ABSTRACTS OF PAPERS

SUBMITTED FOR PRESENTATION TO THE SOCIETY

The following papers have been submitted to the Secretary and the Associate Secretaries of the Society for presentation at meetings of the Society. They are numbered serially throughout this volume. Cross references to them in the reports of the meetings will give the number of this volume, the number of this issue, and the serial number of the abstract.

ALGEBRA AND THEORY OF NUMBERS

103. A. A. Albert: *Algebras derived by non-associative matrix multiplication.*

Let $\mathfrak{C}$ be any J-involutorial algebra and define its isotopes $\mathfrak{C}_\rho$, $\mathfrak{C}_\kappa$, $\mathfrak{C}_{\kappa\rho}$ by $x, y = x(yJ)$, $(x, y) = (xJ)y$, $[x, y] = (xJ)(yJ)$, respectively. When $\mathfrak{C}$ has a unity quantity the algebras $\mathfrak{C}_\rho$, $\mathfrak{C}_\kappa$, $\mathfrak{C}_{\kappa\rho}$ are all semi-simple if and only if $\mathfrak{C}$ is semi-simple, they are simple if and only if $\mathfrak{C}$ is either simple or a direct sum $\mathfrak{S} \oplus \mathfrak{S}J$ where $\mathfrak{S}$ is simple. If $\mathfrak{N}$ is the radical of $\mathfrak{C}$ then $\mathfrak{N} = \mathfrak{N}J$ forms the radical of the isotopes. The structure of row algebras, that is, subalgebras of $\mathfrak{C}_\rho$ is determined. In particular it is shown that there exist real linear spaces of matrices forming algebras under row by row but not under row by column multiplication. They are never semi-simple. (Received January 26, 1943.)

104. Joseph Bowden: *The quaternary permutation function and a generalization of Newton's binomial theorem and Vandermonde's permutation theorem.*

If a and d are any finite numbers, m any integer, r a primary number and ρ an integer whose elements are the primary numbers r_1 and r_2, define $(a, d)P(m, r) = \prod_{k=m+1}^{m+r}(a-(k-1)d)$ and $(a, d)P(m, \rho) = (a, d)P(m-r_2, r_1) : (a, d)P(m-r_2, r_2)$. From these definitions it follows that $(a, d)P(m, 0) = 1$ and $(a, d)P(m, \rho) = : [(a, d)P(m+\rho, -\rho)]$. The operation P is quaternary because it acts on four operands. From the quaternary permutation function $(a, d)P(m, \rho)$ by putting $d=0$ it is found that $(a, d)P(m, \rho) = a^\rho$. By putting $d=1$ or $a=\rho d$ or $m=0$ three ternary functions are obtained. By making two of these three substitutions three binary functions are obtained. In particular if $d=1$ and $m=0$, $(a, d)P(m, \rho) = aP\rho$, the binary permutation function. By making all three of these substitutions $(\rho, 1)P \cdot (0, \rho) = \rho!$ is obtained, the unary factorial function. As examples, it is found that $0! = 1$ and $\rho! = \infty$ if ρ is negative. By mathematical induction the following theorem is proved, of which Newton's binomial theorem and Vandermonde's permutation theorem are special cases: If r is a primary number or zero, a, b, d any finite numbers, except that, if r and d are both zero, neither a nor b nor $a+b$ is zero, and m and n are any integers, then $(a+b, d)P(m+n, r) = \sum_{k=1}^{r+1} rC(k-1) \cdot (a, d)P(m, r-(k-1)) \cdot (b, d)P(n, k-1)$. By putting $d=0$, $(a+b)^r = \sum_{k=1}^{r+1} rC(k-1) \cdot a^{r-(k-1)} \cdot b^{k-1}$, which is Newton's theorem. By putting $d=1$ and $m=0$ $(a+b)Pr = \sum_{k=1}^{r+1} rC(k-1) \cdot aP(r-(k-1)) \cdot bP(k-1)$, which is Vandermonde's theorem. (Received February 1, 1943.)

