BOOK REVIEWS

Topics in topology. By Solomon Lefschetz. (Annals of Mathematics Studies, no. 10.) Princeton University Press, 1942. 139 pp. $2 00.

This book is a companion volume to the author's recently published *Algebraic Topology*. In the latter volume the interest is focused, as the title implies, on the algebraic structure and properties of complexes and on their applications to the study of abstract spaces. Here, however, the interest lies in the topological structure of complexes and related spaces. Indeed, the major part of the book is devoted to a discussion of classes of spaces which are topological generalizations of complexes in much the same way that the abstract complex of *Algebraic Topology* is an algebraic generalization of a finite simplicial complex.

There have appeared in the literature definitions of two such classes which are successful from the point of view of giving rise to interesting and well developed bodies of theorems. The first of these, due to Borsuk, is the class of absolute neighborhood retracts, and the second, due to Lefschetz, is the class of LC* spaces. Both these classes have come to be of considerable topological importance, so the development given here of their essential properties, and of their mutual relations, will be welcomed by all topologists.

There are four chapters. The first is a discussion of various topologies which can be imposed on an infinite complex. A "natural" topology is obtained by taking as a basis for open sets the stars of vertices in all subdivisions. An interesting metric can also be defined and the two topologies are equivalent when the complex is locally finite, the case most used in later applications.

Chapter II is an extension of the discussion in *Algebraic Topology* of singular and continuous cycles. A few new notions along these lines are developed for later use.

In Chapter III a number of topics are dealt with, all dealing with existence theorems for various types of mappings of one space into another. First the Alexandroff theorem on the mapping of a space into the nerve of one of its coverings is generalized by giving necessary and sufficient conditions on the covering for the existence of such a mapping. This has interesting applications to Tychonoff and normal spaces, leading among other things to a characterization of the former.

Next are the proofs of two of the fundamental theorems from dimension theory: any n-dimensional separable metric space can be imbedded in a compact metric space of the same dimension and also in the Euclidean space of dimension $2n+1$.

The chapter concludes with a discussion of Borsuk's theory of retraction. An absolute retract (AR) is a separable metric space such that whenever it is a closed subset of a larger space then there is a continuous mapping (a "retraction") of the larger space onto the subset which leaves fixed each point of the subset. An absolute neighborhood retract (ANR) is defined in the same fashion but with the retraction required to exist only on a neighborhood of the subset. The most useful properties of such spaces stem from their characterizations in terms of extensions of mappings, e.g., a necessary and sufficient condition for a space A to be an AR is that whenever B is a closed subset of C, then every mapping of B into A can be extended to a mapping of C into A.

The fundamental theorem proved here, due originally to the author, is that to any separable metric space there may be added an infinite complex in such a fashion that the resulting union is an absolute retract.

The final chapter is devoted to the author's theory of homotopy local connectedness. Roughly speaking, a space is p-LC if small singular p-spheres in the space can be shrunk to points over small sets. The interesting case is where the space is LC^n, i.e., p-LC for all $p \leqq n$. Such spaces can be characterized in terms of mappings of complexes into the space, and this leads to the slightly stronger notion of an LC^* space. The fundamental theorem is that a compact absolute neighborhood retract is an LC^* space and conversely.

Next the homology theory of such spaces is investigated. It turns out that all types of cycles (e.g., singular, Vietoris, Čech) yield the same homology groups. Furthermore these groups can be no more complicated than those of a finite complex; in particular they have finite bases. Using this result, the fixed point formula is shown to hold for these spaces. As an application the Schauder fixed point theorem for functional spaces and some of its generalizations are derived.

The chapter concludes with a brief outline of the results obtained by generalizing the local connectedness definitions by replacing homotopy by homology, i.e., singular spheres by cycles and shrinking to a point by bounding.

As with all the volumes of this series, this one is well printed and easy to read. It is marred, however, by an unfortunate number of misprints, although few of them will cause the reader any particular difficulty. There are also a few slips. The proof of the last part of the last theorem of Chapter III is incorrect but is easy to remedy. In theorem 23.1 of Chapter IV, the inequality should read "$q \leqq p$." The proof here is not quite complete.

The terminology and notations used here are those of *Algebraic Topology*, so the index and bibliography are brief. There is, however, an additional bibliography of papers dealing with the subjects of retraction and homotopy local connectedness which is quite complete and useful.

EDWARD G. BEGLE

The calculus of extension. By Henry George Forder. (Including examples by Robert William Genese.) Cambridge University Press; New York, Macmillan, 1941. 15+490 pp. $6.75.

It will soon be the hundredth anniversary of the initial publication of Grassmann's monumental work *Die Lineale Ausdehnungslehre Ein Neuer Zweig der Mathematik.* Perhaps this fact will serve to arouse some belated interest in Grassmann's celebrated but otherwise neglected contributions to mathematics. If so, the excellent volume under review should indeed be a welcome addition to the comparatively meagre supply of general works treating this subject.

The history of the *Ausdehnungslehre* is extremely depressing. Grassmann had hoped that he would be able to secure the special opportunities for mathematical research that naturally accrue to a university post. In this he was bitterly disappointed. Such a position was not forthcoming. Cajori writes[1] "At the age of fifty-three this wonderful man, with heavy heart, gave up mathematics, and directed his energies to the study of Sanskrit, achieving in philology results which were better appreciated, and which vie in splendor with those in mathematics." Grassmann presented the *Ausdehnungslehre* in two books. The first, *Die Lineale Ausdehnungslehre*, appeared in 1844 (second edition, 1878) and the second, *Die Ausdehnungslehre, vollständig und in strenger Form bearbeitet*, in 1862. It has been said that only one person had read through the *Ausdehnungslehre* of 1844 eight years after its publication. Apparently, it was too new, general, and abstract to meet with popular approval. The *Ausdehnungslehre* of 1862 is easier to read and gives ample evidence of the wide range of the applications. It fared but little better than the former. In recent times Grassmann's work has been better appreciated. The historians of our subject[2] com-

[1] See Florian Cajori, *A history of mathematics*, New York, 1938.

[2] See, E. T. Bell, *The development of mathematics*, New York, 1940; Florian Cajori, *A history of mathematics*, New York, 1938; J. L. Coolidge, *A history of geometrical methods*, Oxford, 1940.