last theorem of Chapter III is incorrect but is easy to remedy. In theorem 23.1 of Chapter IV, the inequality should read "$q \leqq p$." The proof here is not quite complete.

The terminology and notations used here are those of *Algebraic Topology*, so the index and bibliography are brief. There is, however, an additional bibliography of papers dealing with the subjects of retraction and homotopy local connectedness which is quite complete and useful.

EDWARD G. BEGLE

The calculus of extension. By Henry George Forder. (Including examples by Robert William Genese.) Cambridge University Press; New York, Macmillan, 1941. 15+490 pp. $6.75.

It will soon be the hundredth anniversary of the initial publication of Grassmann's monumental work *Die Lineale Ausdehnungslehre Ein Neuer Zweig der Mathematik.* Perhaps this fact will serve to arouse some belated interest in Grassmann's celebrated but otherwise neglected contributions to mathematics. If so, the excellent volume under review should indeed be a welcome addition to the comparatively meagre supply of general works treating this subject.

The history of the *Ausdehnungslehre* is extremely depressing. Grassmann had hoped that he would be able to secure the special opportunities for mathematical research that naturally accrue to a university post. In this he was bitterly disappointed. Such a position was not forthcoming. Cajori writes[1] "At the age of fifty-three this wonderful man, with heavy heart, gave up mathematics, and directed his energies to the study of Sanskrit, achieving in philology results which were better appreciated, and which vie in splendor with those in mathematics." Grassmann presented the *Ausdehnungslehre* in two books. The first, *Die Lineale Ausdehnungslehre*, appeared in 1844 (second edition, 1878) and the second, *Die Ausdehnungslehre, vollständig und in strenger Form bearbeitet*, in 1862. It has been said that only one person had read through the *Ausdehnungslehre* of 1844 eight years after its publication. Apparently, it was too new, general, and abstract to meet with popular approval. The *Ausdehnungslehre* of 1862 is easier to read and gives ample evidence of the wide range of the applications. It fared but little better than the former. In recent times Grassmann's work has been better appreciated. The historians of our subject[2] com-

[1] See Florian Cajori, *A history of mathematics*, New York, 1938.

[2] See, E. T. Bell, *The development of mathematics*, New York, 1940; Florian Cajori, *A history of mathematics*, New York, 1938; J. L. Coolidge, *A history of geometrical methods*, Oxford, 1940.

mend it, but it has not yet been adopted in America to the extent that it is frequently presented in courses and employed in exposition and research. This is in strong contrast to vector analysis which was "discovered" shortly after its extraction, by Gibbs and others from the *Ausdehnungslehre* and Hamilton's *Quaternions*. Tensor analysis, a second subject which had roots in the *Ausdehnungslehre*, was rather generally discovered twenty years after its formulation by Ricci and Levi-Civita.

Since the *Ausdehnungslehre* is not yet widely known, one or two remarks on its nature are in order. First, as to its formal aspects, this subject may be described as a study of the linear form $\sum x_i e_i$, in which the x's are numbers and the e's (the extensives) are mathematical entities following a prescribed algebra. In the geometrical applications, the extensives are geometrical entities, such as points, vectors, rotors (a rotor is a vector confined to a line through it), circles, etc. The definitions of the sums, differences, and products of the extensives form the means whereby geometrical theorems are reduced to algebraic identities. While the machinery contains numerous minutiae (definitions, rules of operation, etc.) the separate details are quite simple and the resulting theory is extremely powerful. In a word, the general subject matter of *The calculus of extension* is of considerable interest both historically and intrinsically.

With regard to the book itself, the important questions are the following: Does the author present new material and if so is the new material developed with clarity? Is old material rendered more easily attainable for the uninitiated? For what purposes is the book suited? Forder's *The calculus of extension* not only presents a wealth of specific applications of the subject to geometry, that are either new or not readily obtainable elsewhere; but in addition furnishes an admirable and fresh exposition of the *Ausdehnungslehre*. It is very carefully written and the individual proofs are uniformly short and easy to follow. The theorem density is exceptionally high and consequently despite the superior exposition it is not an easy book to work straight through—perhaps the key chapters suffer from lack of recapitulation. Books that contain a wealth of material are never easy to read through, and it is my conviction that *The calculus of extension* provides the best exposition of the fundamental processes of the *Ausdehnungslehre* and the most inclusive treatment of the geometrical applications available at present. It is a book that should be in the library of anyone who is interested in either algebra, the algebraic treatment of geometry, or vector and tensor analysis. Certain chapters could be used effectively as the basis for a college course.

