

$+y_1)/(y_0-y_1)$ . If  $\partial(fQdy)/\partial x$  exists, then each integrating factor of  $Pdx+Qdy=0$  has a similar differentiability property. (Received May 20, 1942.)

244. J. D. Rommel: *On conservative transformations of functions of two variables.*

Necessary and sufficient conditions are given in order that  $y(s, t) = \int_0^\infty \int_0^\infty k(s, t, \sigma, \tau) x(\sigma, \tau) d\sigma d\tau$  shall transform  $x(\sigma, \tau)$  such that  $\lim_{\sigma, \tau \rightarrow \infty} x(\sigma, \tau)$  exists into  $y(s, t)$  such that  $\lim_{s, t} y(s, t)$  exists. From this result a characterization of regularity is obtained. By considering step-kernels and step-functions, regular transformations of double sequences are characterized. (Received April 18, 1942.)

245. Raphael Salem and D. C. Spencer: *The influence of gaps on density of integers*

An infinite sequence of integers is said to have the complete gap property with respect to  $\omega(x)$  ( $\omega(x)$  being a positive non-decreasing function for  $x \geq 0$ ) if in every closed interval  $(a, a+l)$  ( $a \geq 0, l \geq l_0$ ) there exists an open interval of length not less than  $\omega(l)$  which contains no point of the sequence. Such a sequence will be denoted by  $S[\omega(x)]$ . Let  $\nu(m)$  be the number of terms of the sequence not greater than  $m$ . The following theorems are proved: (1) If the integral  $\int_0^\infty (\omega(x)/x^2) dx$  diverges,  $\lim \nu(m)/m = 0$ . (2) If  $\omega(x)$  is such that  $\omega(x)/x^2$  decreases and the integral converges, there exists an  $S[\omega(x)]$  with  $\liminf \nu(m)/m > 0$ . (3) If  $\omega(x) = \theta x$  ( $0 > \theta > 1/3$ ), then  $\nu(m)/m = O(m^{-\alpha})$  where  $\alpha = [\log(1-2\theta)/(1-3\theta)] / [\log 2(1-2\theta)/(1-3\theta)]$ . (4) If  $0 < \theta < 1/3$ , there exists a sequence  $S[\theta x]$  with  $m^{-\alpha} = O(\nu(m)/m)$ . (5) If  $\omega(x) = \theta x$  ( $1/3 \leq \theta < 1/2$ ), then  $\nu(m) = O(\log m)$ . (6) If  $1/3 \leq \theta < 1/2$ , there exists a sequence  $S[\theta x]$  with  $\log m = O(\nu(m))$ . (7) If  $\omega(x) \geq x/2$ , there exists no infinite sequence  $S[\omega(x)]$ . (Received April 30, 1942.)

#### APPLIED MATHEMATICS

246. H. K. Brown: *The resolution of boundary value problems by means of the finite Fourier transformation.*

The finite sine transformation and the finite cosine transformation are defined as the linear functional operations  $\mathfrak{S}\{G\} = \int_0^\pi G(x) \sin nxdx = g_s(n)$  and  $\mathfrak{C}\{G\} = \int_0^\pi G(x) \cdot \cos nxdx = g_c(n)$ , respectively. The inversion of the product of the transforms of  $G$  and  $H$  can be made by means of four Faltung theorems. The finite sine transformation was applied to a problem in general heat flow in one dimension in which the nonhomogeneous linear partial differential equation has coefficients which may be functions of the time. It was proved in detail that this problem can be resolved into a standard heat flow problem which has a differential equation of simpler type and constant coefficients. The Faltung theorems permitted an inversion in closed form of the solution of the transformed boundary value problem. A general heat flow problem of similar type in three dimensions was resolved into the same standard problem in one dimension. By the introduction of a fundamental set of solutions of the transformed problem it was possible to make a resolution of the boundary value problems of a general vibrating string, membrane and beam. (Received April 11, 1942.)