105. L. L. Dines: *On linear combinations of quadratic forms.*

The author considers conditions under which m given quadratic forms in n variables admit a linear combination which is (1) definite, or (2) semi-definite. The paper will appear in full in an early issue of Bull. Amer. Math. Soc. (Received December 10, 1942.)

106. H. Schwerdtfeger: *Identities between skew-symmetric matrices.*

Let P, Q be two $2m$-rowed skew-symmetric matrices, P regular. Put $P^{-1}Q=A$. The determinant $|\lambda P-Q|$ equals $\kappa(\lambda)^2$ with $\kappa(\lambda)=k_0\lambda^m-k_1\lambda^{m-1}+\cdots+(-1)^m k_m$ where k_0, k_m are the pfaffian parameters of P, Q, respectively, and $k_1, \cdots, k_{m-1}$ the rational simultaneous invariants of P and Q. By Cayley's identity one has $\kappa(A)^2=(0)$. By means of known theorems (cf. for example, MacDuffee's *Theory of matrices*, Theorems 32.2, 32.3, and 29.3, or A. A. Bennett, Bull. Amer. Math. Soc. vol. 25 (1919) pp. 455–458) it follows that $\kappa(\lambda)$ has as a factor the highest invariant factor $h(\lambda)$ of $\lambda P-Q$, and thus the minimum polynomial of A. Hence follows $h(A)=(0)$ and $\kappa(A)=(0)$. This identity involving the skew-symmetric matrices P, Q is of geometric interest; if $m=2$ one has, for instance: $k_0QP^{-1}Q=k_1Q-k_2P$ whence the elementary theory of a pair of null systems (linear complexes) in projective 3-space can be derived. (Received January 8, 1943.)

Analysis

107. G. E. Albert: *An extension of Korous' inequality for orthonormal polynomials.*

Let $\{q_n(x)\}$ denote the set of polynomials orthonormal on (a, b) with weight functions $p(x)r(x)$ where $0\leqq p(x)$ and $0\leqq r(x)\leqq M$. If a non-negative polynomial $\pi_m(x)$ of degree m can be found such that the quotient $\pi_m(x)/r(x)$ satisfies a Lipschitz condition on (a, b) and if $\{p_n(x)\}$ denotes the set of polynomials orthonormal on (a, b) with weight function $p(x)[\pi_m(x)]^2$ then if the polynomials $\{p_n(x)\}$ are bounded uniformly with respect to n and x on any subset of (a, b) the same is true of the set $\{q_n(x)\}$. This result follows from an inequality that is established by the same procedure as that used on an equiconvergence theorem by L. H. Miller and the author (abstract 49-3-108). If $r(x)$ is bounded from zero and satisfies a Lipschitz condition on (a, b), the inequality mentioned reduces essentially to an inequality due to Korous (G. Szegö, *Orthogonal polynomials*, Amer. Math. Soc. Colloquium Publications vol. 23, 1939, p. 157). (Received January 19, 1943.)

108. G. E. Albert and L. H. Miller: *Equiconvergence of series of orthonormal polynomials.* Preliminary report.

Walsh and Wiener (Journal of Mathematics and Physics vol. 1 (1922)) found necessary and sufficient conditions for the equiconvergence of the expansions of an arbitrary function in terms of different systems of functions orthonormal on a finite interval. In the present paper these conditions are applied to the study of polynomials orthonormal relative to weight functions satisfying a variety of hypotheses. A remarkably simple proof is obtained for an equiconvergence theorem that includes one published by Szegö (*Orthogonal polynomials*, Amer. Math. Soc. Colloquium Publication, vol. 23, 1938, Theorem 13, 1.2) and the results given by Peebles (Proc. Nat. Acad. Sci. U.S.A. vol. 25 (1939) pp. 97–104). The application of the Walsh-Wiener conditions is based upon the observation that if $K_n^{(1)}(x, t)$ and $K_n^{(2)}(x, t)$ are the