Turning to the negative matter of the accuracy of the work, there is little that needs to be said. It is not feasible, of course, to produce a book without a blemish and I have noticed a few misprints or slips of the pen and one or two places that would benefit by revision. These flaws, however, are remarkably scarce and are not of the type that will cause the reader difficulty, assuming that he is ready for the subject. Consequently, they have but slight bearing on the purpose of this review which is, I presume, to enable the reader to decide whether or not he wishes to examine the book. I should have preferred a more modern treatment of differentiation, but this is not a serious matter since the book is obviously addressed to readers who have the maturity to supply the rigorous formulation. The statement on page 221 that a combination not satisfying $a(b+c)d=abd+acd$ would not be called a product seems a little strong in the light of the dot product of vector analysis; and number 13 on the preceding page needs revision. These are small matters, however, and as I have said the book is unusually accurate.

For the uninitiated who wish to get a general notion of the subject as quickly as possible a careful study of Chapter I, together with a sampling of Chapters II, V, VII, VIII, and IX will suffice. Chapter I (71 pages) begins with a polished treatment of the fundamentals of the algebra of extensives and then proceeds with the interpretation of the primary extensives as points in a plane. The product of a point and a real number k is a point of weight k. The difference $b-a$ of two points of unit weight is the vector $\overrightarrow{ab}$. The sum of a point and a vector is the terminus of the vector when the vector's origin is placed at the point. The sum of two weighted points is a point at their center of gravity with weight equal to the total weights of the original points. The outer product of two points a and b is the rotor obtained by confining the vector $b-a$ to the line through a and b. The outer product of three points $[abc]$ is twice the area of the triangle (a, b, c). The difference of two rotors whose vectors are equal is called a bivector and in accordance with the equation $[(a-b)(p-q)]=[a(p-q)]-[b(p-q)]$ the outer product of two vectors is taken to be a bivector. The inner product of two vectors u and v is defined in terms of the supplement of v, i.e., the vector obtained by rotating v through 90 degrees in the positive direction. Specifically, the inner product of u and v is the outer product of u and the supplement of v. Since a bivector may be identified with its magnitude, namely, the area of an associated parallelogram, the inner product of two vectors is essentially the dot product of vector analysis. On the basis of these and certain other definitions the author establishes a wide variety of geometrical theorems including some in projective geometry.

Chapter II (pp. 71–110) is similar in character to Chapter I except that it is concerned with three-space. Chapter III (pp. 111–155), together with Chapters I, II, and VI, gives a reasonably complete account of projective geometry. Chapter V, Differentiation and Motion (pp. 177–196) is somewhat similar to the development by classical vector analysis. It treats among other things velocity, acceleration, curvature, central motion, and displacements in space. In Chapter VII, The General Theory (pp. 217–249), the central ideas are linear dependence and the various products of extensives. Determinants enter the picture by way of the fact that if $b_1, \cdots, b_r$ are each linearly related to $a_1, \cdots, a_r$, then $[b_1 \cdots b_r]=D[a_1 \cdots a_r]$, D being the determinant of the coefficients in the linear relationship. Chapter VIII (pp. 250–264) gives the application to linear equations and determinants. With regard to linear equations the modus operandi is to multiply each equation of the set $a_c^r x^c = b^r$ by an extensive e_r and add, thus $e_r a_c^r x^c = e_r b^r$. This reduces N equations in N unknowns to a single equation in extensives. The familiar theorems on the solvability of equations follow with marked simplicity. The treatment of determinants like the book as a whole is characterized by the wide scope of the material which is intelligibly presented in a relatively small space. Chapter IX (pp. 265–294) treats transformations, square matrices, and central quadrics—mostly well known material in slightly different dress. The remaining chapters are devoted to: the screw and linear complex, the general theory of inner products, circles (two chapters), the general theory of matrices, quadric spreads, and algebraic products.

HOMER V. CRAIG

Introduction to the theory of relativity. By Peter Gabriel Bergmann with a foreword by Albert Einstein. New York, Prentice-Hall, 1942. 287 pp. $4.50.

Many excellent books have been written on the Theory of Relativity. Although some of them appeared more than twenty years ago they are still read and studied, far from being regarded as antiquated. The books of Weyl, Pauli, Eddington are justly looked upon as classics in this subject. To say that Bergmann's book is in the same class as the books just mentioned means great, but deserved, praise.

Bergmann's book has its own character, and differs from the other books on Relativity Theory which have appeared up to now. First, it is more modern. The application of Relativity Theory to the Theory