#### GEOMETRY

247. C. J. Everett: *Affine geometry of vector spaces over rings.*

Methods of E. Artin's *Coordinates in affine geometry* (Publication of the University of Notre Dame, Reports of a Mathematical Colloquium, (2), Issue 2) are used to show

that a "plane geometry" subject to a weaker set of axioms on "points" and "lines" permits a point coordinatization  $P = (x, y)$  with  $x, y$  in a ring  $R$  with unit, without zero-divisors, and with every two elements  $(a, b) \neq (0, 0)$  having a g.c.r.d.  $= d$  ( $d$  right divisor of  $a, b$  divisible by every right divisor of  $a, b$ ); set of all points on a line:  $(x + ak, y + bk)$ ,  $x, y$  fixed;  $a, b$  fixed, not  $0, 0$ , with g.c.r.d.  $1$ ;  $k$  arbitrary in  $R$ . The intuitive ideas in the "geometry" of the lattice points (integer coordinates) of the affine real plane are thus abstracted in equivalent geometric and algebraic forms. (Received May 20, 1942.)

### STATISTICS AND PROBABILITY

248. Henry Scheffé: *On the theory of testing composite hypotheses with one constraint.*

Suppose that one has a sample from a population whose distribution function depends on  $l$  parameters  $\theta_1, \theta_2, \dots, \theta_l$ , and that on the evidence of the sample it is wished to test the hypothesis  $H_0: \theta_i = \theta_i^0$ . Not only are many statistical problems of practical interest precisely of this type, but the theory of testing such hypotheses with one constraint is intimately related to Neyman's theory of confidence intervals (Proceedings of the Royal Society, 1937). Type B regions for testing  $H_0$  were defined by Neyman (Bulletin Société Mathématique de France, vol. 64, (1935)) who gave a method of finding such regions in the case  $l = 2$ . His theorem is generalized to the case  $l \geq 2$  in the present paper. Type B<sub>1</sub> regions are defined as the natural generalization of the type A<sub>1</sub> regions of Neyman and Pearson (Statistical Research Memoirs, 1936) to permit the presence of the nuisance parameters  $\theta_2, \dots, \theta_l$  in the distribution. Sufficient conditions are given that a type B region be also of type B<sub>1</sub>. The paper will appear in the Annals of Mathematical Statistics. (Received May 29, 1942.)

### TOPOLOGY

249. W. H. Gottschalk: *On  $k$ -to-1 mappings.*

Let  $A$  and  $B$  be compact metric spaces and let the mapping  $T(A) = B$  be continuous and at most  $k$ -to-1, where  $k$  is an integer. Let  $o(x)$  denote the Urysohn-Menger order of the point  $x$ . It is shown that if  $y \in B$  has exactly  $n$  inverse points  $x_1, \dots, x_n$ , then  $\sum_{i=1}^n o(x_i) \leq k \cdot o(y)$ . Suppose  $A$  and  $B$  are continua and  $T$  is (exactly)  $k$ -to-1. It follows from this inequality that if  $y$  is an end point of  $B$ , then each inverse point of  $y$  is an end point of  $A$ . Let  $P$  denote the property of being a subcontinuum of  $A$  on which  $T$  is  $k$ -to-1. It is proved that if  $A$  has (the inducible property)  $P$  irreducibly, then  $B$  contains no end point; and, in case  $k = 2$ ,  $B$  has the stronger property of containing no cut point. It follows that no dendrite is a continuous  $k$ -to-1,  $k > 1$ , image of a continuum; or, equivalently, every continuous  $k$ -to-1,  $k > 1$ , image of a continuum contains a non-cut non-end point. (Received April 11, 1942.)

250. P. R. Halmos and Hans Samelson: *On monothetic groups.*

A topological group is *monothetic* (van Dantzig) if it contains an element (called a *generator*) whose powers are dense in the group. (I) A compact abelian group is monothetic if and only if its character group is algebraically isomorphic to a subgroup of the circle. (II) A compact separable abelian group is monothetic if and only if the quotient group with respect to the component of the identity is such. (Here separability means the existence of a countable dense set.) In particular a compact separable and connected abelian group is always monothetic. (III) Almost every element (Haar