sits next to his wife. By elementary methods, the author obtains the answer $4n\ n!\sum_{k=1}^{n}(-1)^k C_k^{2n-k}(n-k)!/(2n-k)$. (Received June 1, 1943.)

Analysis

182. E. F. Beckenbach and R. H. Bing: *Conformal minimal varieties.*

A set of m real functions of n real variables, $m \geqq n \geqq 2$, defined in a domain D, has been called (for $m=n$ by N. Cioranesco, Bull. Sci. Math. vol. 56 (1932) pp. 55–64) a set of conjugate harmonic functions provided the functions are harmonic and together satisfy the usual conditions for conformality. Such a set of m functions gives a conformal map on Euclidean n-space of a minimal variety V_n immersed in Euclidean m-space. But according to Haantjes, for $n \geqq 4$ the class of conformally flat spaces is quite narrow. It is now shown directly that for $n \geqq 3$ the only sets of conjugate harmonic functions necessarily are constants or linear functions, so that either the V_n is a point or it can be obtained from D by rigid motions, transformations of similitude, and reflections in hyperplanes. (Received May 7, 1943.)

183. R. P. Boas: *Almost periodic functions of exponential type.*

As an extension of a well known result on entire functions of exponential type which are periodic on the real axis, it is shown that an entire function of exponential type which is almost periodic on the real axis has its Fourier exponents bounded. Almost periodicity as general as Besicovitch (order 1) is admissible; in fact, the mere existence of the mean value of $|f(x)|$ implies the existence and vanishing of the mean value of $f(x)e^{i\lambda x}$ when $|\lambda|$ exceeds the type of the function $f(z)$. For the proof, one rotates the line of integration in $\int_0^T e^{i\lambda x}f(x)dx$ through an angle of $\pi/2$, and applies theorems which connect the growth of $|f(x+iy)|$ with that of $|f(x)|$. The estimate $f(x)=o(|x|)$ as $|x|\to\infty$ is a consequence of the existence of the mean value of $|f(x)|$. (Received May 11, 1943.)

184. Vincent Cowling and Walter Leighton: *On convergence regions for continued fractions.*

Let k be any real number greater than 1 and ϵ any number such that $0<\epsilon<k$. If the complex numbers a_{2n} and $a_{2n+1}=r_n e^{i\theta_n}$ satisfy the conditions (1) $|a_{2n}| \leqq k-\epsilon$, $a_{2n}\neq -1$, (2) $r_n \geqq 2[k-\cos\theta_n]$, $0\leqq\theta_n\leqq 2\pi$, then the continued fraction $1+K[a_n/1]$ converges. It follows as a corollary that the continued fraction will converge if conditions (1) and $|a_{2n+1}|\geqq 2(k+1)$ hold. (Received April 28, 1943.)

185. Nelson Dunford: *Spectral theory.* I. *Convergence to projections.*

By a systematic use of an operational calculus suggested by the formula $f(T) =(2\pi i)^{-1}\int_C f(\lambda)(\lambda I-T)^{-1}d\lambda$ conditions are derived which are necessary and sufficient for the convergence of a given sequence $P_n(T)$ of polynomials in a linear operator T (on a complex Banach space X) to a projection on a manifold of the form $\mathrm{M}[P] =E_x[x\in X, P(T)x=0]$. (Received April 24, 1943.)

186. M. R. Hestenes: *On the condition of Weierstrass in the calculus of variations.*

The present paper is devoted to the study of properties of the Weierstrass E-function $E(y, p, \lambda, q)$ associated with the problem of Bolza. It is shown that an extremal C_0 is nonsingular and satisfies the strengthened condition of Weierstrass if and only if there is a constant $b>0$ such that the inequality $E(y, p, \lambda, q) \geqq bE_L(p, q)$ holds for all differentiably admissible elements (y, p), (y, q) with (y, p, λ) in a neighborhood of the values (y, y', λ) on C_0. Here E_L is the E-function for the length integral. Further equivalence relations are given. Moreover, it is shown that if these conditions are satisfied then the integrand $f(y, y')$ can be replaced by a function of form $f+\theta\,(y, y')\,\phi^\beta\phi^\beta_{,}$ such that the above inequality holds without the restriction that $\phi^\beta(y, p) = \phi^\beta(y, q) = 0$. (Received April 23, 1943.)

187. Herman Kober: *On the approximation to integrable functions by integral functions.*

Let $f(t) \in L_p(-\infty, \infty)$, $0<p\leqq\infty$, and suppose that $f(t)$ may be approximated (in mean if $0<p<\infty$, uniformly if $p=\infty$) by integral functions of fixed order not greater than ρ, where ρ is fixed. If $\rho<1$, $f(t)$ is a constant (zero if $p=\infty$). Here 1 is the precise limit and it is always possible to find a sequence of integral functions of order one and normal type approximating $f(t)$ (if $p=\infty$, if and only if $f(t)$ is uniformly continuous in $(-\infty, \infty)$). If the types of the approximating functions have a finite inferior limit α, then $f(t)$ is an integral function of order one and type not greater than α, or a constant. The best approximation by integral functions of order one and given type α can be determined and is given by the Dirichlet singular integral $D_\alpha[f]$ when $p=2$. Approximation by functions of order ρ implies approximation by functions of larger order. If $L_p(-\infty, \infty)$ be replaced by $L_p(0, \infty)$, the order one is to be replaced by the order 1/2 throughout. Extensions to other approximation problems are possible. (Received April 17, 1943.)

