, is an absolute retract, then their sum, $A\cup B$, is an absolute retract if and only if A and B are. They left open the following question: Is $A\cap B$ an absolute retract if A, B and $A\cup B$ are? In this paper an example is presented which answers this question in the negative. On the other hand, it is shown that if A, B and $A\cup B$ are absolute homology retracts, then so is $A\cup B$. (Received March 19, 1942.)

220. M. M. Day: *Compactness of a space with monotone closure.*

A closure c in a set X associates with each $X'\subset X$ a set $cX'\subset X$; c is monotone if $X'\subset X''$ implies $cX'\subset cX''$; c is additive if $c(X'+X'')=cX'+cX''$ for every X', X''. If c is monotone there is a unique largest additive c' such that $c'X'\subset cX'$ for every X'. A set X is called compact under c (J. W. Tukey, *Convergence and Uniformity in Topology*), if given a family of subsets of X such that finite intersections of sets of the family are non-empty, there is a point common to the closures of all sets of the family. The following theorem holds: X is compact under a monotone c if and only if it is compact under c'. The proof falls into two parts: The conditions given by Tukey for compactness of a space with additive closure still hold for monotone c; in particular, X is compact under c if and only if every ultraphalanx in X converges (in the topology of c) to a point of X. The theorem quoted can then be proved by showing that any ultraphalanx has the same limit points in the topologies of c and c'. (Received April 4, 1942.)

221. R. C. James: *Normability of topological abelian groups.*

In this paper the properties of convexity, boundedness, and normability of topological abelian groups are considered. The existence of a bounded, convex neighborhood of the identity is found to be a necessary condition for normability, and a sufficient condition if taken with connectedness or one of several generating properties. The representation of convex and normable topological abelian groups as subgroups of linear topological spaces is investigated. (Received March 14, 1942.)

222. Samuel Kaplan: *Homologies in metric separable spaces.*

It is shown that the denumerable star-finite coverings form a complete family. By developing a method originating with Čech, homology properties of arbitrary subsets can be obtained by use of "neighborhood coverings." A new type of bounding called Čech bounding is defined for Vietoris cycles as follows: To each Vietoris cycle of the space there corresponds a Čech cycle, determined up to homology; the Vietoris cycle is said to Čech bound if its corresponding Čech cycle bounds. By way of application, the following duality theorem is proved (in the case of closed sets reducing to the Alexander duality): If A is an arbitrary subset of the n-sphere S^n, and there are m k-dimensional Vietoris cycles independent in A relative to Čech homologies, then in S^n-A there are m $(n-k-1)$-dimensional Vietoris cycles independent relative to Čech homologies. Among other results are dualities established between the Čech homology properties of arbitrary subsets of S^n and the Vietoris homology properties of their complements, as well as miscellaneous results as: If a Vietoris cycle z^k fails to Čech bound in a metric separable space, there exists a homeomorphic remetrization in which z^k fails to ϵ-bound for some ϵ. (Received March 19, 1942.)

223. A. N. Milgram: *Dimension of the points with at most n rational coordinates.*

In this paper it is shown that the points in Hilbert space at most n of whose coordinates are rational is an $(n+1)$-dimensional set. This is an extension of the theorem that the points in Hilbert space all of whose coordinates are irrational form a one-dimensional set, which follows directly from the proof by Erdös for the set with all rational coordinates. (Received March 26, 1942.)

224. J. H. Roberts and Paul Civin: *Sections of continuous collections.*

The following theorem is proved: If G is a continuous collection of closed and compact sets filling a separable metric space X, and if G, regarded as a decomposition space, has dimension n, then there exists in X a closed set C such that for any g in G the set $g \cdot C$ is nonvacuous and consists of at most $n+1$ points. (Received March 20, 1942.)

