we note the omission of cosmological applications (treated so well in Tolman's book) and of time-space measurements in connection with the experimental verifications of General Relativity Theory. On the other hand much space is devoted to the equations of motion as deduced from the field equations.

The third part (pp. 245–279) is of much more special character and deals with the unification of the gravitational and electromagnetic field. Here we find an exposition of Weyl's and Kaluza's theories and of their generalizations on which the author collaborated with Einstein. This part will rather interest specialists than students.

The book is well designed. The title of the book is too modest. It is not an introduction; it is an excellent book on the principles of Relativity Theory.

L. INFELD

Degree of approximation by polynomials in the complex domain. By W. E. Sewell. (Annals of Mathematics Studies, no. 9.) Princeton University Press, 1942. 9+236 pp. $3.00.

The main subject of this book is the relation of the analytic and continuity properties of a function inside and on the boundary of a region, and the degree of approximation by polynomials. It is related to the analogous questions for functions of a real variable and approximation by trigonometric sums, as presented in treatises of C. de la Vallée Poussin, S. Bernstein, D. Jackson and G. Szegö. The problems of the present work originate with the fundamental theorem that a function analytic in a Jordan region and continuous in the closed region can be uniformly approximated by polynomials. This was proved by J. Walsh in 1926. His treatise *Interpolation and approximation by rational functions in the complex domain* (Amer. Math. Soc. Colloquium Publications, vol. 20, 1935) is an important forerunner to the present book.

This book is divided into two parts. Part 1 (Chapters II, III, and IV) is devoted to a study of the relation between the degree of convergence of certain sequences of polynomials $p_n(z)$ to a function $f(z)$ on a point set E in the z-plane on the one hand and the continuity properties of $f(z)$ on the boundary C of E on the other hand (Prolem α). Recent contributions to Problem α are due primarily to J. Curtiss, Sewell, and Walsh.

In Part II (Chapters V–VIII) a more delicate problem is considered. Let E with boundary C be a closed limited set, whose complement K is connected and regular; thus a function $w=g(z)$ maps K conformally on the exterior of the unit circle $|w|=1$ in the w-plane so that the points at infinity in the two planes correspond to each other. Denote by C_ρ the image, under the conformal map, in the z-plane of the circle $|w|=\rho>1$. Let $f(z)$ be given analytic interior to some fixed C_ρ and let its continuity properties on C_ρ be prescribed. Problem β is the study of the relation between the degree of convergence of various sequences $p_n(z)$ to $f(z)$ on E on the one hand and the continuity properties of $f(z)$ on C_ρ on the other hand. Special cases of this problem have been studied by Bernstein, G. Faber, and de la Vallée Poussin. The present work discusses in detail more recent material primarily due to Sewell and Walsh.

Two types of results are considered in either Problem α or β. A direct theorem is one in which the properties of $f(z)$ are in the hypothesis and the degree of convergence of $p_n(z)$ to $f(z)$ is the conclusion. An indirect (generally far more difficult) theorem is in the converse direction, that is the degree of convergence of $p_n(z)$ to $f(z)$ is in the hypothesis and the properties of $f(z)$ form the conclusion. Such properties are: (a) A Lipschitz condition of order α is satisfied by $f(z)$ on a set E; that is $|f(z_1)-f(z_2)|\leqq L|z_1-z_2|^\alpha$, where α is a fixed number, $0<\alpha\leqq 1$, z_1 and z_2 are arbitrary points of E, and L is a constant independent of z_1 and z_2. (b) $f(z)$ belongs to the class $L(\kappa, \alpha)$ on a Jordan curve C (boundary of E), if $f(z)$ is analytic in the interior points of E, is continuous in the closed set E, and the κth derivative $f^{(\kappa)}(z)$ exists on C in the one-dimensional sense and satisfies a Lipschitz condition of order α on C. (c) $f(z)$ belongs to the class log $(\kappa,1)$ on C if $f(z)$ is analytic in the interior points of E, is continuous on the closed set E, and $f^{(\kappa)}(z)$ exists on C in the one-dimensional sense and satisfies the condition

$$\left|f^{(\kappa)}(z_1) - f^{(\kappa)}(z_2)\right| \leqq L\left|z_1 - z_2\right|\left|\log\left|z_1 - z_2\right|\right|,$$

where z_1, z_2 are arbitrary points of C, $|z_1-z_2|$ sufficiently small ($<1/2$ say); L is a constant.

Occasionally an integrated Lipschitz condition is also considered. The degree of convergence or approximation is measured:

1. In the sense of Tchebycheff, that is by

$$\underset{z \text{ in } E}{\text{l. u. b.}}\left|f(z) - p_n(z)\right|.$$

2. By a line integral:

$$\int_C \Delta(z)\left|f(z) - p_n(z)\right|^q |dz|,$$

where $\Delta(z)$ is a non-negative weight function, q is a fixed positive number.

As the author points out, many of the results are not final; at the end of each chapter possible extensions are indicated. The bibliography is restricted to a selected list. A few scattered misprints and errors can be easily corrected. For example on p. 13 the inequality $|\partial P/\partial\theta| \leqq A/(1-r)$ is false, but the proof of the theorem can be completed. On p. 126, where a beautiful theorem of Hardy and Littlewood is quoted, reference should be made to a paper of E. S. Quade, Duke Math. J., vol. 3 (1937), pp. 529–543.

On the whole the author deserves credit for his valuable contribution which will serve to stimulate further research on this important subject.

OTTO SZÁSZ

Table of arc-tan x. Federal Works Agency, Works Projects Administration for the City of New York, 1942. 25+173 pages. $2.00.

"This table of arc-tan x is believed to be the most comprehensive so far published, in respect both to the number of decimal places given and to the smallness of tabular interval. It forms the first contribution to what it is hoped will be a complete set of tables of the inverse trigonometric and hyperbolic functions." The tables are computed to twelve decimals. The interval between successive arguments is 0.001 for $0 \leqq x \leqq 7$, it is 0.01 for $7 \leqq x \leqq 50$, 0.1 for $50 \leqq x \leqq 300$, 1 for $300 \leqq x \leqq 2{,}000$ and 10 for $2{,}000 \leqq x \leqq 10{,}000$. Linear interpolation provides for the whole range an accuracy of six decimal places. For interpolation to twelve places the second central differences are tabulated. The introduction gives the necessary formulae; correspondingly tables to six places for $p(1-p)$ and $1/6\ p(1-p^2)$ are given for the range $0 \leqq p \leqq 0.5$ and $0 \leqq p \leqq 0.999$. Tables for the conversion of radians into degrees and conversely are added. The introduction gives the necessary information for the use and scope of the tables, and ends in a report about the method of checking by a sixth difference test. The bibliography contains also a new list of errors in the tables of Hayashi.

O. NEUGEBAUER