188. E. R. Lorch: *The theory of analytic functions in normed abelian vector rings.*

The development of an analytic function theory for these rings $\mathfrak{R}$ has been announced previously (Bull. Amer. Math. Soc., abstract 46-11-469). Certain applications are here considered. The function exp (z) is simply periodic if and only if $\mathfrak{R}$ is irreducible; alternatively, if and only if the spectrum of each $a \in \mathfrak{R}$ is a continuum or a point. The periods of exp (z) are $2\pi i\sum_1^n \pm j_k$ where the j_k are any idempotents. The function log z exists for all z in the principal component of the topological group of non-singular elements in $\mathfrak{R}$. An $a\in\mathfrak{R}$ may be embedded in a continuous group, $a^s\cdot a^t = a^{s+t}$, if and only if log a exists; then $a^s = \exp(s \log a)$ where the range of s may be taken to be $\mathfrak{R}$. All points in the frontier of the spectrum of $a\in\mathfrak{R}$ are permanent singularities (cannot be removed by ring extensions). If $\mathfrak{R}$ is irreducible, a simple closed rectifiable curve C in $\mathfrak{R}$ generates a closed set of singular elements, an open set of exterior elements, and an open set of interior elements. The open sets need not be connected. In case C has a tangent at one point, the set of interior points is not empty. (Received April 19, 1943.)

189. George Piranian: *On the order of analytic functions in the sense of Hadamard.*

If in the region $|z|>R$ the function $f(z)=\sum a_n/z^n$ is holomorphic except for poles of aggregate multiplicity P, if D_{nP} is Hadamard's determinant of order $P+1$ and rank n for $f(z)$, and if the order of the function $F_P(z)=\sum D_{nP}/z^n$ on its circle of convergence is ω, then the order of $f(z)$ on the circle $|z|=R$ is at least ω. Moreover, if the order of $f(z)$ on $|z|=R$ is $\omega'>\omega$, there either exist at least two points of order ω' on this circle or the singularity of order ω' is non-Fuchsian. By means of an extension of Mandelbrojt's method for finding the singularities of an analytic function on its circle of convergence, the theorem gives a formula for the order of every pole lying outside of the convex hull of non-polar singularities of $f(z)$, and for the order of every Fuchsian singularity on the boundary V of the convex hull, provided the singularity is not an interior point of a straight-line segment of V. (Received April 12, 1943.)

190. Harry Pollard: *A new criterion for completely monotonic functions.*

The Bernstein criterion for completely monotonic functions states that if (i) $f(0+)$ exists and (ii) $(-1)^k\Delta_\delta^k f(x)\geqq 0$ for $k\geqq 0$, $\delta>0$, $x>0$, then $f(x)$ is completely monotonic in $0\leqq x<\infty$. It is established in this paper that (ii) can be weakened to $(-1)^k\Delta_{\delta_k}^k f(x)\geqq 0$ for a suitable sequence $\{\delta_k\}$. (Received April 10, 1943.)

191. W. J. Thron: *A general theorem on convergence regions for continued fractions $b_0+K(1/b_n)$.*

Let the regions B_0 and B_1 be defined by: $r\cdot e^{i\theta}\in B_0$ if $r>(1+\epsilon)\cdot f(\theta)$, $r\cdot e^{i\theta}\in B_1$ if $r>(1+\epsilon)g(\theta)$, where ϵ is an arbitrary small positive number and the functions $f(\theta)$ and $g(\theta)$ are positive in the interval $[0, 2\pi]$. If it is required that the complements of the regions B_0 and B_1 be both convex and if $f(\theta)\cdot g(\pi-\theta)\geqq 4$, then the continued fraction $b_0+K(1/b_n)$ converges if $b_{2n}\in B_0$ and $b_{2n+1}\in B_1$, that is B_0 and B_1 are twin convergence regions for the continued fraction. The condition $f(\theta)g(\pi-\theta)\geqq 4$ is a necessary condition for two regions to be twin convergence regions. (Received April 19, 1943.)

192. W. J. Thron: *Convergence regions for the general continued fraction.*

It is shown that the continued fraction $K(a_n/b_n)$ converges if all $a_n=r\cdot e^{i\theta}$ lie in a bounded part of the parabola $r\leqq a^2/2(1-\cos(\theta-2\gamma))$, and if all b_n lie in the half-plane $R(b_n e^{i\gamma})\geqq a+\epsilon$. Here $a>0$ and ϵ is an arbitrary small positive number. (Received April 24, 1943.)

193. Hassler Whitney: *On the extension of differentiable functions.*

Let A be a bounded closed set in Euclidean space E. Suppose that for some number ω any two points of A are joined by an arc in A of length not more than ω times their distance apart. Then any function of class C^m in A which, with derivatives through the mth order, is sufficiently small in A, may be extended throughout E so as to be small, with its derivatives. (Received May 11, 1943.)

Geometry

194. T. C. Doyle: *Tensor theory of invariants for the projective differential geometry of a curved surface.*

This paper completes the explicit determination of differential invariants of all orders for a curved two dimensional surface begun by E. J. Wilczynski, *Projective differential geometry of curved surfaces (fourth memoir)*, Trans. Amer. Math. Soc. vol. 10 (1909) pp. 176–200. The Lie theory of groups serves to determine the number of exist-