225. R. L. Swain: *Proper and reductive transformations.*

A (single-valued) transformation T of a point set M is called: *proper* if for each $K \subset M$, $T(K)$ is closed and compact whenever K is; *strong proper* if, in addition, T preserves connectedness; *complete proper* (CP) if an additional condition concerning contiguous points is satisfied. A strong proper transformation of a compact continuous curve is continuous. In spaces which may contain contiguous points (and which satisfy R. L. Moore's axiom set Σ_c), for M to be a compact continuous curve it is necessary and sufficient that M should be the image of an ordinary arc under some CP transformation; also a CP transformation of an arc into itself either leaves a point fixed or interchanges two contiguous points. A transformation T of M is called *reductive* provided that if $X \in T(M)$, and $V \subset M$ is such that X is a limit point of $T(V)$, then $T^{-1}(X)$ contains a limit point of V. A reductive transformation is continuous on the nucleus of a compact closed point set. In a compact space the class of reductive transformations equals the class of proper transformations. Using reductive transformations, one may associate with each point set M an abstract measure called its topological power. (Received March 17, 1942.)

226. G. T. Whyburn: *Coherent and saturated collections.*

A theory of coherence and saturation relative to a given collection G of subsets of a connected separable metric space M is developed. Conditions are found under which the collection G will generate a unique upper semi-continuous decomposition of M and under which the associated transformation will be monotone or non-alternating. These are applied to find necessary and sufficient conditions for the mappability of M onto an arc or a circle by a monotone transformation. As a further application, an existence theorem for saturated collections of continua in a locally connected continuum L is proved which yields in a new way the known (Moore-Roberts) monotone mappability of L into a regular curve. (Received March 4, 1942.)

227. G. T. Whyburn: *On the interiority of mappings.*

A previous theorem of the author (as yet unpublished) is extended to yield the following theorem: Let $f(M)=K$ be continuous where M is locally connected separable and metric and let D be the closure of the set of all points of K of Menger-Urysohn order greater than 2. There exists a countable subset C of $K-D$ such that f is interior at every point of $M-f^{-1}(D+C)$. (Received March 4, 1942.)

228. G. T. Whyburn: *Unitary subcontinua.*

A subcontinuum N of a compact metric continuum M will be called unitary proved that if K is any subcontinuum of M intersecting N we have either $K \supset N$ or

$K \subset N$. For example, M itself (and also any single point of M) is unitary. A study of such subcontinua is made and it is shown, for example that: (1) the property of being a unitary subcontinuum containing a given subset X of M is inducible; (2) for any two given unitary subcontinua A and B there exists a unique least unitary subcontinuum $A+B$ containing both A and B; (3) no nondegenerate proper unitary subcontinuum can have a cut point; (4) the property of being a unitary subcontinuum of M is a monotone and also an interior invariant; (5) if M is irreducible between two points, any monotone interior mapping of M into the interval maps unitary subcontinua into single points of the interval. (Received March 4, 1942.)

229. J. W. T. Youngs: *On parametric representations of surfaces. II.*

This paper is a continuation of an earlier study (abstract 46-5-352). A continuous transformation $R(A)=B$ from the 2-sphere A to a set B in 3-space is called a representation. By the Eilenberg-Whyburn factor theorem, $R(A)=L(M(A))$, where $M(A)=\Sigma$ is monotone and $L(\Sigma)=B$ is light. Two representations $R_1(A)$ and $R_2(A)$ are K-equivalent (Kerekjarto, Acta Szeged, vol. 3 (1927), pp. 49–67) if there is a homeomorphism $H(\Sigma_1)=\Sigma_2$ such that $L_1(\Sigma_1)=L_2(H(\Sigma_1))$. They are F-equivalent if the Fréchet distance between them is zero. The prime end theory is used to define what is meant by the statement that R_1 and R_2 are consistent. The concept has to do with orientation. Suppose that R_1 and R_2 are themselves monotone. Then, if they are F-equivalent, they are K-equivalent and consistent. Moreover the converse is true without the restriction that R_1 and R_2 be monotone. The converse is important from the point of view of applications to the theory of area. (Received March 18, 1942.)