

condition) is linearized. It is shown that, in this case, the above problem reduces to finding a solution  $F(x, y)$  for the hyperbolic equation under given Cauchy data. Next, it is shown that for certain types of boundary conditions, the stresses corresponding to the linear problem are equal to the stresses corresponding to the original nonlinear problem over a series of equally spaced lines parallel to the boundary ( $x$ -axis). The situation is analogous to two surfaces which do not coincide but do intersect in curves whose projections on the  $xy$ -plane are parallel lines. The method may be modified to give approximate solutions of the nonlinear problem throughout the plate. Some examples are worked. An indication is given of how the method may be applied to the finite rectangular plate. (Received January 2, 1943.)

118. H. J. Greenberg: *Application of a summability method in solving boundary value problems.*

In order to apply the method of particular solutions for solving boundary value problems for the elliptic linear partial differential equation  $L(U)=0$ , S. Bergman (Duke Math. J. vol. 6 (1940) p. 541) has introduced the complete set  $P_{2n-\alpha}(z)$  of such solutions, by means of which every function  $U$ ,  $L(U)=0$ , regular in the circle  $x^2+y^2 < R^2$ ,  $R > 0$ , can be developed in the series  $S: U(z) = \sum a_k P_k(re^{i\theta})$ . By studying the "associated function"  $\sum a_k \Gamma(1/2) \cdot \Gamma(k+1/2) z^k / \Gamma(k+1)$  necessary and sufficient conditions are given for the series  $S$  to be convergent on  $|z|=R$  and, what is more important for applications, for  $S$  to be  $(C, 1)$  summable on this circle of convergence. Conditions are given in each case under which the values thus obtained are the boundary values of the function  $U(z)$ . Using these results a method is given for the actual solution of boundary value and characteristic value problems for the equation  $L(U)=0$ . (Received January 29, 1943.)

#### GEOMETRY

119. John DeCicco: *Extensions of certain dynamical theorems of Halphen and Kasner.*

The theorem of Halphen which characterizes central fields of force and the general theorem of Kasner concerning one-third the curvatures is extended to generalized fields of force in space, which depend upon the position of the point and direction. The number of generalized fields of force whose dynamical trajectories are all plane curves is  $\infty^{f(2)+f(4)+2f(6)}$ . The  $\infty^5$  generalized trajectories consist of  $\infty^2$  systems of  $\infty^3$  generalized plane trajectories, each system lying in a plane tangent to a given surface  $\Sigma$ . In an arbitrary positional field of force, Kasner showed that the rest trajectory and line of force through a given point  $O$  have the same osculating plane and that the ratio  $\rho$  of the curvature of the rest trajectory to that of the line of force is  $1/3$ . For generalized fields of force this theorem is no longer valid. All generalized fields of force such that the rest trajectory and the line of force through any point  $O$  of the space have the same osculating plane are determined; and, also in this class, the subclass of all generalized fields of force for which  $\rho=1/3$ . (Received January 2, 1943.)

120. Edward Kasner and John DeCicco: *Generalized dynamical trajectories in space.*

The differential geometry of positional fields of force has been developed in *Differential-geometric aspects of dynamics*, Amer. Math. Soc. Colloquium Publications vol. 3, 1913. In abstract 48-11-329, the authors began the study of generalized fields

of force in the plane which depend not only upon the position of the point but also upon the direction through the point. In this paper, the geometry of the dynamical trajectories of such generalized fields of force in space is studied. It results that the  $\infty^5$  dynamical trajectories are completely characterized by two geometric properties as follows. For each of the  $\infty^1$  curves of the  $\infty^5$  trajectories which pass through a given lineal element  $E$ , construct the osculating plane and sphere at  $E$ . The two properties are: (I) The  $\infty^1$  trajectories all have the same osculating plane, and (II) The locus of the centers of the osculating spheres is a straight line. Finally velocity systems are defined and shown to be curvature trajectories. (Received December 28, 1942.)

### STATISTICS AND PROBABILITY

121. Henry Scheffé: *On solutions of the Behrens-Fisher problem, based on the  $t$ -distribution.*

The problem is to find confidence intervals for the difference of the means of two normal populations when the ratio of their variances is unknown. Certain disadvantages inhere in solutions hitherto proposed: Some data are completely discarded when the sample sizes are unequal (Neyman), or the confidence coefficient is not known exactly (Fisher), or existing tables are inadequate if the confidence coefficient is to be 95 or 99 per cent (Wilks). The present solution is as follows: Let the samples be  $(x_1, x_2, \dots, x_m)$  and  $(y_1, y_2, \dots, y_n)$ , where the  $x$ 's and  $y$ 's are mutually independently normally distributed, the former with mean  $a_x$  and variance  $\sigma_x^2$ , the latter with mean  $a_y$  and variance  $\sigma_y^2$ . Assume  $m \leq n$ . Let  $\bar{x}$ ,  $\bar{y}$  be the sample means, and  $\delta = a_x - a_y$ . Then  $(\bar{x} - \bar{y} - \delta)/Q^{1/2}$  has the Fisher  $t$ -distribution with  $m-1$  degrees of freedom if  $m(m-1)Q = \sum_{i=1}^m (u_i - \bar{u})^2$ ,  $u_i = x_i - (m/n)^{1/2}y_i$ ,  $\bar{u} = \sum_{i=1}^m u_i/m$ . This leads immediately to confidence intervals for  $\delta$ , and it is shown that of a certain class these have the minimum expected length. (Received January 7, 1943.)

### TOPOLOGY

122. M. G. Ettliger: *On irreducible continuous curves.*

It is proved that, in a connected space satisfying Axioms 0-2 of R. L. Moore's *Foundations of point set theory* (Amer. Math. Soc. Colloquium Publications, vol. 13, 1932), every compact and closed point set is a subset of a compact continuous curve, and every compact and closed point set having no continuum of condensation is a subset of a compact hereditary continuous curve. It is shown that if, in a space satisfying Axioms 0-1,  $M$  is a locally compact continuous curve which is an irreducible continuum about a closed subset  $T$  of  $M$ , then every continuum of condensation of  $M$  is a continuum of condensation of  $T$ . It is then demonstrated that in a connected space satisfying Axioms 0-2, every compact and closed point set having no continuum of condensation is a subset of a compact continuous curve having no continuum of condensation. (Received January 21, 1943.)

123. O. G. Harrold: *A decomposition theorem for certain compacta.*

Let  $\psi_n$  denote the property of being locally connected in dimension  $n$  in the sense of homotopy ( $n-LC$ ) at a point  $p$  of a compactum  $X$ . Let  $\Delta_n$  denote the  $\psi_n$ -singular points of  $X$ . Let  $\mathfrak{C}_n^{n+1}$  denote the class of compacta which are  $LC^n$  ( $i-LC$ ,  $i=0, 1, \dots, n$ ) and such that small singular  $(n+1)$ -spheres bound. If  $X \in \mathfrak{C}_n^{n+1}$ , either  $\overline{\Delta_{n+1}}$  is vacuous or contains nondegenerate connected sets. Also, analogous to